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Contents

MODELING THE EUROPEAN CENTRAL BANK OFFICIAL RATE: A STOCHASTIC APPROACH

Maria Francesca CARFORA
PhD ,Researcher at the Institute for Mathematics Applications (IAC), Naples
Italian National Research Council (CNR), Italy

Luisa CUTILLO
PhD, Assistant Professor, Faculty of Economics,
University of Naples Parthenope, Italy

Albina ORLANDO
PhD ,Researcher at the Institute for Mathematics Applications (IAC), Naples
Italian National Research Council (CNR), Italy

E-mail: a.orlando@iac.cnr.it

Abstract
Following its main task of price stability in the euro area, the European Central Bank (ECB) increases or decreases interest rates in order to cool inflation or respectively to support economic growth. Monetary policy shows delayed effects on inflation and thus the ECB modifies interest rates on the basis of forecasts about the state of economy over the coming quarters. Aim of our contribution is to provide a stochastic model for the ECB official rate taking into account the expectations on the future state of economy. We propose a non homogeneous Poisson process to describe the intervention times of the ECB. In particular the jump process parameters depend on the evolution of the economic cycle as modeled by a MS-AR model. We show an application on suitably aggregated European data.

Keywords: ECB rates, Markov-switching, business cycle, non-homogeneous Poisson process

1. Introduction

The European Central Bank (ECB) is the central bank for Europe single currency, the euro. The ECB main task is to maintain the euro purchasing power and thus the price stability in the euro area. The euro area comprises the 17 European Union countries that have introduced the euro since 1999.

The ECB monetary policy operates by steering short-term interest rates, thereby influencing economic developments for the euro area over the medium term. Monetary policy decisions are taken by the ECB's Governing Council that meets every month to analyze and assess economic monetary developments and to decide the appropriate level of key interest rates, based on the ECB strategy. The Governing Council of the ECB sets the key interest rates for the euro area: the interest rate on the main refinancing operations (MRO), which provide the bulk of liquidity to the banking system; the rate on the deposit facility, which banks may use to make overnight deposits with the Eurosystem; the rate on the marginal lending facility, which offers overnight credit to banks from the Eurosystem.

Short-term interest rates set by central banks have a large impact on the pricing of financial assets and on the broader economy, which in turn affect prices of shares and corporate debt securities. Many authors focus their research on modeling the evolution of Central Banks official rates. One of the most popular approaches is the "Taylor rule" [19]. It describes the behavior of a central bank by means of a policy reaction function. The interest rate is the policy instrument, depending on both inflation and current output gap. The Taylor rule is based on the assumption that interest rate follows a linear, continuous process and such an assumption is not in line with its discrete changes. Indeed central banks announce interest rates changes during their regular meetings and define somewhat an upper limit on possible changes during a year; usually adjustment occur in a series of small steps (25 basis points). As a consequence the interventions can be well represented by applying discrete choice models. Among the others, for the ECB rates [6], [16] and [10], whereas [7] looks at Federal Reserve's reaction function.

In line with the discrete approach, in our work we refer to [3] where they present the techniques they employ to simulate the future behavior of interest rates. Observing that any rate (regardless of its maturity) has a strong correlation with the ECB rate, they develop an original approach to the generation of future term structure scenarios considering the fluctuations of each rate with respect to the ECB official rate. To this aim they assume that the interventions of the ECB can be represented as a stochastic jump process. Some features of this process are readily apparent by looking at the evolution of the ECB official rate since January 1999: there have been about three interventions per year until today, in each intervention the rate jumps by either 25 or 50 basis points. However they do not include in the model any variable linking the ECB official rate to the evolution of macroeconomic indicators.

While ECB reacts to many factors and staff assess literally hundreds of time series of data in preparing the background material for policy meetings, empirically it looks like only a few data series are needed to capture a central bank's policy decisions [10]. In particular the ECB interventions are due to: real economic activity expected growth; money growth which is an indicator of inflation pressure; exchange rate appreciation or depreciation which influence inflation directly through import prices and indirectly by affecting competitiveness of the euro area and the demand for euro area goods; finally, to current inflation.

Following its main task, ECB increases interest rates when the economy is in an expansion phase to cool inflation and, vice versa, decreases interest rates when the economy is in a recession phase to support economic growth. Monetary policy shows its effects on inflation some time later (one year and over). On the other hand the effects on output are immediate and temporary, being the monetary policy neutral in the long run. As a consequence, monetary policy must anticipate economic cycle to be effective. That is why

ECB modifies interest rates on the basis of forecasts about the state of economy over the coming quarters.

In the present paper our aim is to improve the simple jump process proposed by [3] taking into account the macroeconomic indicators that impact on ECB interventions on interest rates. We are aware that ECB interventions respond to several macroeconomic indicators (real economic activity expected growth, money growth, exchange rate appreciation or depreciation, current inflation). Nevertheless as a first step of our research we focus on the link between the ECB rates and the expectations on the growth of real economic activity. We look at this macroeconomic variable basing on the evidence that the editorials by the ECB's Governing Council contain frequent statements about development in real economic activity presumably because it has an impact on the rate of inflation with a lag [10].

We propose a stochastic model for the ECB interventions able to link the reference rates to the predicted states of the economy, that is to the forecast probability of expansion of real economic activity. We choose to describe the economic cycle via a Markov switching Auto Regressive model (MS-AR model) proposed first in Hamilton's seminal article [12] and we consider two possible states of the economy: recession and expansion. The MS autoregressive model allows us to estimate the filtered probability of being in each of the states. To link the rates dynamics to this probabilities we propose an empirical classification of economic cycle phases basing on some features of ECB's behavior in steering interest rates such as the asymmetry in the number and timing of ECB interventions between the two economic regimes.

Then we model the rates dynamics through a jump process whose parameters depend on the predicted states of the economy estimated by the proposed classification. Indeed, a non homogeneous Poisson process is often appropriate for the modeling of a series of events (in our case the ECB interventions) that occur over time in a non-stationary fashion, since its intensity function may vary with time. We assume that this intensity varies according to the real economic cycle phases, being constant as long as the economy remains in the same state. The proposed methodology is empirically validated on the time series of ECB interventions.

The paper is organized as follows. Section 2 introduces the adopted methodology, by describing in details the Markov Switching model of the business cycle, the non-homogeneous Poisson model for the ECB rates and the empirical classification rule we adopted. Section 3 briefly presents the data, while Section 4 is devoted to the description and discussion of the results. Section 5 concludes.

2. The Model

2.1. Modeling the Business Cycle

As first suggested by [12], we model the business cycle as a Markov switching process. Hamilton's work gave rise to a considerable number of papers that also use Markov switching models to capture regime changes in a diverse set of macroeconomic and financial time series. Indeed, many economic time series occasionally exhibit dramatic breaks in their behavior, associated with events such as financial crises or abrupt changes in government policy. In particular, many authors have successfully used Hamilton's model to characterize and explain business-cycle fluctuations. These studies were primarily motivated by a belief

that recessions and expansions are distinct phases or regimes that make economic fluctuations a fundamentally asymmetric phenomenon. Because such models, yet still very tractable, allow for nonlinear dynamics and sudden changes, so matching many stylized facts about the business cycle, this approach has become an important alternative to linear, autoregressive structures. The following brief description helps us to establish the notation.
The most general form of a Markov-switching autoregressive (MS-AR) process of order p is given by [13, cap. 22]

$$y_t = \mu(s_t) + A_1(s_t)y_{t-1} + \ldots + A_p(s_t)y_{t-p} + \varepsilon_{t.} \qquad (1)$$

Here ε_t is a Gaussian error term conditioned on s_t:

$$\varepsilon_t | s_t \sim NID\big(0, \sigma(s_t)\big);$$

while the parameter vector shift function $\mu(s_t)$ and the autoregressive coefficients $A_1(s_t),\ldots A_p(s_t)$ describe the dependence of the time series y on the regime variable $s_t \in \{1,\ldots M\}$, which represents the probability of being in a particular state of the world. We assume that s_t follows an ergodic Markov chain, so that the transition probability matrix will be

$$p_{j,k} = Prob(s_t = j \,| s_{t-1} = k), \qquad j,k \in \{1, \ldots M\} \qquad (2)$$

with $\Sigma_k p_{j,k} = 1$ for $j \in \{1,\ldots M\}$. If the process is governed by regime $s_t = j$ at date t, then for $j = 1,\ldots M$ the conditional density of y_t is assumed to be given by

$$f(y_t | s_t = j, \Upsilon_{t-1}; \theta),$$

where $\theta = (\mu, A_1,\ldots A_p, \sigma)$ is the vector of parameters characterizing the conditional density and Υ_t is a vector containing all observations obtained through date t.

To estimate both the parameters vector θ and the transition probabilities $p_{j,k}$, Hamilton proposed a filtering algorithm to iterate through the observations while making and updating inferences about the probability of being in a given state.

The filtered probability can be understood as an optimal inference on the state variable at time t using only the information up to time t:

$$Prob(s_t = j \,|\, \Upsilon_t), \qquad j \in \{1, \ldots M\}.$$

From this probability we obtain the forecast probability

$$\mathcal{P}_t = Prob(s_{t+1} = j \,|\, \Upsilon_t) = \sum_{i=1}^{M} p_{i,j} Prob\,(s_t = i \,|\, \Upsilon_t), \qquad j \in \{1, \ldots M\}. \qquad (3)$$

In this study, we consider a two-state model $(M = 2)$, that is, we use observations of a single variable y_t to estimate and forecast the probability of being in one of the two given states, that we identify as Expansion and Recession.

2.2. Classification of business cycle phases

The MS autoregressive model described in the previous Section allows us to estimate the filtered probability of being in each of the states.

To link the rates dynamics to this probability we must take into account some additional features such as the asymmetry in the number and timing of ECB interventions between the two economic regimes.

Then we build an empirical classification rule basing on the following three assumptions:

- ECB "upward" interventions are limited to stable and certain expansion phases, as identified by a forecast probability of expansion above a fixed threshold α_E and very slightly oscillating;
- on the contrary, "downward" interventions are often realized not only when a certain recession is expected (forecast probability of expansion below a second fixed threshold α_R), but also in uncertain (oscillating) situations, while leaving an expansion phase;
- moreover, when leaving a recession phase, the ECB tends to wait. In this case, counterbalancing interventions are delayed until a certain expansion state is reached.

Thus, the empirical rule we propose relies on the evaluation of the forecast probability of expansion $P_t(E)$ as defined by (3) identifying the ECB intentions at time t as

- counterbalancing expansion when $P_t(E) > \alpha_E$;
- counterbalancing recession when $P_t(E) < \alpha_R$ or $\alpha_R < P_t(E) < \alpha_E$ while leaving an expansion period;
- waiting when $\alpha_R < P_t(E) < \alpha_E$ while leaving a recession period.

Application of this rule leads us to partitioning the considered time interval in non-overlapping subintervals, each of them classified as an expansion, recession or uncertainty period. Clearly, isolated single points are reclassified to agree with their neighbors classification. Results of the application of our classification rule to the probability estimated by a two states MS model using business cycle indicators data will be shown in Section 4.

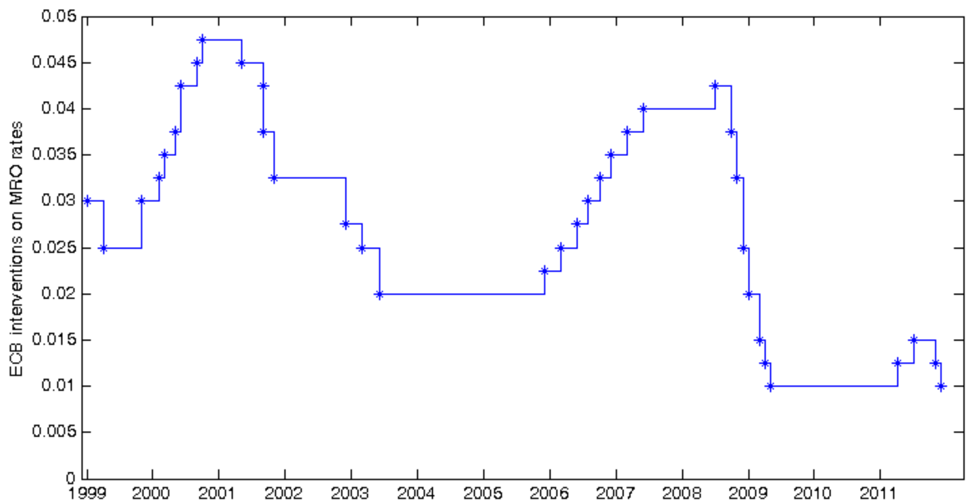

Figure 1. Historical value of the MRO rates as fixed by ECB; stars represent ECB interventions

2.3. Modeling the ECB official rate

By considering the empirical evidence of the historical observed rate (as shown in Figure 1) we note that the ECB official rate time series starts in January 1999, thus it is much shorter than any available business cycle indicators data series, and there have been about three interventions per year until today; moreover, in each intervention the rate jumps by either 25 or 50 basis points. These starting observations suggest that the ECB rate dynamics should be defined, following the approach proposed by [3], by a jump model:

$$ECB_t = ECB_0 + \sum_{h=1}^{N_t} a_h b_h \quad (4)$$

where N_t represents the total number of interventions up to time t, while $a_h \in \{0, 0.25, 0.50\}$ and $b_h \in \{-1, +1\}$ are the width and the direction of the intervention h, respectively. In particular, we assume that the number of ECB interventions is a counting process that can be modeled as a non homogeneous Poisson process: its intensity function $\lambda(t)$ may vary with time and the cumulative intensity function $\Lambda(t) = \int_0^t \lambda(\tau) d\tau$ gives the expected number of events by time t.

