ESTIMATING THE PROJECT'S UTILITY BY MEANS OF HARMONIZATION THEORY

Avner BEN-YAIR[1,2]
PhD, Department of Industrial Engineering and Management,
Sami Shamoon College of Engineering, P.O.Box 950, 84100, Beer-Sheva, Israel

E-mail: avnerb@sce.ac.il

Dimitri GOLENKO-GINZBURG[3]
PhD, Department of Industrial Engineering and Management (Professor Emeritus),
Ben-Gurion University of the Negev, P.O.Box 653, 84105, Beer-Sheva, Israel
& Department of Industrial Engineering and Management,
Ariel University Center in Samaria, Ariel, 44837, Israel

E-mail: dimitri@bgu.ac.il

Doron GREENBERG[4]
PhD, Department of Economics and Business Administration,
Faculty of Social Science, Ariel University Center in Samaria, Ariel, 44837, Israel

E-mail: dorongreen@gmail.com

Zohar LASLO[5]
PhD, Department of Industrial Engineering and Management,
Sami Shamoon College of Engineering, P.O.Box 950, 84100, Beer-Sheva, Israel

E-mail: zohar@sce.ac.il

Abstract: *This paper is a generalization of our previous publications [2,19-20] where some newly developed models of the harmonization theory, together with various practical applications, have been outlined. The goal of the paper under consideration is to describe in depth both the general concepts of the harmonization theory and the fitness of the latter by citing an example from a widely used class of stochastic network projects (PERT-COST models).*

Harmonization theory is a multi-parametric optimization model in order to maximize the system's utility as a generalized quality measure of the system's functioning. We have implemented all the principles of the developed theory in a PERT-COST project and have outlined all the mechanisms to evaluate the project's utility. A numerical example is presented.

Key words: *System's utility; Multi-parametrical harmonization theory; PERT-COST network projects with random activity durations; Independent and dependent basic parameters; Two-level optimization algorithm; Partial harmonization model*

1. Introduction

In recent years problems associated with developing various quality concepts have been discussed extensively in scientific literature. However, numerous publications refer mostly to quality control, which is usually applied to products and services. As a matter of fact, the existing quality techniques, including the developed utility theory [13-15, 21][6], are not applicable to technical and organization systems, which are actually supervising and monitoring the process of the systems' functioning: all those models are restricted to solving market competitive problems alone. Thus, nowadays, the existing utility theory centers on analyzing the competitive quality of organization systems' outcome products rather than dealing with the quality of the systems' functioning, i.e., with organization systems in their entirety. This may result in heavy financial losses, e.g. when excellent project objectives are achieved by a badly organized project's realization.

Thus, a conclusion can be drawn that the existing utility theory cannot be used as the system's quality techniques. In order to fill in the gap, we have undertaken research in the area of estimating the quality of the system itself, e.g. the system's public utility. We will consider a complicated organization system which functions under random disturbances. Such a system usually comprises a variety of qualitative and quantitative attributes, characteristics and parameters, which enable the system's functioning. The problem arises to determine a generalized (usually quantitative) value which covers all essential system's parameters and can be regarded as a system's qualitative estimate. We will henceforth call such a generalized value the system's utility.

The backbone of this paper is:

- *to formalize the multi-parametric harmonization model in order to maximize the system's utility as a generalized quality measure of the system's functioning, and*
- *to develop the techniques of the harmonization problem to estimate the stochastic network project's utility.*

To develop the corresponding techniques we suggest to take into account the basic parameters, which actually form the utility of the system - validity, reliability, flexibility, cost, sensitivity, forecasting (timeliness), etc. We suggest subdividing the basic parameters into two sub-sets:

- independent parameters, where for each parameter its value may be preset and may vary independently on other parameters' values, and
- dependent parameters whose values may not depend uniquely on the values of independent parameters.

We suggest a multi-stage solution of harmonization problems. At the first stage a look-over algorithm to examine all feasible combinations of independent basic values, is implemented. The independent parameters' values obtained at that stage are used as input values at the second stage where for each dependent parameter a local subsidiary optimization problem is solved in order to raise the system's utility as much as possible. Solving such a problem enables the solely dependence of the optimized value on any combination of independent input parameters. At the next stage the system's utility value is calculated by means of basic parameters' values obtained at the previous stages, with subsequent search for the extremum in order to determine the optimal combination of all basic parameters' values delivering the maximum to the system's utility.