Moreover, we aim at improving the simple jump model in [3] by linking the intensity function λ to the predicted state of the economy. To this purpose, after having partitioned the entire time interval in m subintervals $I_1, \ldots, I_m$, each of them classified as described in the previous Subsection, we allow $\lambda(t)$ to be piecewise constant on each subinterval. Thus, its Maximum Likelihood estimator is the average number of events that occurred on the interval I_j, normalized to the length of that interval

$$\hat{\lambda}(t) = \frac{n_j}{|I_j|} \qquad t \in I_j. \quad (5)$$

As a consequence, if no events are observed on an interval, then the intensity function estimate is zero on that interval.

In a similar way, to assign a value for the parameter b_h in (4), we look at the time t_h of the intervention and set $b_h = +1$ in expansion subintervals, while $b_h = -1$ in recession subintervals.

Finally, to reduce the model parameters we fix a_h=0.25; this choice is not restrictive, provided that any ECB intervention modifying rates of 50 basis points (0.50) is accordingly counted as a multiple intervention.

3. The data

Business cycles are usually measured by considering the growth rate of real gross domestic product (GDP). However GDP data are published with a lag of several quarters and are typically revised several times, occasionally by large amounts.

The Directorate General for Economic and Financial Affairs (DG ECFIN) conducts regular harmonised surveys for different sectors of the economies in the European Union (EU) and in the applicant countries. They are addressed to representatives of the industry (manufacturing), the services, retail trade and construction sectors, as well as to consumers. These surveys allow comparisons among different countries' business cycles and have become an indispensable tool for monitoring the evolution of the EU and the euro area economies, as well as monitoring developments in the applicant countries. Survey measures are typically available with very short lags and never updated. Moreover it is well known that editorials in the ECB's *Monthly Bulletin* frequently comment on business and consumer confidence and survey measures of expected output growth.

For these reasons in the following we model the business cycle by means of a survey measure as a proxy of the real GDP. Among the several proposed survey indicators (see [18] for a review and, more recently, [9, 4, 5]), we choose the Economic Sentiment Indicator(ESI) that pertains to the euro area and is based on a large survey of firms and consumers. It has a number of features that make it suitable for our analysis: it is strongly correlated with data on the real state of economy, it is available monthly instead of quarterly as is the case for real GDP, it is available much faster than the GDP data and move in advance of the output gap picking up business cycle turning points more rapidly than real GDP does. Furthermore, according to several authors [11, 5], this indicator is much more significant in the regressions than output gaps that are traditionally used to capture the state of the economy.

ESI is a composite indicator made up of five sectorial confidence indicators with different weights: Industrial confidence indicator, Services confidence indicator, Consumer confidence indicator, Construction confidence indicator, Retail trade confidence indicator. Confidence indicators are arithmetic means of seasonally adjusted balances of answers to a selection of questions closely related to the reference variable they are supposed to track. Surveys are defined within the Joint Harmonised EU Programme of Business and Consumer Surveys. The ESI is calculated as an index with mean value 100 and standard deviation of 10 over a fixed standardized sample period. Long time series of the ESI and confidence indicators are available at the Survey database in the DG ECFIN website http://ec.europa.eu/economy_finance/db_indicators/surveys/index_en.htm.

Figure 2, where GDP growth rate and ESI rate are simultaneously plotted at monthly frequency, confirms not only the strong agreement between the two data series, and so the ability of ESI in capturing the state of the business cycle, but also the fact that ESI moves in advance, picking up business cycle turning points more rapidly than GDP growth rate.

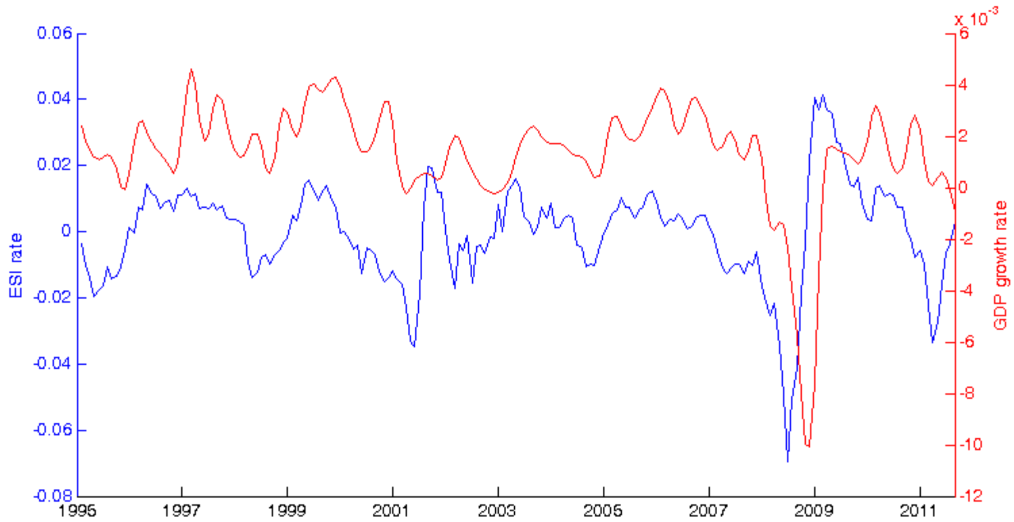

Figure 2. ESI rate (blue) and GDP growth rate (red) data starting from January 1995. Monthly GDP data are obtained by linear interpolation of quarterly data

4. Results

The data used here are a third order moving average of the monthly ESI rates from 1985:1 to 2012:2 as drawn from the EUROSTAT database. We estimated the transition matrix p_{ij} and the parameters σ, A_{11}, … A_{1p}, A_{21}, … A_{2p} of the MS-AR model (1) in the case of two regimes with order *p* ranging from 1 to 3 by means of a Matlab package [17]; we outline that for such data $\mu_1 = \mu_2 = 0$. The estimation results are reported in Table 1, where Regime 1 corresponds to growth, while Regime 2 represents recessions. It is evident from these results that all of the models for the considered orders gave us exactly the same transition matrix and just slightly different values of the parameters. Even though they are essentially equivalent in estimating the forecast probability, nevertheless we choose the model with the highest Likelihood value (*p* = 3).

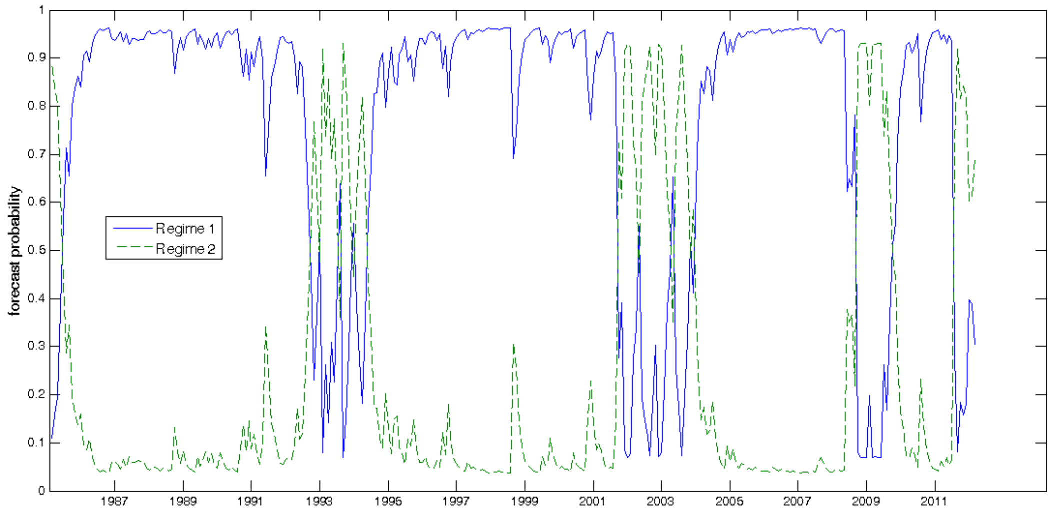

Figure 3. Forecasted probability of being in Regime 1 and 2 as estimated by the MS-AR model with three lags in the period 1985:3 to 2012:3

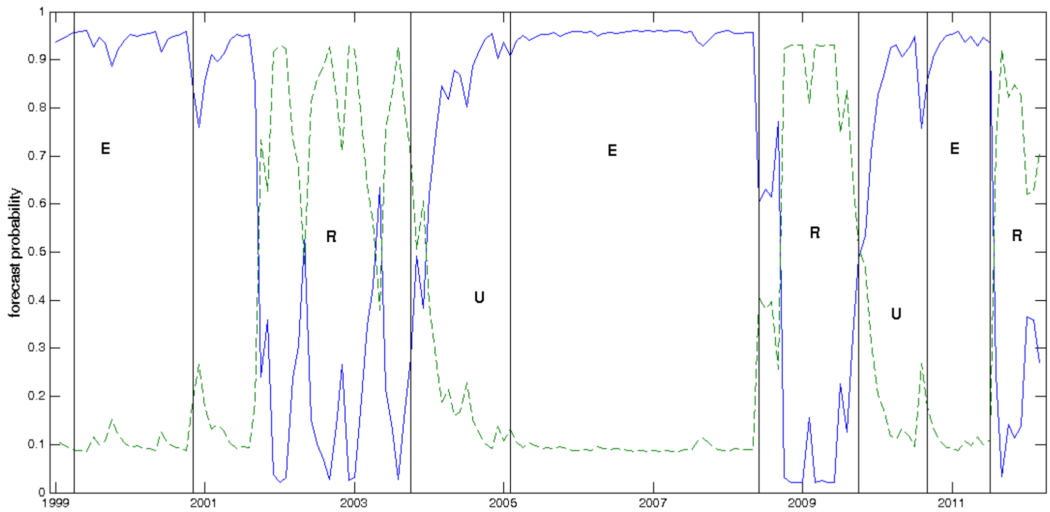

Figure 4. Forecasted probability of being in Regime 1 and 2 as estimated by the MS-AR model with three lags in the period 1999:1 to 2012:3; vertical lines mark the Expansion (E), Recession (R) and Uncertainty (U) subintervals as identified by the proposed classification rule

The time paths of the forecasted probability (3) are depicted in Figure 3 for the entire time period from 1985 to 2012; the following Figure 4 presents the same probability from January 1999, when ECB started its activity in fixing rates, along with the results of the classification rule proposed in Section 2. Then, Figure 4 helps us to clarify the rationale for our classification rule: stable periods of expansion (marked with an E label) can be easily recognized in the plot; moreover, we labeled with an R not only the intervals where the forecast probability is below the recession threshold α_R, but also the intervals where this probability is below the certain expansion threshold α_E and moving towards a certain recession; finally, we denoted as uncertain (U label) the intervals following a recession, when a stable expansion phase is not yet reached. As long as the choice of the parameters α_R, α_E is concerned, basing on these considerations we adopted for the former the ``natural" value 0.5, while for the latter we choose the expected value of the probability considering only the values above the recession threshold α_R, obtaining $\alpha_E = 0.9$.

To check the robustness of this choice we repeated the classification while allowing the parameter α_E to vary in the range 0.85 - 0.95. For each repetition, to evaluate the success of our classification we considered the series of ECB interventions and counted the matches between increasing (resp. decreasing) rates interventions and the corresponding classification of that month as belonging to an expansion (resp. recession) interval. Clearly, the lowest value of the parameter ($\alpha_E = 0.85$) leads to a minor sensitivity to the detection of uncertainty periods, so that the classification error increases in these intervals. On the other hand, the highest value ($\alpha_E = 0.95$) excessively penalizes the expansion periods, leading to a minor expected number of interventions. Table 3 summarizes the classification results for any chosen α_E, while Figure 5 shows the classification corresponding to $\alpha_E = 0.9$, $\alpha_R = 0.5$. In the same Figure we also report the real ECB rates to visually confirm the good classification results. Indeed, for this choice of the parameters there is only one real ECB intervention (July 2008) which is misclassified since it increases the rates while classified as belonging to a recession interval.

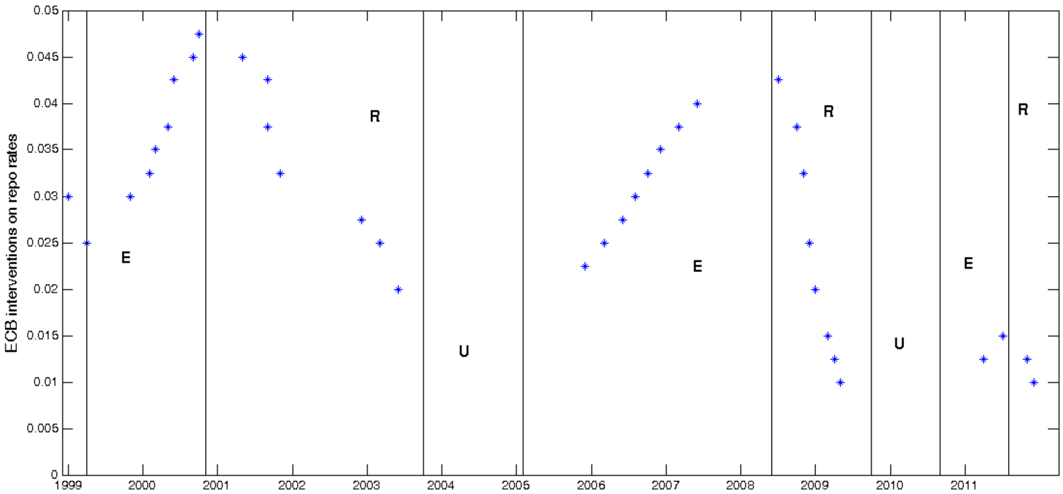

Figure 5. Classification results for the considered time period (1999 to 2012) compared with ECB decisions. Stars represent time of the real ECB interventions and corresponding value of the rates. Vertical lines mark the Expansion (E), Recession (R) and Uncertainty (U) subintervals as identified by the proposed classification rule.

Classification results are detailed in Table 2, which also reports the rates variation in each time interval and the estimated intensity function for the Poisson process modeling the ECB rates. Indeed, in each of the intervals we estimated the intensity function λ of the Poisson process as given by Eq. 5.

Finally we validate our model for the ECB rates by simulating their dynamics with the estimated intensity functions over the entire period 1999--2011. Specifically, for each time point *t* we estimate the intensity function according to Eq. 5 in the subinterval ending at *t-1* and generate the corresponding value of the rate for time *t*. In Figure 6 we show the average rates dynamics over 5000 simulations along with an estimate of the confidence interval corresponding to the 10th and 90th percentiles. Red stars represent the real ECB interventions. Such a short period simulation confirms the good agreement between the average trend of the simulated Poisson process and the real ECB rates dynamics.

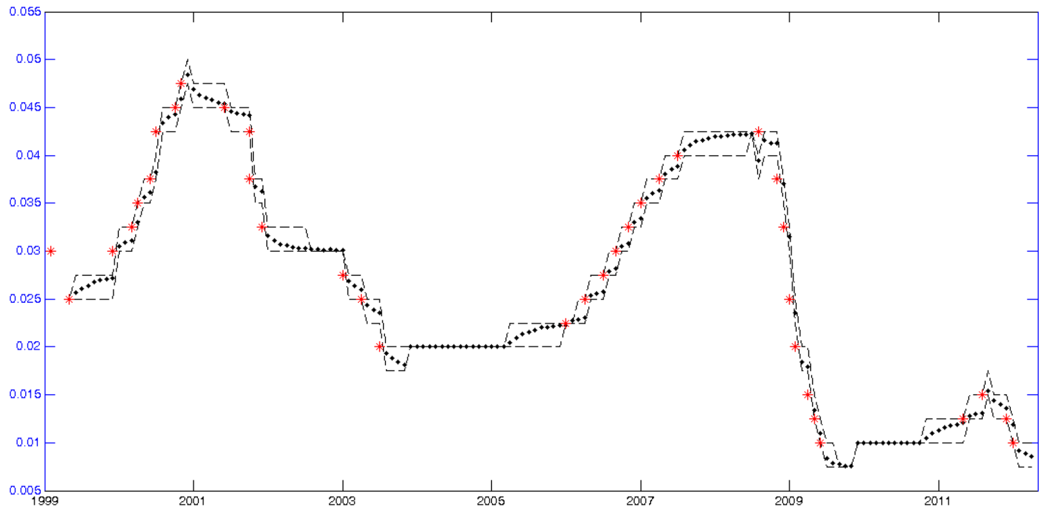

Figure 6. Short term simulation results for the considered time period (1999 to 2012) compared with ECB decisions. Red stars represent time of the real ECB interventions and corresponding value of the rates. The dotted line represents the average value of the ECB rates over 5000 simulations, while the dashed lines represent the 10th and 90th percentiles.

5. Conclusion

In the present paper we propose a stochastic model for the ECB interventions able to link the reference rates to the states of the economy. Basing on the empirical evidence that a jump process is suitable to describe ECB interventions, we aim at improving the simple jump model in [3] by linking the intensity function to the predicted state of the economy. The first step is to model the economic cycle. We choose a two-state MS-AR model and develop an empirical classification algorithm of the business cycle phases, basing on the ECB's interventions since 1999. The empirical rule relies on the evaluation of the forecast probability of expansion as estimated by the two-state MS model and on the comparison of this probability with a fixed threshold. Application of the classification rule leads us to partitioning the considered time interval in non-overlapping subintervals. Referring to ECB interventions each interval is than classified as an expansion, recession or uncertainty period.

We define the rates dynamics through a stochastic jump process whose parameters depend on the predicted states of the economy as defined by our classification rule. The overall proposed methodology is empirically validated on the series of the ECB interventions. Our work shows that an MS model (using ESI as the only explanatory variable) and a classification rule relying on such a model is completely coherent with the ECB choices in fixing the interest rates, allowing us to model the time series of ECB interventions.

We are aware that ECB interventions respond not only to real economic activity expected growth but also to other macroeconomic indicators of the business cycle evolution. Indeed future investigation related to our work should consider other recently proposed survey indicators of the economic variables [4, 9] and generalize our univariate model by considering the joint effect of several relevant indicators of the Business Cycle, hence modeling the business cycle via a multivariate model (MS-VAR). Another proper improvement of our contribution should be the extension of the ECB rate jump model to a full doubly stochastic Poisson process where the intensity is assumed to be a generic function of time. Finally, providing that many contributions in the economic literature [2, 8] give theoretical and empirical evidence that the term structure of interest rates is a leading indicator of the business cycle, we intend to explore the use of our model to link the economic cycle to the term structure of interest rates via official ECB rate.

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Appendixes

Table 1. Estimation results for the MS-AR models with 1,2,3 lags. For each coefficient, standard values are reported in parenthesis, ( ) and p-values in brackets,[ ].

		p = 1			p = 2			p = 3		
Regime 1	A_{11}	0.89	(0.03)	[0.00]	1.01	(0.07)	[0.00]	1.01	(0.07)	[0.00]
	A_{12}				-0.13	(0.07)	[0.05]	-0.10	(0.09)	[0.32]
	A_{13}							-0.04	(0.06)	[0.54]
	$\sigma*10^{-4}$	0.2	(0.0)	[0.00]	0.2	(0.0)	[0.00]	0.2	(0.0)	[0.00]
Expected duration (months)			50.1			48.2			47.5	
Regime 2	A_{21}	0.92	(0.05)	[0.00]	1.32	(0.09)	[0.00]	1.21	(0.11)	[0.00]
	A_{22}				-0.46	(0.11)	[0.00]	-0.13	(0.21)	[0.55]
	A_{23}							-0.26	(0.15)	[0.08]
	$\sigma*10^{-4}$	1.0	(0.2)	[0.00]	0.8	(0.2)	[0.00]	0.7	(0.2)	[0.00]
Expected duration (months)			14.8			14.8			14.5	
LogLikelihood			1244.22			1254.96			1257.59	
	p_{11}	0.98	(0.06)	[0.00]	0.98	(0.06)	[0.00]	0.98	(0.06)	[0.00]
	p_{12}	0.07	(0.05)	[0.18]	0.07	(0.05)	[0.20]	0.07	(0.05)	[0.20]
	p_{21}	0.02	(0.02)	[0.22]	0.02	(0.02)	[0.23]	0.02	(0.02)	[0.25]
	p_{22}	0.93	(0.11)	[0.00]	0.93	(0.11)	[0.00]	0.93	(0.11)	[0.00]

Table 2. Results of the classification rule for $\alpha_R = 0{:}5$, $\alpha_E = 0{:}9$; for each time interval the estimated intensity of the Poisson process is also reported in the last column

Time interval	Estimated Regime	ECB rates variation	Estimated Intensity
1999:4 to 2000:10	Expansion	+2.25	0.47
2000:11 to 2003:9	Recession	-2.75	0.31
2003:10 to 2005:1	Uncertainty	--	--
2005:2 to 2008:5	Expansion	+2.25	0.23
2008:6 to 2009:9	Recession	-3.25	0.81
2009:10 to 2010:8	Uncertainty	--	--
2010:9 to 2011:6	Expansion	+0.5	0.22
2011:7 to 2012:3	Recession	-0.5	0.25

Table 3. Sensitivity of the classification rule to the threshold for the probability of a certain expansion α_E: classification error for different values of the parameter, measured as the percentage of ECB interventions falling in a misclassified time interval. The second column gives the total error (T), while the third and fourth columns refer to wrong direction of the intervention error (S) and missed intervention error (M), respectively.

α_E	T error (%)	S error (%)	M error (%)
0.85	15.2	15.2	--
0.90	2.2	2.2	--
0.95	10.8	6.5	4.3

JOURNAL OF APPLIED QUANTITATIVE METHODS

MEASURING THE EFFECTS OF USABILITY ISSUES AFFECTING AN ENJOYABLE LEARNING EXPERIENCE – A PATH ANALYSIS APPROACH

Costin PRIBEANU[1]
National Institute for R&D in Informatics – ICI Bucharest, Romania

E-mail: pribeanu@ici.ro

Abstract

Educational applications based on augmented reality (AR) are integrating real objects into a computer environment thus creating a more engaging and enjoyable learning environment for students. The interaction with real objects in an AR setting may lead to usability problem that affect the extent to which the user learning experience is perceived as enjoyable. The objective of this paper is to measure and analyze the negative influence of usability issues on the perceived enjoyment of using a Chemistry AR-based application. In order to better understand and explain students' perceptions by measuring both direct and indirect effects a two-step approach was taken. During the first step a prediction model is developed based on multi-level multiple linear regression. In the second step the model is refined and estimated using path analysis. The results revealed four factors having a significant influence: quality of the see-through screen, ease of reading the information on the screen, ease of Chemical elements selection, and fatigue.

Keywords: *e-learning, augmented reality, perceived enjoyment, user experience, path analysis usability, ergonomics*

1. Introduction

Augmented reality (AR) technology has a great potential to support novel approaches to education [4], [12], [13], [21]. Students can touch and hold real objects, control the learning process, and build knowledge by themselves. Apart from their pedagogical value, AR-based applications are increasing the intrinsic motivation by creating engaging and enjoyable learning environments for students [3], [14], [18], [21]. Implementation of the AR concept in desktop settings may create perceptual problems [2], [10], [11] which may lead to specific usability problems in terms of ease of learning how to operate, ease of use, and fatigue [15]. In turn, these negative effects may affect the extent to which the user experience with an AR-based application is perceived as enjoyable.

This work reports on the negative influence of usability problems on the user learning experience with an Augmented Reality Teaching Platform (ARTP). The platform has been developed in the ARiSE European project having as main objectives to assess the pedagogical effectiveness of introducing the AR technology in primary and secondary schools and to increase students' motivation to learn [22]. Three research prototypes (applications) implementing three learning scenarios have been developed on the ARTP. The application analyzed in this study implemented a Chemistry learning scenario and was targeted at understanding the periodic table of Chemical elements, the structure of atoms / molecules, and the chemical reactions.

The objective of this work is to measure and analyze the negative influence of usability problems on the perceived enjoyment of learning with ARTP. In a previous paper [16] a multiple linear regression analysis was carried on to identify the main factors affecting the learning experience. The results revealed three predictors: accuracy of visual perception, Chemical elements selection, and fatigue. In this paper we extend the analysis in order to better explain students' perceptions by measuring and analyzing both the direct and the indirect effects. The approach is based on a method consisting in two steps: (1) multilevel multiple linear regression yielding a prediction model and (2) refinement and estimation of the prediction model using AMOS for Windows [1].

The rest of this paper is organized as follows. In section 2, previous and related work is discussed with a focus on usability evaluation and specific issues related to the ergonomics of ARTP. The method and experiment are presented in section 3. In the next section, the predictive model is presented and the estimation results are discussed. The paper ends with conclusion in section 5.

2. Related work

2.1. Perceptual issues and usability problems in AR environments

Designing AR applications for usability is not easy since each real object requires careful implementation of specific interaction techniques. As Dunser & Billighurst [6] pointed out AR is an emerging technological field so many technological issues have to be solved in order to create usable applications. They are advocating for a user centered-design approach of AR systems and for evaluating applications with actual users.

Gabbard & Swan [7] proposed a usability engineering approach that is based on user-cantered design, user-based studies, and iterative evaluation. They illustrated their approach with a case study of analyzing how users perceive text in an outdoor AR setting. The authors are advocating for small experimental designs that focus on main issues.

Kruijff et al. [11] classified and analyzed the perceptual issues on a range of AR platforms with a focus on the correct perception of augmentations. Each category of platforms has specific perceptual issues. Their study concludes that while perceptually correct augmentation remain a difficult problem that could only be accomplished through improved hardware and software, more work on evaluation is needed to better understand the specific issues on different platforms.

Bai & Blackwell [2] analyzed several usability issues in AR systems based on the papers published by ISMAR conferences (International Symposium of Mixed and Augmented Reality). Their study highlights four areas of interest for usability evaluation: performance, perception and cognition, collaboration, and user experience. Regarding the user experience, the study mentioned the accuracy of visual perception and the accuracy and quality of sound. Regarding the user experience, the main issues are related to discomfort, headaches and eye pains.

2.2. Previous work in ARTP evaluation

During the ARiSE project several usability evaluations have been carried on for the Biology and Chemistry applications. The data samples collected in 2007 and 2008 included usability reports (prioritized list of usability problems), measures of effectiveness and efficiency, and answers to questionnaire. The answers to questionnaire included both qualitative and qualitative data (answers to open questions). The qualitative data helped evaluators to better understand the usability issues and helped developers to fix most of them.

In a previous work [14], the motivational value of the Chemistry application was evaluated based on the analysis of answers to the questionnaire. Students found the Chemistry scenario interesting, captivating and enjoyable. However, the analysis of quantitative data revealed several items with a low mean value (i.e. bellow 3.50 on a scale from 1 to 5) which suggest some usability problems related to the accuracy of the visual perception.

Iordache & Pribeanu [9] reported on the formative evaluation of the Biology application implemented on ARTP. In order to increase confidence in results, a comparison between quantitative and qualitative data has been done. The results showed that most of the usability problems were related to selection, accuracy of visual perception, and comfort in use (eye pains).

In a recent work [15] the ergonomic quality of the Chemistry application was analyzed by using a model with causal indicators that are influencing the perceived ease of learning how to use, perceived ease of use, and comfort in use. The results showed that the most important indicators for the ergonomic quality are the ease of reading the information on the screen and the ease of selecting a chemical element.

3. Method and empirical study

3.1. Method

The method used in this study is based on Cohen's path analysis [5]. Cohen noticed that a multiple linear regression leads to a flat model that only takes into account predictors having a direct effect on the dependent variable. Therefore he used the multi-level multiple regression in order to develop causal models that could better explain the correlation between variables. In a previous work [17] this approach was used in order to refine the causal model and assess its quality. The method has two steps:

- Development of the causal model based on multi-level multiple linear regressions.
- Model estimation, model refinement, and analysis of direct and indirect effects.