The structure of the paper is as follows. In *Section 2* the general concepts and definition of the harmonization theory are presented, while *Section 3* considers the harmonization's model optimality. *Section 4* presents the PERT-COST project's description. In *Section 5* harmonization model for PERT-COST projects is formulated, while *Sections 6* and *7* provide both the model's heuristic solution and the partial harmonization model. In *Section 8* a step-wise procedure to control a PERT-COST network project by means of harmonization is presented. *Section 9* presents applications in a real design office while in *Section 10* conclusions are outlined. In the Appendix, terms to be used in the paper, are presented.

2. General Concepts and Definitions

Consider a complicated organization system which functions under random disturbances. Such a system usually comprises a variety of qualitative and quantitative attributes, characteristics and parameters, which enable the system's functioning. The problem arises to determine a generalized (usually quantitative) value which covers all essential system's parameters and can be regarded as a system's qualitative estimate, namely, the system's utility.

We will require some new definitions.

Definitions

I. Call the system's model M_s a formalized description of the system's structure as well as the system's functioning. M_s usually comprises the logical links between the system's elements, decision-making rules, various random parameters, etc. For project management systems various M_s may be used, e.g. PERT-COST models [6, 9-12], GANTT chart models [21], CPM models [4, 6], GERT models [6], etc. PERT-COST network models which are widely used in project management [11-12], are used as M_s in our paper. Such a network model is actually a graph type simulation model comprising activities with random durations. The p.d.f. of each activity duration depends parametrically on the budget value assigned to that activity.

M_s usually comprises all the basic parameters (see below) which have an influence on the system's utility.

II. Call a quantitative parameter entering the system a basic parameter on condition that changes in the parameter result in changing the system's utility. Note that the restriction value for any basic parameter is, actually, the worst permissible value that may be implemented into the system. The set of basic parameters, together with the corresponding restriction values, are externally pregiven.

III. Call the system's utility which corresponds to the pregiven restriction values for all basic system's parameters, the basic utility. Denote henceforth the basic utility by U_0. Value U_0 is externally pregiven as well.

IV. Call the direction of changing a basic parameter's value which results in increasing the system's utility, a positive direction, and vice versa. Call the change of the system's utility caused by altering a parameter by its unit value in the positive direction, a local parametric utility. Denote henceforth the additional local parametric utility for the k-th basic parameter by $\alpha_k > 0$. Parametric utility values are also pregiven externally.

Denote henceforth the pregiven restriction values for each basic k-th parameter R_k, $1 \le k \le n$, by R_{k0}, correspondingly.

V. Note that to solve the harmonization problem, we need to define for each k-th basic parameter its best values which by no means can be refined. Denote those values which are externally pregiven, by R_{k00}, correspondingly.

VI. Call the basic n_1 system's parameters which can be pregiven independently from each other, independent basic parameters.

VII. Call other $n_2 = n - n_1$ basic system's parameters dependent basic parameters. Thus, the basic parameters can be subdivided into two groups: independent and dependent parameters. The latter do not depend uniquely on the preset values of independent parameters. Moreover, a combination of independent parameters may correspond to numerous different values (sometimes to an infinite number) of a certain dependent parameter. If, for example, a PERT-COST network project is carried out under random disturbances, setting the cost value (assigned for the project) and the time value (in the form of the project's due date) does not define solely the value of the project's reliability, i.e., its probability to meet the deadline on time. This is because the budget value C assigned to the project has to be reallocated beforehand among the project activities in order to start processing the latter. Each budget reallocation results in a certain project's reliability and, thus, different feasible (but non-optimal!) reallocations correspond to different non-optimal reliability values. However, for the same preset independent basic parameters - cost and time values - it is possible to maximize the project's reliability by means of optimal budget reallocation among the project's activities.

Thus, we suggest to implement a solely dependency of each dependent basic parameter on the combination of independent input values by means of a subsidiary optimization procedure (heuristic, simulative, approximate) in order to maximize the system's utility for the fixed combination of independent parameters and the optimized dependent parameter.