Cohen's path analysis enables the development of causal models in which the predictors of a dependent variable become dependent variable on the next level. This multi-level model has an increased explanatory power than a regression model since it shows how some causal influences are mediated by endogenous variables [5]. There is one limitation of the development path: since the model is developed level by level is possible to miss relationships between variables that are not on consecutive levels.

The second step enables an easy specification of the model and an automate computation of the direct and indirect effects. Model estimation allows checking the significance of causal relationships. The examination of modification indices makes it possible to identify missing relationships so the model could be further improved. This is an important advantage that overcomes the limitation mentioned above. Also, the model estimation with AMOS for Windows [1] makes it possible to assess the quality of the model (model fit with the data).

3.2. Equipment and data sample

ARTP is a desktop AR platform that has been registered by Fraunhofer IAIS under the trade mark Spinnstube®. The experiment has been carried on using an ARTP with 4 independent modules organized around a table on which real objects are placed [20]. A remote controller Wii Nintendo has been used as interaction tool for selecting a menu item.

The learning scenario for Chemistry has an introduction and three lessons, each lesson having several exercises. More details about the pedagogical goals, lessons, and exercises could be found in [19]. The Chemistry scenario integrates two kinds of real object: a periodic table and a set of colored balls (4 colors) symbolizing atoms. Each workplace has its own periodic table. By placing a ball over a chemical element it becomes an atom of that element and could be further used to form molecules. Previously created molecules could be further used for simulating Chemical reactions.

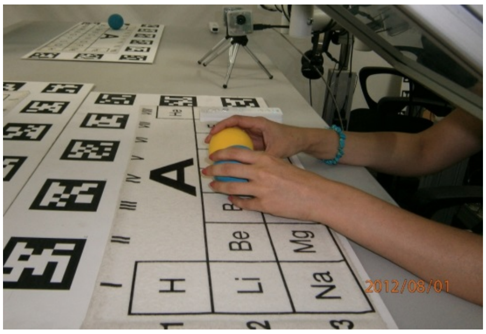

Figure 1. Student using the Chemistry learning application on the ARTP

The data was collected in 2012-2013. A total of 186 students (13-15 years old) from several schools in Bucharest tested the Chemistry scenario during a session of 30 minutes. None of them was familiar with the AR technology. After testing, the students were asked to answer a questionnaire by rating the items on a 5-point Likert scale. The data sample is gender balanced (96 boys and 90 girls). A data analysis has been carried on that revealed multivariate outliers (based on Mahalanobis distance, $p<.001$). Therefore two observations were eliminated which resulted in a working sample of 184 observations.

3.3. Variables

The variables used in this study, the mean value and standard deviation are presented in Table 1.

Table 1. Variables

No.	Item	M	SD
ERG1	Observing the real objects through the screen is clear	3.36	0.90
ERG2	Understanding the augmentation of a real object was easy	3.22	1.11
ERG3	Selecting a Chemical element is easy	3.28	1.35
ERG4	Selecting a menu item is easy	4.47	0.99
ERG5	Vocal explanations are clear an understandable	4.41	0.90
ERG6	Reading the information on the screen is easy	4.08	0.88
CONF1	I felt tired after using the system	1.99	1.32

CONF2	After using the system I experienced headaches	1.73	1.19
CONF3	After using the system I experienced eye pains	1.91	1.24
PE1	Using ARTP is an enjoyable learning experience	4.27	0.91

First 6 items refer to ergonomic and usability aspects of the ARTP. Next three items refer to the comfort in use and the last item is the dependent variable in this study. The relatively high mean value of PE1 shows that students perceived the interaction with ARTP as an enjoyable learning experience.

First three items have low mean values suggesting that some usability problems are related to the clarity of the see-through screen, accuracy of the augmentation, and ease of selecting of Chemical elements. Menu selection and accuracy of vocal explanations were highly rated by students. As regarding the comfort in use (high mean value means lack of comfort) the students mainly complained about fatigue, then about eye pains and less about headaches.

A correlation analysis has been carried on based on Pearson's correlation coefficients. The correlation table is presented in Table 2. All independent variables are significantly correlated with the dependent variable.

Table 2. Correlation table

	ERG1	ERG2	ERG3	ERG4	ERG5	ERG6	CONF1	CONF2	CONF3	PE1
ERG1	1									
ERG2	.45**	1								
ERG3	.36**	.38**	1							
ERG4	ns	.17*	.15*	1						
ERG5	.17*	.18*	ns	.42**	1					
ERG6	.36**	.25**	.28**	.27**	.35**	1				
CONF1	ns	ns	ns	ns	-.16*	-.21**	1			
CONF2	ns	ns	ns	-.17*	ns	-.22**	.62**	1		
CONF3	ns	ns	ns	-.18*	ns	-.16*	.60**	.59**	1	
PE1	.36**	.26**	.35**	.15*	.17*	.36**	-.27**	-.15*	-.17*	1

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

Significant correlation coefficients are ranging from -0.27 to 0.62. Items related to the visual perception (ERG1, ERG2, ERG6) are inter-correlated, with correlation coefficients varying from 0.26 to 0.45. Items related to the comfort in use are highly inter-correlated (0.59-0.62).

4. Analysis and results

4.1. Predictive model development

The target variable PE1 (enjoyable learning experience) has been used as starting dependent variable by regressing on it the rest of variables. In the next levels only causal relationships that make sense have been introduced. For example, ERG5 can only be a predictor and is independent from ERG1, ERG2, and ERG6. Overall, eight multiple linear regressions have been made. The summary of results is presented in Table 2. The regressions have been done by using SPSS for Windows. In all cases the multiple correlation coefficients were significant.

Table 2. Results of regression analysis

Dependent variable	Adjusted R^2	F	Sig.	D-W
PE1	0.253	16.507	0.000	1.994
CONF1	0.462	79.523	0.000	2.012
CONF2	0.359	52.309	0.000	1.903
CONF3	0.028	6.240	0.013	1.776
ERG3	0.195	15779	0.000	1.827
ERG6	0.211	25.425	0.000	2.077
ERG4	0.171	38.791	0.000	1.599
ERG2	0.199	46.584	0.000	1.868

The correlations between the independent variables are not too high. VIF values are well bellow 10 with the highest value of 1.541 so there is no collinearity within our data. The Durbin-Watson test value is ranging between 1.599 and 2.077 so we can conclude that the residuals are uncorrelated. Table 3 displays the standardized regression coefficients, t-values, and significance

Table 3. Regression analysis – coefficients

Dependent variable	Predictor	β	t	sig.
PE1	ERG1	.196	2.730	.007
	ERG3	.220	3.165	.002
	ERG6	.183	2.579	.011
	CONF1	-.213	-3.255	.001
CONF1	CONF2	.403	5.988	.000
	CONF3	.363	5.394	.000
CONF2	CONF3	.572	9.537	.000
	ERG6	-.125	-2.092	.038
CONF3	ERG4	-.182	-2.498	.013
ERG3	ERG2	.260	3.477	.001
	ERG1	.190	2.447	.015
	ERG6	.145	2.020	.045
ERG6	ERG1	.311	4.672	.000
	ERG5	.300	4.506	.000
ERG4	ERG5	.419	6.228	.000
ERG2	ERG1	.151	6.825	.000

The results of the first regression show that only four independent variables have a direct effect on the dependent variable. Three of them are positively related to the visual perception of the real object, the Chemical element selection, and the ease of reading the information on the screen. The last predictor has a negative influence showing that the more tired the student feels the less enjoyable is the learning experience.

4.2. Causal model estimation

The causal model developed by multilevel multiple linear regression was specified and estimated in AMOS for Windows [1]. This step enables to analyze the interaction between factors and to explain the contribution of each factor. In Figure 2 the causal model is presented with standardized regression coefficients and explained variance for each endogenous variable. The model includes the error term for the variables that are not inter-correlated.

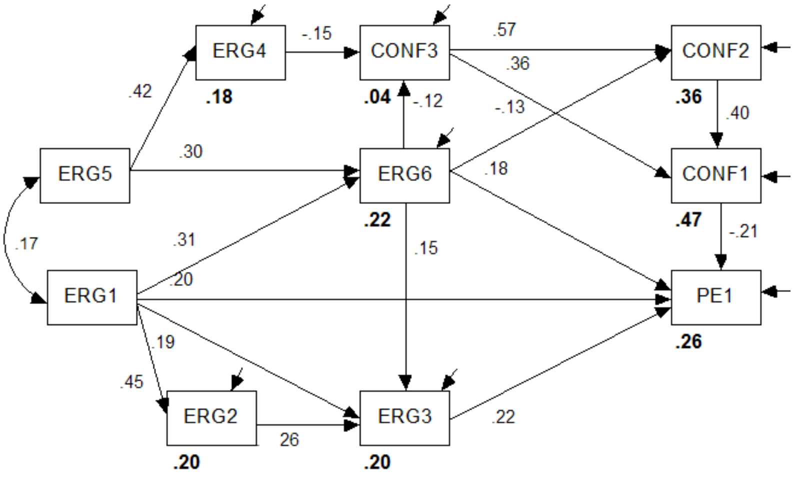

Figure 2. Initial causal model - estimation results

The indices of the model fit with the data are good, over the cut-off values recommended by de Hair et al. [8]: χ^2=24.739, DF=27, p=.589, χ^2/DF=.916, CFI=1.000, TLI=1.009, GFI =0.974, RMSEA =0.000, SRMR=0.0506.

The analysis of estimated regression coefficients shows that all causal relationships are significant (p< 0.05) except for ERG6 > CONF3. The model could be simplified by specifying ERG2 and ERG4 as exogenous variables. The refined model is presented in Figure 3.

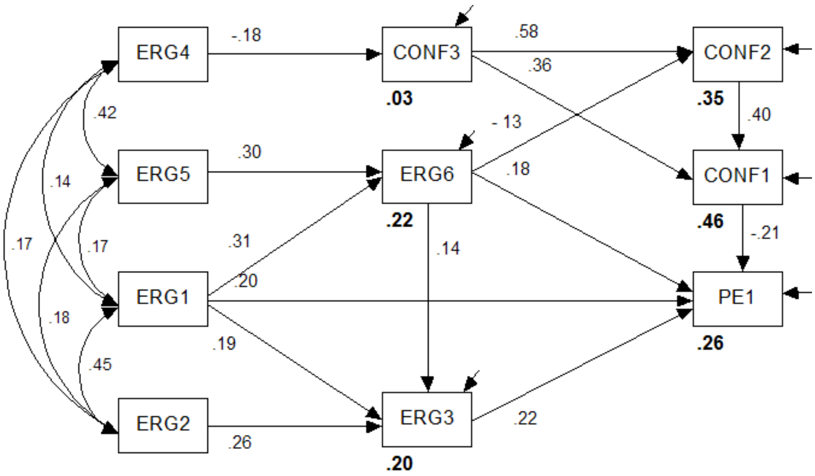

Figure 3. Refined causal model - estimation results

All causal relationships are significant at p<0.05 level. The indices of the model fit with the data are very good (slightly better than those of the initial model), over the cut-off values: χ^2=22.784, DF=25, p=.590, χ^2/DF=.911, CFI=1.000, TLI=1.010 GFI =0.977, RMSEA =0.000, SRMR=0.0492. The results show the benefits of the second step of the method: the refined model is simpler, the fit indices are better, and a non-significant causal relationship has been removed.

The estimation in AMOS enables the automate computing of direct, indirect, and total effects. In Table 4 the standardized total effects are presented. Results show that all 9 indicators are useful to explain the effects of usability issues on the perceived learning experience.

Table 4. Standardized total effects

	ERG1	ERG2	ERG3	ERG5	ERG4	ERG6	CONF3	CONF2	CONF1
ERG6	0.311	0	0	0.300	0	0	0	0	0
ERG3	0.235	0.261	0	0.044	0	0.145	0	0	0
CONF3	0	0	0	0	-0.182	0	0	0	0
CONF2	-0.039	0	0	-0.038	-0.105	-0.127	0.577	0	0
CONF1	-0.016	0	0	-0.015	-0.109	-0.051	0.596	0.401	0
PE1	0.310	0.058	0.221	0.068	0.023	0.228	-0.127	-0.086	-0.214

As regarding the direct effects, the most important predictors are ERG3 and CONF 1. The selection of Chemical elements was sometimes difficult, as shown by the results of a previous empirical study [14]. Also, the relatively low mean value of 3.28 (SD=1.36) shows that students found it difficult to select Chemical elements. The fact that the items ERG1 and ERG2 have also low mean values suggests that students experienced usability problems with the visual perception onto ARTP. Both variables are predictors of ERG3 thus indirectly influencing the dependent variable PE1.

A second factor affecting the learning experience is fatigue. The students felt tired after using the system. Some of them also complained about headaches and eye pains. The influence of selection techniques onto the user experience is not surprising since these are closely related to the real objects integrated by the application. The fourth factor is the ease of reading the information displayed on the see-through screen.

Besides the direct effect, reading the information on the screen (ERG6) is also an important mediator of indirect effects of ERG1 and ERG5 on the enjoyable learning experience. The indirect effects of ERG1 are also mediated by ERG3. If we take into account both direct and indirect effect of ERG1 it seems that the clarity of the see-through screen is a very important predictor. This finding shows the usefulness of the path analysis in explaining the users' perceptions. Overall, the results show that all 9 indicators are useful to explain the effects of usability issues on the perceived learning experience.

5. Conclusions

The enjoyable learning experience is influenced by two categories of factors. First category includes specific AR features that proved to enhance the user experience in e-learning [4], [9], and [17]. The second category includes ergonomic aspects of the AR platform and usability of the interaction techniques that are undermining the user experience.

In this work we measured the extent to which the specific ergonomic and usability issues are affecting an enjoyable user experience with an AR-based learning application. In order to measure both direct and indirect effects a method that is based on path analysis has been carried on. The results revealed four factors having a direct influence: the quality of the see-through screen (hardware issue), the ease of reading the information on the screen (software issue), the ease of Chemical elements selection (interaction technique), and fatigue after using ARTP (ergonomic issue).

These findings have several implications for developers. First, the visual perception in AR settings is a critical issue that may affect task effectiveness, visual fatigue, and perceived enjoyment. Second, the vocal explanations for user guidance are a useful feature that reduces the amount of information displayed on the see-through screen. Third, the selection techniques that are closely related to the real objects are specific to each AR application and should be carefully designed.

The results of this study show that all factors together account for 26% of the variance in the perceived enjoyable learning experience. This value is pretty high for a negative influence and suggests that developers should pay more attention to ergonomics and usability aspects that may undermine their effort to create enjoyable applications.

The method used in this study integrates and extends Cohen's path analysis. Causal models provide more insights in understanding the user perceptions than a multiple linear regression can do. Summing up the advantages, this two-step method enables: (1) estimating both direct and indirect effects, (2) revealing the variables acting as mediators, (3) analyzing the interaction between factors, and (4) refining and assessing the quality of the causal model.

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[1] Costin Pribeanu is Senior Researcher I at ICI – Bucharest. His researches are focused on user interfaces, model-based design, usability and user experience evaluation, accessibility, and AR-based educational systems.

RANKING CAPITAL MARKETS EFFICIENCY: THE CASE OF TWENTY EUROPEAN STOCK MARKETS

Olivia Andreea BACIU
PhD Candidate, Babes Bolyai University, Cluj Napoca, Romania

E-mail: oli_baciu@yahoo.com

Abstract
This study investigates the deviations from efficiency of twenty European Stock Markets using three measures: long-term memory, fractal dimension and efficiency index of Kristoufek and Vosvrda (2012). The markets are ranked according to each measure on a period of fifteen years but also on three sub-periods of time. The sub-periods are chosen depending on the phenomena that has marked the stock market evolution. The results show that the developed markets are closer to efficiency than the emerging ones. Overall, the most efficient markets are the ones of UK, Sweden, Switzerland or France while among the least efficient ones can be mentioned the markets of Bulgaria, Czech Republic, Greece or Slovakia.

Keywords: *capital market, market efficiency, long-range dependence, fractal dimension, efficiency index*

1. Introduction

Market efficiency is one of the most important topics in finance and the subject of numerous studies. The notion itself is abstract and although its simplicity, there is not a straight forward pattern to prove the efficiency or not of a market. An efficient market is assumed to incorporate immediately all the available information and such way, there are no possibilities to earn profits based on past information. Proving that the market does not follow a random walk is not synonymous with market inefficiency, while the reciprocal holds.

In the literature of market efficiency, lots of studies are focused on long-range dependence. If the return series exhibits long-range dependence then the week form of the market efficiency is violated. The presence of long-range dependence or long memory in the return series benefits the technical analysis strategies (Cerqueti and Rotundo, 2014). Based on past informations one can make predictions for future returns. If the dependencies are tracked long time back in the past, the time horizon of the predictions can be larger. Among the first studies of long range dependence are the ones of Mandelbrot and Wallis (1969) or Mandelbrot (1971, 1972) in which was suggested that in the presence of the long memory may exist profit opportunities. Green and Fielitz (1977) identified long memory on 200 daily US stock returns using the rescaled range analysis. The series of papers of Cajueiro and Tabak have used the long-range dependence to rank the efficiency of different markets. Cajueiro and Tabak (2004) ranked the three Asian markets of Hong-Kong, Singapore and China using as measure of the long memory the Hurst exponent, both on the return series and on rolling windows. Same authors (2005) went further and using the rolling sample approach to estimate the Hurst exponent, they ranked the efficiency of the emerging equity markets. It turned out that the Asian countries are more efficient than the ones of Latin America, excepting Mexico. In the article of 2007, when the long-range dependence was searched in the stocks of the Dow Jones Average Industrial Index, the stocks presented an anti-persistent behavior although the Index did not exhibit long memory. Kristoufek (2010) or Ausloos (2012), using for assessing the long range dependence the generalized Hurst exponent proposed by Di Matteo et al. (2003), detrending moving average and detrended fluctuations analysis , contradicts the findings of Serletis and Rosenberg (2009), who found an anti-persistent behavior in the returns of the most important US stock indices by wrong application of the detrended moving average .

On the European stock markets, Cajueiro and Tabak (2006) or Dumitrana *et al*. (2010) or Ausloos (2002) found evidence of long range dependence for all the investigated European stock indices. Moreover, their findings support previous research that the emerging markets present short or long term memory (Rotundo, 2006). Kasman et al (2009) found long memory on the markets of Hungary, Slovakia and Czech Republik but not on the Poland one. Necula and Radu (2012) concluded that the markets of Romania, Hungary, Czech Republik, Poland, Bulgaria, Slovenia, and Croatia exhibit long-range dependence.

Most of the studies have ranked the efficiency of different stock markets based on a specific measure. Kristoufek and Vosvrda (2012) introduced a new measure of the market efficiency based on a combination of several measures. Assuming no correlation structure on an efficient market, the efficiency index incorporates measures of long and short term memory but also of the local herding behavior. When ranking 41 stock markets around the world, the European markets turned out to be the most efficient ones while the markets of Latin America, Asia, Oceania were the least efficient ones. Kristoufek and Vosvrda (2013) utilized a different version of the efficiency index in ranking 38 stock markets. In the composition of the efficiency index were considered the long term memory, fractal dimension and approximate entropy. The analysis revealed that the most efficient markets are the ones of Netherlands, France and Germany while the least efficient ones were the markets of Venezuela and Chile.

Other studies use different efficiency measures to assess the market efficiency. Lagoarde-Segot and Lucey (2006) or Ristea *et al.*, (2010) use an efficiency index based on several statistical tests of the random walk behavior and technical analysis and concluded that in seven emerging Middle-Eastern North African stock markets the weak form efficiency is explained by the differences in stock market size and corporate governance factors. Mensi (2012) used a modified version of the Shannon entropy on a rolling sample approach and a data time window for 100 days together with a symbolic time series analysis to rank the markets and identify the impact of the financial crisis on the level of efficiency. For the 26 analyzed stock markets it was found out that the market efficiency not only changes in time, but is also evolving.

In this study is analyzed the efficiency of twenty European stock markets. These markets are ranked depending on long-range dependence, fractal dimension and the Efficiency Index of Kristoufek and Vosvrda (2012).

The main contribution of this study is that it ranks the efficiency of most of the European stock market indices using three measures of efficiency in which one is an index of several measures. Another contribution is that the rankings were performed not only on a period of fifteen years but also on three sub-periods divided according to the main phenomena that affected the economical, political and social life and implicitly the stock market.

The paper is structured as follows. Second part introduces the methodology and presents the measures that are used in the analysis. In Section 3 is presented the data used. In Section 4 are exposed the results and finally Section 5 concludes the paper.

2. Methodology

In assessing the long-range dependence of the return series are used two approaches. First, the Hurst exponent is computed using the classical R/S analysis and then a rolling sample approach is used to identify if the long-range dependence can be assessed for the entire period of time or if it changes in time. 300 observations are used to create each window and a Hurst exponent is composed for each one. The sample is then rolled one more observation and the first one is dropped. The rolling is stopped when the last sample observation is introduced in the window. In this manner are created several Hurst exponents, specifically the number of observations minus 300, the length of the window. Of course, not all the values of the Hurst exponent can be used in the analysis but instead is chosen the median value. The analyzed indices are ranked according to the median Hurst exponent.

For the fractal dimension are used the Hall-Wood and Genton estimators. The ranking of the indices is based on the average value of the two estimators.

The last ranking of the twenty European stock indices is according to the efficiency index introduced by Kristoufek and Vosvrda (2012):

$$EI = \sqrt{\sum_{i=1}^{n}\left(\frac{\widehat{M_i}-M_i^*}{R_i}\right)^2}, \qquad (1)$$

where $\widehat{M_i}$ is an estimate of the i^{th}measure of efficiency, M_i^* is the expected value of the i^{th} measure in an efficient market and R_i is the range for each measure. For the purpose of the analysis are considered here three measures of the efficiency: the average of Hurst exponent and the median value of the Hurst exponents in rolling windows approach, the average of Hall-Wood and Genton estimator and first order autocorrelation. In an efficient market, the expected value of the Hurst exponent is 0.5 and for the fractal dimension is 1.5. The range is standardized to 1 for Hurst exponent and fractal dimension and 2 for the first order autocorrelation to keep the same maximum deviation for all measures.

In an efficient market EI=0, while in the least efficient market EI=$\frac{\sqrt{n}}{2}$. In our case, with three measures of efficiency, the index of the least efficient market would be 0.86.

2.1. Measure of the long range dependence

The long range dependence measure used is the Hurst exponent. The Hurst exponent is computed using the classical R/S analysis using two different approaches: on the return series and also by means of the rolling sample approach. The return is computed as the difference between natural logarithm of current day closing price and natural logarithm of previous day closing price.

The R/S statistic, as described in Cajueiro and Tabak (2004), is computed as:

$$\left(\frac{R}{S}\right)_\tau = \frac{1}{s_\tau}\left[max_{1\le t\le \tau}\sum_{t=1}^{\tau}(R(t)-\bar{R}) - min_{1\le t\le \tau}\sum_{t=1}^{\tau}(R(t)-\bar{R})\right] \qquad (2)$$

where, $\bar{R}$ is the average return in the considered period

s_τ is the standard deviation estimator, $s_\tau = \sqrt{\frac{1}{\tau}\sum_t (R(t) - \bar{R})^2}$

According to Hurst (1951), the following relation holds:

$$(R/S)_\tau = (\tau/2)^H \quad (3)$$

Hurst exponent takes values between 0 and 1. A value of 0.5 corresponds to a series that does not exhibit long range dependence. Values lower than 0.5 are evidence of long-range dependence with an anti-persistent behavior, while values larger than 0.5 are evidence of long-range dependence with a persistent behavior.

2.2. Measures of the fractal dimension

Fractal dimension is used as a measure of the local memory of the series. The deviations of the short-term trends from a random behavior are captured in the roughness of the series. For a market with no deviations from a random behavior, the fractal dimension measure is expected to be 1.5. Ranging from 1 to 2, a fractal dimension value lower than 1.5 characterizes a series with local persistence while values larger than 1.5 characterizes a rougher series with local anti-persistence.

As measures of fractal dimension, in this study are used the Hall-Wood and Genton estimators, two of the estimators used by Kristoufek and Vosvrda (2012) in computing the efficiency index.

Hall-Wood estimator

As presented by Kristoufek and Vosvrda (2013), the Hall-Wood estimator is given by:

$$\widehat{D_{HW}} = 2 - \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log A\widehat{(l/n)})}{\sum_{l=1}^{L}(s_l - \bar{s})}, \quad (4)$$

where L≥ 2, $s_l = \log(l/n)$, $\bar{s} = \frac{1}{L}\sum_{l=1}^{L} s_l$ and the absolute deviations between steps are:

$$A\widehat{(l/n)} = \frac{1}{n}\sum_{i=1}^{[n/l]} |x_{\frac{il}{n}} - x_{(i-1)l/n} \quad (5)$$

To minimize the bias, Hall and Wood (1993) suggested L=2, which gives the below estimator:

$$\widehat{D_{HW}} = 2 - \frac{\log A(2/n) - \widehat{\log A(1/n)}}{\log 2}. \quad (6)$$

Genton estimator

This method implies the use of the Genton variogram introduced by Genton (1998) and which is defined as:

$$\widehat{V_2(l/n)} = \frac{1}{2(n-l)}\sum_{i=l}^{n}(x_{i/n} - x_{(i-l)l/n})^2. \quad (7)$$

The Genton estimator is then given by:

$$\widehat{D_G} = 2 - \frac{\sum_{l=1}^{L}(s_l - \bar{s})\log V_2\widehat{(l/n)})}{2\sum_{l=1}^{L}(s_l - \bar{s})}, \quad (8)$$

where L≥ 2, $s_l = \log(l/n)$, $\bar{s} = \frac{1}{L}\sum_{l=1}^{L} s_l$. For the same purpose as above, L=2 and the Genton estimator is

$$\widehat{D_G} = 2 - \frac{\log V_2\widehat{(2/n)} - \log V_2\widehat{(l/n)}}{2\log 2} \quad (9)$$

3. Data

In this study are comprised twenty indices of the European stock markets that are covering not only the most developed markets but also the emerging ones. Data consists of daily closing prices and was gathered starting 1999 until 2013, with the exception of few indices that were founded later than 1999.

During the analyzed fifteen years, the stock markets went through different behaviors. If between 2003 and 2007 the markets went through constant increases and development, the economical crisis that affected Europe in 2007 has turned the gains into important looses with short periods of slight recoveries. Until the end of 2013 the markets did not manage to recover completely from the effects of the persistent crisis. Due to these important behaviors between 1999 and 2013, the analysis is also performed on the three sub-periods: 1999- 2006, 2007- 2008, 2009- 2013.

4. Results

Table 1 presents the absolute deviations from the expected values in an efficient market of the Hurst exponent and fractal dimension and the values of the efficiency index. All markets exhibit long-range dependence with a persistent character.

The markets of Bulgaria, Slovakia and Greece present the strongest deviations from efficiency if we take a look at the efficiency index, which is also confirmed by large deviations on the Hurst exponent and fractal dimension. On the other side, the most efficient market is the one of UK.