VIII. Call a partial harmonization problem PHM_j an optimization problem (analytic, simulative, heuristic) which on the basis of preset independent basic parameters delivers an optimum value to a dependent basic parameter R_j in order to maximize the conditional system's utility. Thus, a PHM enables the solely dependence of a dependent parameter from independent ones.

IX. We suggest to calculate the system's utility by

$$U = \sum_{i=1}^{n_1} \alpha_i^{(ind)} \cdot R_i^{(ind)} + \sum_{j=1}^{n_2} \beta_j^{(dep)} \cdot R_j^{(dep)}, \qquad 1 \le i \le n_1, \tag{1}$$

$1 \le j \le n_2 = n - n_1$,

where

$R_1^{(ind)},\dots, R_{n_1}^{(ind)}$ - independent basic parameters;

$R_1^{(dep)},\dots, R_{n_2}^{(dep)}$ - dependent basic parameters.

Denoting by $PHM_j\left\{\vec{R}_i^{(ind)}\right\} = R_j^{(dep)}$, $1 \le j \le n_2$, a partial harmonization model, we finally obtain

$$U = \sum_{i=1}^{n_1} \alpha_i^{(ind)} \cdot R_i^{(ind)} + \sum_{j=1}^{n_2} \beta_j^{(dep)} \cdot PHM_j\left\{\vec{R}_i^{(ind)}\right\}. \tag{2}$$

Value U may comprise both analytic PHM_j as well as PHM_j based on simulative modeling. In some cases PHM_j can be based on subjective decision-making.

3. Harmonization's Model Optimality

The harmonization problem is as follows: determine optimal values R_k, $1 \le k \le n$, to maximize the system's utility

$$\max_{\{R_k\}} U = U_0 + \sum_{k=1}^{n} \alpha_k \cdot \left| R_k - R_{k0} \right| \tag{3}$$

subject to

$$Min\ \{R_{k0}, R_{k00}\} \le R_k \le Max\ \{R_{k0}, R_{k00}\}. \tag{4}$$

Since U_0 remains constant, the objective can be simplified as follows

$$\underset{\{R_k\}}{Max} \sum_{k=1}^{n} \{\alpha_k \cdot |R_k - R_{k0}|\} \tag{5}$$

subject to (4).

Problem (3-5) is a very complicated optimization problem which usually does not provide analytical estimates.

Let us analyze the general harmonization problem in greater detail. Since independent basic parameters $R_i^{(ind)}$ serve as input values which can be optimized by means of a search algorithm, the harmonization problem's solution suggests itself as a combination of two sequential problems:

- to determine an optimal combination of independent basic values $\{R_i^{(ind)(opt)}\}$ by means of a lookover algorithm that checks the feasibility of each possible combination (*Sub-Problem I*),
- to solve all the partial harmonization problems by means of $PHM_j\{R_i^{(ind)}\}$ (*Sub-Problem II*), and
- to facilitate a search for the extremum in order to maximize utility value (2).

Theorem

Optimal values $R_k^{(opt)}$, $1 \le k \le n$, in problem (4-5) satisfy

$$\{R_k^{(opt)}\} \equiv \{R_i^{(ind)(opt)}\} \cup PHM_j\{R_i^{(ind)(opt)}\}. \tag{6}$$

Proof

Assume that $\{R_k^{(opt)}\}$ dose not satisfy (6), i.e., there exists a combination

$$\{R_k'\} \equiv \{R_i^{(ind)'}\} \cup \{R_j^{(dep)'}\} \tag{7}$$

which satisfies (5) and does not coincide with (6). Note, first, that relation

$$\{R_j^{(dep)'}\} \equiv PHM_j\{R_i^{(ind)'}\} \tag{8}$$

holds, otherwise the combination $\left\{R_k'\right\}$ may be improved by substituting $R_j^{(dep)\,'}$ for $PHM_j\left\{R_i^{(ind)\,'}\right\}$. This, in turn, contradicts relation (5). Secondly, relation

$$\left\{R_i^{(ind)\,'}\right\} \equiv \left\{R_i^{(ind)(opt)}\right\} \tag{9}$$

holds as well, since values $\left\{R_i^{(ind)(opt)}\right\}$ have been obtained by means of an optimal lookover algorithm which checks all possible combinations $\left\{R_i^{(ind)}\right\}$, including $\left\{R_i^{(ind)\,'}\right\}$. Thus, our assumption proves to be false and combinations (6) and (7) fully coincide.