Table 1. Hurst exponent, Fractal dimension and Efficiency Index

Index	Country	Hurst	Fractal dimension	Efficiency Index
ATX	Austria	0.089	0.065	0.116
BEL20	Belgium	0.076	0.02	0.079
CAC40	France	0.038	0.025	0.067
DAX	Germany	0.076	0.045	0.089
OMX	Sweden	0.046	0.125	0.140
SMI	Switzerland	0.060	0	0.065
FTSE100	UK	0.027	0	0.034
GD.AT	Greece	0.118	0.155	0.199
FTSEMIB	Italy	0.072	0.015	0.073
SOFIX	Bulgaria	0.188	0.12	0.231
XU100	Turkey	0.065	0.01	0.067
BUX	Hungary	0.066	0.04	0.081
PX	Czech Republic	0.105	0.02	0.107
SAX	Slovakia	0.109	0.18	0.211
BET	Romania	0.122	0.07	0.148
HEX	Finland	0.074	0.03	0.089
WIG20	Poland	0.085	0	0.086
AEX	Netherlands	0.059	0.005	0.069
OBX	Norway	0.067	0.02	0.070
IBEX	Spain	0.053	0.02	0.068

Figure 1 shows the deviations from efficiency of the analyzed markets for the entire period of time, 1999- 2013. Acording to the median Hurst exponent, the closest to an efficient market is the France market followed by the markets of UK, Sweden and Spain. On the other hand, the least efficient market is the one of Bulgaria. The Romanian, Czech

Republic and Greek markets also present strong deviations from efficiency. It seems that the developed countries exhibit less long-range dependence than the emerging ones. All the indices present a persintent behavior, meaning that the future trend of the index is expected to follow the past behavior. Used as a measure of the local memory of the series, the fractal dimension deviations shown above indicate that the markets of Switzerland, UK and Poland do not exhibit local memory. This indices do not have a local trending behavior. The majority of the indices exhibit negative deviations from an efficient market, suggesting a persistent behavior. The most severe deviations are indentified in the case of Slovakia and Greece.

Applying the efficiency index, the top three efficient markets are UK, Switzerland and Turkey while the markets of Bulgaria, Slovakia and Greece present the highest deviations from an efficient market. It can be noticed that most of the markets have similar, but not high, deviations from the efficiency.

The three measures used to rank the European stock markets for the entire period of time show that the highest deviations from efficiency are presented in the emergent markets, among which are the markets of Bulgaria, Slovakia, Greece, Romania, while the most developed ones present slight deviations from efficiency. The most efficient market seems to be the one of UK, with a low long range dependence and no fractal dimension.

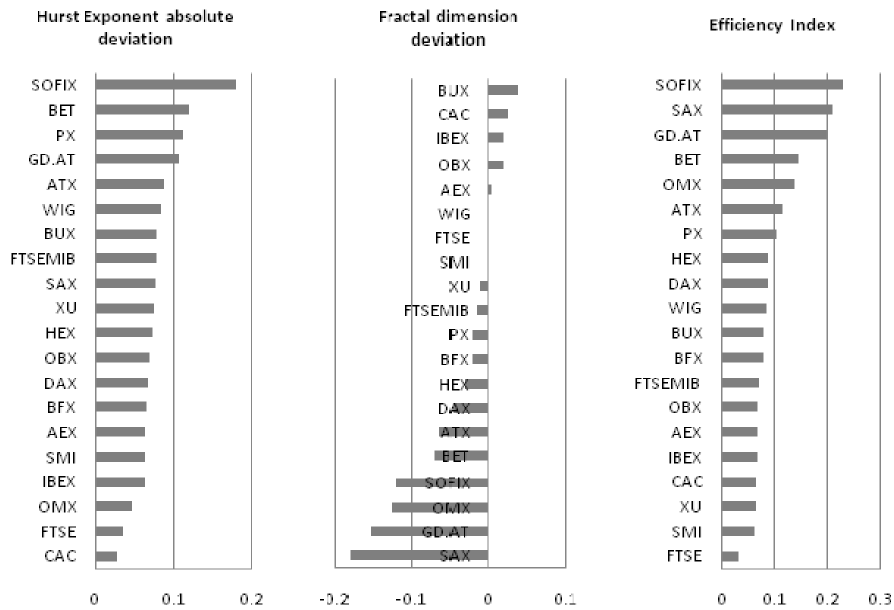

Figure 1. Ranked stock indices according to Hurst Exponent, Fractal dimensionand Efficiency Index between 1999- 2013

Between 1999 and 2006 the stock market has known a period of important increases. Through this period of time, most of the analyzed indices manifest higher deviations from the expected Hurst exponent, as it can be seen in Figure 2. The ranking based on the Hurst exponent is similar to the one of the entire period of time.

The markets of Bulgaria, Slovakia, Greece, Czech Republik or Romania are among the ones that present the strongest deviations from efficiency. The most efficient markets are the ones from UK and France.

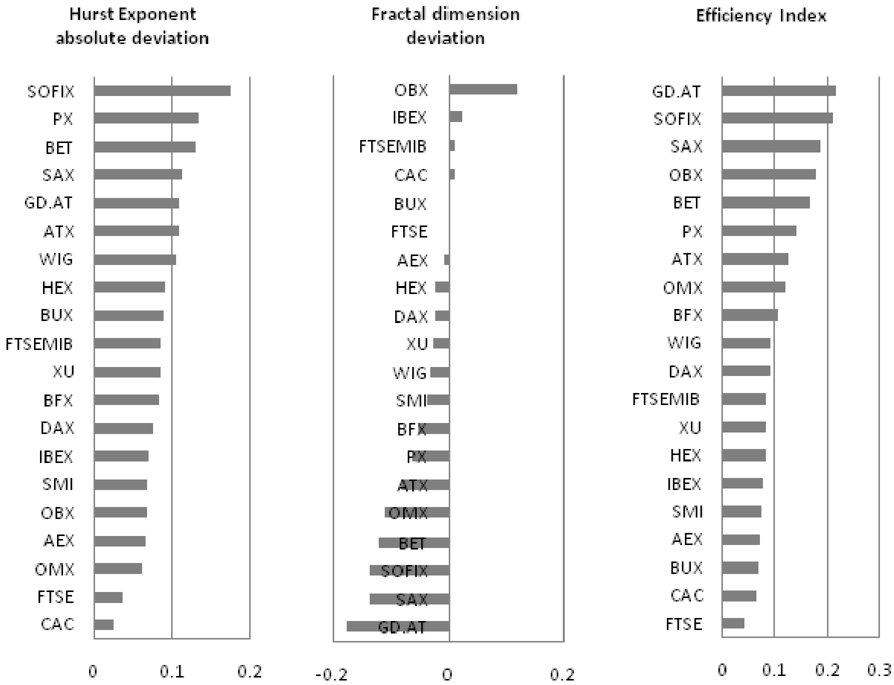

Figure 2. Ranked stock indices according to Hurst Exponent, Fractal dimension and Efficiency Index between 1999- 2006

Characterized by instability, the years of 2007 and 2008 are marked by important looses on the stock market. This behavior is due to the social, political and economical crisis that has affected the entire world. Although the index of France does not exhibit long-range dependence, according to the Efficiency Index, the French market is not efficient. Only the index of Slovakia presents much higher deviations from efficiency, while others, like OMX of the Swedish market describes a more efficient market than in the previous years.

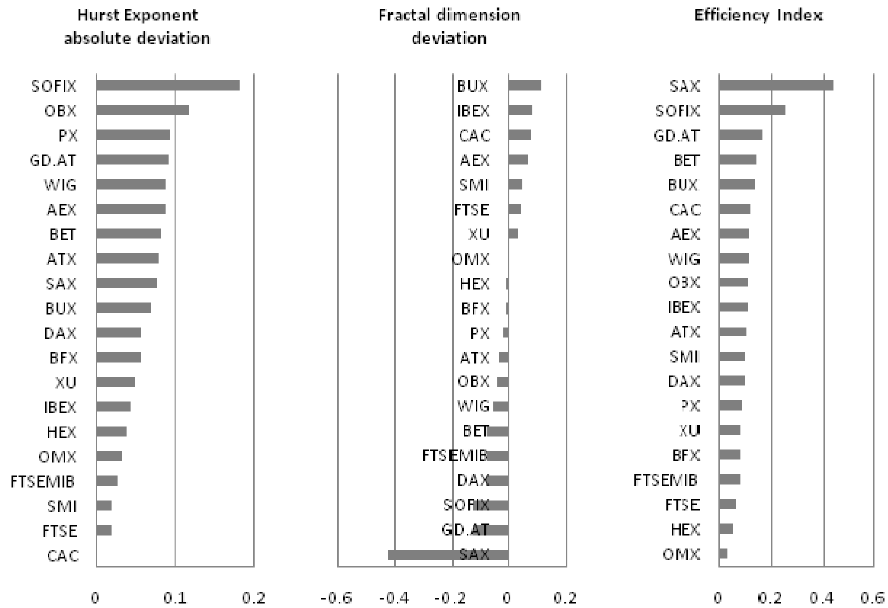

Figure 3. Ranked stock indices according to Hurst Exponent, Fractal dimension and Efficiency Index between 2007-2008

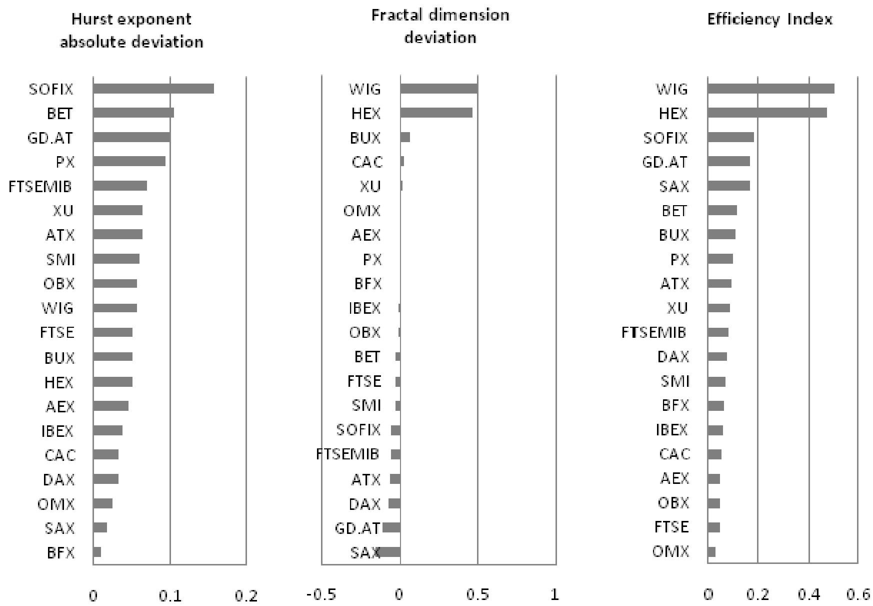

Figure 4. Ranked stock indices according to Hurst Exponent, Fractal dimension and Efficiency Index between 2009-2013

Although the years of 2009-2013 were expected to get the market out of the crash, the expectations din not become reality. The markets presented small deviations from efficiency, with the exception of Finland and Poland, according to the rankings based on fractal dimension and efficiency index, as seen in Figure 4.

In the three sub-periods of time, all markets exhibit long-range dependence, which suggest the predictability of the future returns. The presence of the long-range dependence benefits the technical analysis strategies.

The largest value of the Hurst Exponent was identified for the entire period of time, 0.18 for the index SOFIX of Bulgaria and the smallest value was 0.004 for OMX of Sweden between 2009 and 2013. For fractal dimension, the values covered a larger range, especially between 2007-2008 and 2009-2013, with absolute deviations of 0.495 for WIG of Poland or 0.425 for SAX of Slovakia. In the same time, indices like SMI and FTSE100 do not possess the fractal dimension between 1999 and 2013.

Overall, the indices of the main European stock markets come to support previous findings of Cajueiro and Tabak (2004,2005) or Risso (2008) that the developed markets are more efficient than the emerging ones.

5. Conclusions

Three measures of the market efficiency were used to rank twenty European stock market indices. Long-range memory and fractal dimension were used independetly to rank the markets but also were used as input variables in the Efficiency Index, together with the first order autocorrelation value. The rankings of the three measures were related, showing that the more developed were the markets, the closer were to efficiency. The analysis was performed over a period of fifteen years and also on three sub-periods of time chosen depending on the market behavior.

All the indices present long range dependence, with the exception of the index CAC of France between 2007 and 2008. The markets have a persistent behavior. Looking at the fractal dimension, not all the markets have deviations of the short term trends. Depending on the considered time period, markets like UK, Sweden or Switzerland show zero deviations. As top efficient European stock markets can be mentioned the markets of UK, Sweden, Switzerland or France while among the least efficient ones are the markets of Bulgaria, Czech Republic, Greece or Slovakia.

As further research, the analysis should be extended to other markets and identify the possible causes of the deviations from efficiency and how this benefits the profitability of the technical analysis strategies.

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The use of the Premium calculation principles in actuarial pricing based scenario in a coherent risk measure

Hernández Solís MONTSERRAT[1]
PhD, University Assistant, Faculty of Economics,
Universidad Nacional de Educación a Distancia (UNED), Madrid, Spain

E-mail: montserrath@cee.uned.es

Abstract
Calculation principles enable the development of the actuarial pricing process in the insurance sector both of life and of non-life. Among all principles assessed in this article we have chosen those verifying the so-called coherency criterion (Artzner, P. Delbaen, F. Eber, JM. Heath, D. (1999)), performing the theoretical-mathematic reasoning of such coherency criterion for all of them. Once those principles, more specifically two – the principle of net premium and the principle based on the distortion function in the form of power - are applied for the calculation of the risk premium both to a general and specific extent they will be applied to the Makeham Law for insurances with death cover: the whole life insurance.

Keywords: *Distortion function; coherent risk measure; Risk Premium; Insurance*

Introduction. Premium Calculation Principles based on Risk Measures

All companies assume risks that can potentially jeopardize their economic situation even forcing them into bankruptcy. The word risk goes normally hand in hand with luck, with uncertainty, so it is therefore related to the randomness of its occurrence and to the amount of the loss. It can be defined as the uncertainty regarding the onset of an event at a certain time and under specific conditions, generating therefore quantifiable loses. It is necessary to analyze the risks threatening life insurers with a view to optimize their management as risk assessment does not limit to their quantification (measurement) but also to achieve optimal protection against them and to try to avoid them. In this case we will focus on life insurances.

The main risks insurance companies have to deal with are mentioned in the following table. Among all of them, the death risk is the one used in this research paper, as it is the risk insurance companies have to face in insurances with death cover.

Table 1. Actuary risks

Market risk	**Observed in both areas, life and non-life**
Liquidity risk	Observed in both areas, life and non-life
Credit risk	Observed in both areas, life and non-life
Operational risk	Observed in both areas, life and non-life
Decrease in portfolio: - **Rescue** - **Reduction**	Specific for life insurance
Biometric risk: - **Mortality** - **Longevity**	Specific for life insurance

Source: Prepared by the author on the basis of Sandell, R (2003); Vegas Asensio, J (2000).

In order to implement an efficient risk management policy it will be necessary to previously quantify the risk using a tool for risk measurement. Therefore the next step is to define what a risk measure is. It is a functional (a function of functions) that assigns to a certain risk X a real non-negative number M(X) representing the additional amount that has to be added to the X loss in order to be acceptable by the insurance company (Gómez Déniz, E. Sarabia, JM.(2008)).

Thus, it will be rated (using a premium calculation principle) based on a risk measure as it adapts to the definition provided for this last one, as the premium assigns a real number to a random variable, which in the life insurance area represents the updated value of the product in question (for example, death insurance).

By definition, a premium calculation principle is a function H(X) assigning a real number to a risk X. Such real number is the premium. In practice, the premium calculation principle will depend on the distribution function F(X) which follows the random variable X, so instead of talking of a function H(X) we should talk about the functional H[F(X)] (Gerber, H. (1979)).

2. Coherent axiom for a premium calculation principle and its verification

The coherency criterion refers to the criterion granting economically rational contributions to the risk. Such coherency criterion has to provide correct information on financial assets, allowing its appropriate management (Tasche, D. (2000)).

Such coherency criterion is related to the fulfillment of four properties in a way that any premium calculation principle meeting such properties will be considered appropriate and optimal for a correct risk management, as it will perform an efficient allocation of the premium to the risk random variable (Artzner, P. Delbaen, F (1999); Landsman, Z. Sherris, M. (2001); Dhaene, J. Laeven, R. (2008)).

Table 2. Properties of a coherent risk measure

Positive coherency	M(aX)=aM(X). a≥0.
Invariance to translations	**M(X+a)=M(X)+a.**
Monotony	**Sea X1(w) and X2(w), with w ЄΩ, and where X1(w) ≤ X2(w), then M(X1)≤M(X2)**
Sub-additive	**M(X1+X2)≤M(X1)+M(X2)**

Source: Prepared by the author on the basis of Artzner, P. Delbaen, F (1999); Landsman, Z. Sherris, M. (2001); Dhaene, J. Laeven, R.Vanduffel, S. (2008).

Below we will mathematically develop for each and all of the existing premium calculation principles, the fulfillment of the four properties necessary to meet the coherency criterion. H(X) is the premium calculation principle.

2.1. Principle of the expected value and its particular case: the net premium principle

$H(X)=(1+\theta)E[X] \quad \theta>0$, where θ is the surcharge factor.

This premium calculation principle shows a premium with an explicit surcharge.

(i) Sub-additive property:

Given any two risks X_1 and X_2:

$$H(X_1+X_2)=H(X_1)+H(X_2)$$

$$H(X_1+X_2)=\left[(1+\theta)E(X_1)+(1+\theta)E(X_2)\right]=(1+\theta)\left[E(X_1)+E(X_2)\right]=$$

$$=(1+\theta)E(X_1)+(1+\theta)E(X_2)=H(X_1)+H(X_2)$$

Therefore this property is met.

(ii) Property of positive coherency:

Given a parameter $c\geq 0$ and a variable Y:

$$Y=cX$$

$$H(Y)=H(cX)=(1+\theta)E(cX)=c(1+\theta)E(X)=cH(X)$$

Therefore this property is met.

(iii) Property of monotony

Given any two risks X_1 and X_2, verifying that $X_1\geq X_2$ and for $\theta>0$:

$$E\left[X_1\right]\geq E\left[X_2\right]$$

$$H(X_1)=(1+\theta)E\left[X_1\right]\geq(1+\theta)E\left[X_2\right]=H(X_2)$$

$$H(X_1)\geq H(X_2)$$

Therefore the monotony property is met.

(iv) Property of invariance to translations

Given a parameter $c\geq 0$ and a variable $Y=c+X$:

Verifying that $H(Y)=H(X+c)=H(X)+c$

$$H(Y)=(1+\theta)E(c+X)=(1+\theta)\left[c+E(X)\right]=$$

$$=(1+\theta)c+(1+\theta)E(X)=(1+\theta)c+H(X)$$

As observed, it does not meet the property above mentioned.

This principle of the expected value does not meet the four desirable properties to be considered a coherency risk measure.

$\theta=0$: *Principle of net premium*

$$H(X)=(1+\theta)E\left[X\right]$$

$$\theta=0$$

$$H(X)=E(X)$$

(i) Property of sub-additive

Given any two risks X_1 and X_2:

$$H(X_1+X_2)=E(X_1+X_2)=E(X_1)+E(X_2)=H(X_1)+H(X_2)$$

Therefore this property is met as the expectancy of the addition is the addition of expectancies.

(ii) Property of Positive coherency

Given a parameter $c\geq 0$ and a variable Y:

$Y = cX;$

$H(Y) = H(cX) = E(cX) = cE(X)$

Therefore the property is met.

(iii) Property of Monotony

Given any two risks X_1 and X_2, verifying that $X_1 \geq X_2$:

$E[X_1] \geq E[X_2]$

$H(X_1) = E[X_1] \geq E[X_2] = H(X_2)$

$H(X_1) \geq H(X_2)$

Therefore this property is met.

(iv) Property of Invariance to translations.

Given a parameter $c \geq 0$ and a variable $Y = c + X$:

$H(X + c) = E(X + c) = E(X) + c = H(X) + c$

The property abovementioned is met.

This net premium principle meets the four properties desirable to be considered a coherent risk measure.

2.2. Variance premium principle

$H(X) = E[X] + \alpha V[X]$ $\alpha \geq 0$ where α is the surcharge factor and $V[X]$ is the variance. This risk measure incorporates the safety surcharge factor in order to face random deviations of the random variable loses or loss rates. In this premium expression, the surcharge factor is proportional to the variance and shows an explicit surcharge of the premium.

(i) Property of sub-additive

Given any two risks X_1 and X_2:

$H(X+Y) = E[X+Y] + \alpha V[X+Y] \nleq E[X] + E[Y] + \alpha [V[X] + V[Y] + 2Cov(X;Y)]$

It does not meet the sub-additive principle unless both variables are independent.

(ii) Property of positive coherency

Given a parameter $c \geq 0$ and a variable Y:

$Y = cX;$

$H(Y) = E[Y] + \alpha Var[Y] = E[cX] + \alpha V[cX] =$

$= cE[X] + \alpha c^2 V[X] = c[E[X] + \alpha c V[X]] \neq cH(X)$

It does not meet the positive coherency principle.

(iii) Property of monotony

Given any two risks X_1 and X_2, verifying that $X_1 \geq X_2$:

$E[X_1] \geq E[X_2]$

$H(X_1) = E[X_1] + \alpha V[X_1] \ngeq E[X_2] + \alpha V[X_2]$

In this case it is not necessary to meet that the premium calculation principle of the first risk has to be higher or equal to the premium calculation principle of the second risk. Thus, it does not meet this property.

(iv) Property of invariance of translations.

Given a parameter $c \geq 0$ and the variable $Y = c + X$:

$$H(X+c) = E(X+c) + \alpha Var(X+c) = c + E(X) + \alpha Var(X) = c + H(X)$$

Therefore the principle of invariance to translations is met.

This premium calculation principle does not meet the four properties to be considered as a coherent risk measure.

2.3. Exponential premium

$H(X) = \frac{1}{\alpha} LogE\left[e^{\alpha X}\right] \quad \alpha \rangle 0$, where α is the so called Arrow-Pratt measure or absolute risk aversion associated to the person, showing an explicit surcharge of the premium.

(i) Property of invariance to translations

Given a parameter $c \geq 0$ and a variable $Y = c + X$:

$$H\left[x+c\right] = H\left[x\right] + c$$

$$H\left[X+c\right] = \frac{LogE\left[e^{\alpha(x+c)}\right]}{\alpha} = \frac{LogE\left[e^{\alpha x}e^{\alpha c}\right]}{\alpha} = \frac{Log\left[E\left(e^{\alpha x}\right)e^{\alpha c}\right]}{\alpha} =$$

$$= \frac{LogE\left[e^{\alpha x}\right] + Loge^{\alpha c}}{\alpha} = \frac{LogE\left[e^{\alpha x}\right]}{\alpha} + \frac{\alpha c}{\alpha} = H\left(x\right) + c$$

Therefore the invariance property to translations is met.

(ii) Property of positive coherency

Given a parameter $c \geq 0$ and a variable Y :

$$H(X) = \frac{LogE\left[e^{\alpha X}\right]}{\alpha}$$

$$H(Y) = H(cX) = cH(x)$$

$$H\left[cX\right] = \frac{LogE\left[e^{\alpha cX}\right]}{\alpha} \neq cH\left[X\right]$$

Therefore it does not meet the positive coherency property.

(ii) Property of Monotony

Given any two risks X_1 and X_2, verifying that $X_1 \leq X_2$:

$$H\left[X_1\right] = \frac{LogE\left[e^{\alpha(X_1+c)}\right]}{\alpha} = \frac{LogE\left[e^{\alpha X_1+\alpha c}\right]}{\alpha}$$

$$H\left[X_2\right] = \frac{LogE\left[e^{\alpha(X_2+c)}\right]}{\alpha} = \frac{LogE\left[e^{\alpha X_2+\alpha c}\right]}{\alpha}$$

$$H\left[X_1\right] = \frac{LogE\left[e^{\alpha(X_1+c)}\right]}{\alpha} \leq H\left[X_2\right] = \frac{LogE\left[e^{\alpha(X_2+c)}\right]}{\alpha}$$

Therefore this principle of monotony is met.

(iii) Property of sub-additive.

Given any two risks X_1 and X_2:

$$H(X_1)=\frac{LogE\left[e^{\alpha X_1}\right]}{\alpha};$$

$$H(X_2)=\frac{LogE\left[e^{\alpha X_2}\right]}{\alpha};$$

$$H(X_1+X_2)=\frac{LogE\left[e^{\alpha(X_1+X_2)}\right]}{\alpha}=\frac{LogE\left[e^{\alpha X_1}e^{\alpha X_2}\right]}{\alpha}\nleq\frac{LogE\left[e^{\alpha X_1}\right]}{\alpha}+\frac{LogE\left[e^{\alpha X_2}\right]}{\alpha}$$

The principle of sub-additive is met only when both risks analysed are independent (Gómez Déniz, E. Sarabia, JM (2008)).

This principle of net premium does not meet the four properties desirable to be considered a coherent risk measure.

2.4. Esscher premium principle

$$H(X)=\frac{E\left[Xe^{\alpha X}\right]}{E\left[e^{\alpha X}\right]} \quad \alpha\rangle 0$$

This principle shows an explicit surcharge of the premium.

(i) Property of invariance to translations

Given a parameter $c\geq 0$ and a variable $Y=c+X$:

$$H(X+c)=\frac{E\left[(X+c)e^{\alpha(X+c)}\right]}{E\left[e^{\alpha(X+c}\right]}=\frac{E\left[(X+c)e^{\alpha(X+c)}\right]}{E\left[e^{\alpha X}e^{\alpha c}\right]}=\frac{E\left[Xe^{\alpha(X+c)}+ce^{\alpha(X+c)}\right]}{E\left[e^{\alpha X}\right]\left[e^{\alpha c}\right]}=$$

$$=\frac{E\left[Xe^{\alpha X}e^{\alpha c}\right]+cE\left[e^{\alpha X}e^{\alpha c}\right]}{e^{\alpha c}E\left[e^{\alpha X}\right]}=\frac{e^{\alpha c}E\left[Xe^{\alpha X}\right]+ce^{\alpha c}E\left[e^{\alpha X}\right]}{e^{\alpha c}E\left[e^{\alpha X}\right]}=$$

$$=\frac{e^{\alpha c}E\left[Xe^{\alpha X}\right]}{e^{\alpha c}E\left[e^{\alpha X}\right]}+\frac{ce^{\alpha c}E\left[e^{\alpha X}\right]}{e^{\alpha c}E\left[e^{\alpha X}\right]}=H(X)+c.$$

Therefore it meets the invariance property to translations.

(ii) Property of positive coherency

Given a parameter $c\geq 0$ and a variable $Y=cX$:

$$H(Xc)\neq cH(X)$$

$$\frac{E\left[cXe^{\alpha Xc}\right]}{e^{\alpha Xc}}\neq c\frac{E\left[Xe^{\alpha X}\right]}{e^{\alpha X}}$$

Therefore it does not meet the positive coherency principle.

(iii) Property of Monotony

Given any two risks X_1 and X_2, verifying that $X_1\leq X_2$:

$$H(X_1) \le H(X_2)$$

$$H(X_1) = \frac{E\left[(X_1 e^{\alpha X_1})\right]}{E\left[e^{\alpha X_1}\right]} \not\le H(X_2) = \frac{E\left[(X_2 e^{\alpha X_2})\right]}{E\left[e^{\alpha X_2}\right]}$$

Where the first risk is lower or equal to the second risk, this property is not always met.

(iv) Property of sub-additive

Given any two risks X_1 and X_2:

$$H(X_1+X_2) \le H(X_1)+H(X_2)$$

$$\frac{E\left[(X_1+X_2)\,e^{\alpha(X_1+X_2)}\right]}{E\left[e^{\alpha(X_1+X_2)}\right]} = \frac{E\left[(X_1\,e^{\alpha X_1}\,e^{\alpha X_2})+(X_2\,e^{\alpha X_1}\,e^{\alpha X_2})\right]}{E\left[e^{\alpha X_1}\,e^{\alpha X_2}\right]} \not\le \frac{E\left[(X_1\,e^{\alpha X_1})\right]}{E\left[e^{\alpha X_1}\right]} + \frac{E\left[(X_2\,e^{\alpha X_2})\right]}{E\left[e^{\alpha X_2}\right]}$$

In this case the sub-additive property is not met.

This principle does not meet the four properties desirable to be considered as a coherent risk measure.

2.5. Wang distortion functional principle

$$H(X) = \int_0^\infty g\left(S_X(x)\right)dx = \int_0^\infty \left(S_X(x)\right)^{\frac{1}{\rho}} dx$$

This expression is the so-called Proportional Hazards Premiums Principle. Therefore, the distortion functional "g" is a tool used to build risk measures.

In cases where parameter ρ is valued as 1, the particular case of the risk measure takes place based on the principle of the net premium explained above.

It is worth highlighting that Wang, S (1995) demonstrated the four properties and therefore such demonstration is not going to be repeated. Regarding the last property, sub-additive, Wang, S (1995) demonstration for the case $\rho \ge 1$ is quite interesting. For $\rho \ge 1$ values, the premium calculation principle based on Wang distortion functional represents a coherent risk measure. And for the values of parameter $\rho \le 1$ it also represents a coherent risk measure being the sub-additive property demonstrated by Hernández, M (2013). This principle is considered valid to be applied to within the life insurance scope as it verifies the coherence properties.

3. Application of the Net Premium Principle and the Principle of the Distortion Functional to Obtain the Single Risk Premium in a Death Insurance

The single risk premium is going to be expressed for insurances with death cover, the whole life insurance, on the grounds of the principles for the calculation of premiums considered as coherent risk measures (explained in section 2). In this insurance, the insurer undertakes to pay to the beneficiary of the policy the amount insured upon the death of the holder (Bowers, JR. Newton, L. Gerber, H. Jones, D. (1997)). In order to be entitled to the amount agreed, the holder will have to pay to the insurance company the amounts of the premiums, either on a periodic basis or by means of a single premium upon the date of the subscription of the insurance contract. In this case the random variable is the variable of residual life or the time to life from age x, T_x .

3.1. Calculation of the single risk premium with the net premium principle

The general expression of the premium for this type of insurance is (Bowers, JR. Newton, L. Gerber, H. Jones, D. (1997)).):

$$P = \int_0^{\infty} v^t d\left(G_x(t)\right)dt \, ,$$

$$_t p_x = P\left(X > x + t / X > x\right) = 1 - G_x(t) = S_x(t)$$

$$P = \int_0^{\infty} v^t d(1 - S_x(t))dt = -\int_0^{\infty} v^t dS_x(t)$$

The following table shows single risk premiums for the law on survival used in this article. Their mathematical development can be seen in the doctoral dissertation of the author (Hernández, M. (2013)).

Table 3. Single risk premium by application of the Net premium principle

Single risk premium	Single risk premium. Makeham Law
$P = 1 - \int_0^1 S_x\left(\frac{\text{Ln}\, z}{\text{Ln}\, v}\right)dz$	$P = \frac{g^{C^x}\left(\text{LnS} + \text{Lnv} + C^{x+1}\text{Lng}\right) - \text{Lnv}}{g^{C^x}\left(\text{LnS} + \text{Lnv} + C^{x+1}\text{Lng}\right)}$

Source: Prepared by the author on the basis of the doctoral dissertation of the author.

3.2 Calculation of the single premium with the distortion functional principle

The distortion functional transforms the survival function through the operator g, based on the expression of the single risk premium calculated in section 3.1. It is expressed as power, being ρ parameter considered as the parameter of risk aversion (Tse, Y-K (2009)).

$$P = \int_0^{\infty} g\left(S_X(x)\right)dx = \int_0^{\infty} g\left(S_X(x)\right)^{\frac{1}{\rho}} dx$$

The following table shows the single risk premiums for each of the survival laws used in this article. Their mathematical development can be studies in the doctoral dissertation of the author (Hernández, M. (2013)).

Table 4. Single risk premium surcharged by application of the distortion functional principle

Single risk premium	Single risk premium. Makeham Law
$P_{rec} = 1 - \int_0^1 S_x\left(\frac{\text{Lnz}}{\text{Lnv}}\right)^{\frac{1}{\rho}} dz$	$P_{rec} = \frac{g^{C^x \frac{1}{\rho}}\left(\frac{1}{\rho}\text{LnS} + C^{x+1}\frac{1}{\rho}\text{Lng} + \text{Lnv}\right) - \text{Lnv}}{g^{C^x \frac{1}{\rho}}\left(\frac{1}{\rho}\text{LnS} + C^{x+1}\frac{1}{\rho}\text{Lng} + \text{Lnv}\right)}$

Source: Prepared by the author on the basis of the doctoral dissertation of the author

4. Conclusions

This article has analysed the main premium calculation principles within the actuarial area. Such analysis has consisted on the mathematical development, for each and all of them, of the properties defining coherence. Among all premium calculation principles chosen -the net premium principle, the expected value principle, the variance principle, the exponential premium principle, the Esscher premium principle as well as the distortion functional principle- only two of them verify the coherence axiom. The net premium principle is a particular case of the expected value and therefore represents a coherence risk measure. The disadvantage is that it provides a premium free of surcharges and therefore insurance companies have to work with outdated death tables to use death risk under the rates of the human group considered in such tables. In turn, and this is the main contribution of this research work, the principle of the distortion function has been applied to date within the scope of general insurances. In this study, it is applied for the first time to the calculation of the single risk premium in death insurances for the life insurance scope (Hernández, M (2013)). This is a principle that represents a coherent risk measure for parameter values $\rho \leq 1$, which are the values that have to be verified in the type of insurance used in order to obtain a premium higher than the net premium (that obtained by the first of the principles).

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[1] **Hernández Solís Montserrat**: PhD. in Economic Sciences and Business Studies specialized in actuarial models by Universidad Complutense de Madrid, Spain. Full-time assistant of the Business Economics and Accounting Department of the Faculty of Economics and Business Studies of the Universidad Nacional de Educación a Distancia (UNED).

GEOSTRATEGIC RESOURCES – NON-FERROUS METALS – REQUIRE AN APPROPRIATE MANAGEMENT

Gheorghe VLASCEANU

PhD, University Professor, Bucharest University of Economic Studies

E-mail: gheorghevlasceanu@rei.ase.ro

Abstract:

Natural resources, in general, hold a geostrategic connotation due to their importance. The geostrategic interest mainly targets oil, natural gases and non-ferrous ore deposits. These resources can be found in Romania's subsoil in various quantities, and their exploitation and processing started early on. Non-ferrous deposits stand out as a particular case, which also explains the start of this industry in Romania simultaneously with those around the world.

Key words: strategic resources; geopolitical; non-ferrous metals; resource management

The management of geostrategic resources includes radioactive ores, especially uranium, the importance of which does not mainly reside in its use as fuel for nuclear power plants, but in its use in the production of nuclear bombs. Given this military priority, detailed information regarding the geographical distribution, nature and size of the deposits falls under the law of national safety.

Radioactive deposits are not an exception to the rule of subsoil resources in Romania, in the sense that they are not very large and their distribution is spread across the Carpathian regions: Bihor, Almaj, Bistritei, Giurgeu and others. With the exception of some deposits in the Banat and Apuseni Mountains, known since before the Second World War and mercilessly exploited until 1955 ("to be processed by the ex-USSR"), all the others began to be exploited only after 1965.

The geopolitical context after 1990, with the fall of the Iron Curtain, reconfigured both the strategic role and the management of non-ferrous resources in the Eastern European space [1].

Romania's intentions to produce electricity through nuclear technology materialized when the nuclear power plant at Cernavodă started to be put to use in 1996.

After 1990, there were no foreign companies in Romania that wanted to exploit radioactive deposits. The interest that foreign investors and companies have for oil is not equaled by radioactive, gold or silver ores [3].

Gold has been and tends to remain one of the attractive resources in these parts. The Greek historian Herodotus mentioned the gold in the Apuseni Mountains for the first time when he refers to the war between the Persians led by King Darius and the Scythians in 513 B.C., in the region of the Maris (Mures) river. Presumably, Darius was drawn by the gold riches of the Scythian Agatârș tribe.

Some historians claim that the gold of the Dacians (estimated at over 150 000 kg) was one of the main reasons of the Roman invasion of Dacia. After 106 B.C., gold exploitation intensifies, the Romans bring specialized miners, especially from Dalmatia. Among the more important mining centers, all of them found in the Apuseni Mountains, are: Roșia Montana, Bucium – Alburnus Major and Zlatna – Ampelum [8]. Given the importance of the deposits, exploitation was initially managed by the officers of the 13th Gemina Legion, and then by a "procurator aurarium", mining specialist, who resided in Zlatna (Ampelum) and by the general procurator of the province, who resided at Alba Iulia (Apulum).

The Roman retreat and the invasions of the migratory peoples slowed the systematical exploitation down for a time, without stopping it permanently. It was continued on a reduced scale by the locals.

Over the centuries, the Hungarian Arpadian kings brought Saxon colonists specialized in mining into the region. It is worth pointing out that, through the strengthening of the Catholic Church, most of the mines in the Apuseni Mountains became the property of the roman-catholic bishop at Alba Iulia [4].

With the end of the First World War, the Romanian administration regains its rights. As such, in 1929 the "Mica" Company builds the first gold flotation plant at Brad, followed by the one in Dealul Crucii. The direct involvement of the state and the financial policy of the National Bank encouraged the modernization of exploitations and an increase in production: 730 kg of fine gold in 1919, 1000 kg in 1921, 1300 in 1925 and in 1937 – 5355 kg, a number representing the maximum quantity extracted before the Second World War.

The main gold producers before the War were: the "Mica" Society, producing 55% of the Romanian gold at an exploitation complex in the Apuseni Mountains (the Musariu, Bradisor, Valea-Morii mines and the processing plant at Gura Barza) and owning a total surface of 5375 acres; the "RIMMA" society – "Autonomous Administration of Mining and Metallurgical Enterprises in Ardeal", managing the state mines, the second most important gold producer (21.9%) and owning several areas: Valea Roșie (2557.7 acres) in the Baia Mare region, Dealul Crucii (2500 acres) in the same region, Baia Sprie (387.5 acres), Baiut-Văratec (662.5 acres) and Rsoia Montana in the Apuseni Mountains (1115 acres), Sacaramb (6632.5 acres) and Valea lui Stan (1500 acres), to which we should add the processing plants at Firiza and Zlatna. The "Aurum" company, the third gold producer with a production of 6.5%, had 180 acres leased in the Baia Mare region. Among these big producers were societies like "Petrosani", "Industria aurului", "Toplita-Magura-Concordia", "Albini" and small producers, usually forming an association, who were mostly working in the Apuseni Mountains and very few in the Baia Mare region. The only company with foreign capital was "The French Gold Mining Society in Transilvania", that came into being after 1930, owning 1982.5 acres in lease in Baița and Nistru (Baia Mare) and producing 166 kg of gold and 483 kg of silver during 1937.

The silver producers were the same as the gold producers, with an exception: the "Phonix" company, processing the argentiferous lead at Herja. Until 1933, obtained silver was left at the producers' disposition. After this date, it was bought by the National Bank. In the case of silver, taking 1937 again as a reference point, total production had reached 25,645 kg, of which "Phonix" had produced 9184 kg and "RIMMA" 7430 kg.

The year 1940 temporary brings a Hungarian management of the gold mines, which will operate during the beginning of the Second World War. Data regarding the occupation period is confusing and difficult to confirm: the end of the war and the nationalization in 1948 change the property of the mines, seeing them pass in the Romanian state's possession [5].

The management of the gold and silver deposits was done until 1990 without showing great leaps in production or the discovery of new resources.

The interest for Romanian gold was felt after 1995, when the Deva Autonomous Administration of Copper and the Canadian company "Gabriel Resources Ltd." started negotiations regarding the discovery, exploitation, extraction and processing of gold and silver deposits at Roșia Montană and the Bucium Complex in the Apuseni Mountains. In 1997, the "Euro Gold Resources", a Romanian-Canadian company, was founded. The feasibility study showed that Roșia Montană had the largest gold deposit in Europe – 8 million ounces of gold (approximately 300 tons of gold and 1600 tons of silver at a present value of over $3 billion, the exploitation taking place over a period of 11 years). Afterwards, the company changed its name to the "Roșia Montană Gold Corporation", with the following stockholders: "Gabriel Resources Ltd." (holding 80%), "Minivest Deva" (18.5%), Cartel Bau SA Cluj (0.5%), Commat SA Trading Bistrița (0.5%) and Foricon SA Deva (0.5%).

In order to capitalize on the deposit, management aims to uncover large areas and move the Roșia Montană village (population of 4125) altogether, along with the two churches buit in the 16th century and declared architectural monuments. The excavation of the over 200,000 tons of ore (with a content of 1.9 grams of gold / ton) would upset the balance of the ecosystem (even though the company commits itself to rebuilding the landscape), which is the reason why at the present time the project is still full of controversy in the eyes of various environmental NGOs, including Greenpeace.

On the same note, other companies are interested in the extraction of gold and silver from the sterile dumps.

The Romanian state funds gold exploitation with approximately 35 million RON yearly, exploitation which at the current moment is the exclusive work of the state company "Minivest Deva"; the Romanian-Canadian "Roșia Montană Gold Corporation" is only prospecting and exploring.

The Romanian state spends $32,000 for the production of one kilogram of gold (including subventions); the stock market values gold at $12,500, so the losses are very big.

On a worldwide scale, countries in South America are important gold producers along with China, Australia, South Africa, USA and Russia. India, Japan, Turkey are great gold consumers.

Romania's membership in NATO and the EU has caused radical changes in the field of the country's geostrategic resources. The reasons are most importantly linked to reserves, which are usually small and any other future discoveries cannot be spectacular. Romania enters the aforementioned organizations as a growing consumer and importer, not by any means as a supplier of geostrategic resources.

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