The proved theorem enables solution of problem (4-5) by means of a sequential solution of *Sub-Problems I* and *II*. However, if, due to the high number of possible combinations $\left\{R_i^{(ind)}\right\}$, solving both problems on a lookover basis requires a lot of computational time, we suggest a simplified heuristic algorithm as follows.

Since practically most partial harmonization models PHM_j (see, e.g. [1]) for organization systems are complicated non-linear functions of independent parameters $\left\{R_i^{(ind)}\right\}$, determining the optimal system's utility results in implementing the theory of unconstrained optimization for non-linear problems. As outlined in [18], the most effective and widely known methods for maximizing a non-linear function of several variables, e.g. the gradient method, the Newton's method, the conjugate direction method, etc., cannot be carried out without determining the gradient vector at each search step. However, solving the gradient equation for partial harmonization problems based on simulation models comprising stochastic programming constraints leads usually to futile computational efforts.

Thus, a conclusion can be drawn that more attractive and at the same time more realistic approximated algorithms have to be implemented. According to the general recommendations outlined in [18] we have replaced the precise lookover algorithm (*Sub-Problem I*) by the cyclic coordinate search algorithm (CCSA). The latter optimizes the non-linear function of independent parameters cyclically, with respect to coordinate variables. The cyclic coordinate algorithm has been applied in many Production and Project Management problems [1, 11-12, 19].

4. The PERT-COST Project's Description

A PERT-COST project $G(N, A)$ [1, 9, 11-12, 21] is characterized by the following parameters:

- the budget C assigned to the project which has to be redistributed among the project's activities;

- the due date D for the project to be accomplished;

- the project's reliability R, i.e., the probability of meeting its due date on time subject to the pre-assigned budget C.

It can be assumed that for each activity time duration its density function depends parametrically on the budget which is assigned to that activity.

A conclusion can be drawn from various studies in PERT-COST [3-7, 9, 12, 16-17, 23-25] that for most activities (i, j) entering the network model, their random time duration t_{ij} is close to be inversely proportional to the budget c_{ij} which is assigned to that activity. Thus, three different distributions may be considered:

- random activity durations are assumed to have a beta-distribution, with the probability density functions (p.d.f.) as follows:

$$p_{ij}(t) = \frac{12}{\left(b_{ij} - a_{ij}\right)^4}\left(t - a_{ij}\right)\left(b_{ij} - t\right)^2, \tag{10}$$

where $b_{ij} = \frac{B_{ij}}{c_{ij}}$ and $a_{ij} = \frac{A_{ij}}{c_{ij}}$, A_{ij} and B_{ij} being pregiven constants for each activity (i, j) entering the PERT-COST network model.

- random activity durations are assumed to be normally distributed with the p.d.f. $N\left(a, \sigma^2\right)$

$$p_{ij}(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{2(x-a)}{2\sigma^2}} dx, \tag{11}$$

where the mean value a and the variance σ^2 are calculated by

$$a = 0.5 \cdot \frac{A_{ij} + B_{ij}}{c_{ij}}, \qquad \sigma = \frac{B_{ij} - A_{ij}}{6c_{ij}}. \tag{12}$$

- random activity durations are assumed to be distributed uniformly in the interval $\left(\frac{A_{ij}}{c_{ij}}, \frac{B_{ij}}{c_{ij}}\right)$, with the p.d.f.

$$\frac{c_{ij}}{B_{ij} - A_{ij}} = \frac{1}{b_{ij} - a_{ij}} . \qquad (13)$$

In the problem under consideration all those cases will be examined.

The following restrictions will be implemented in the model:

- $C \leq C_0$, where C_0 is the maximal permissible budget to be assigned to the project;
- $D \leq D_0$, where D_0 is the maximal permissible due date to be accepted by the project management;
- $R \geq R_0$, where R_0 is the least permissible reliability of meeting the project's deadline on time, i.e., the minimal probability of accomplishing the project before its due date.

Besides those worst permissible pregiven values C_0, D_0 and R_0, one can define the best pregiven possible correspondent values - the minimal budget C_{00} to be assigned to the project, the earliest due date D_{00} (there is no need in accomplishing the project before D_{00}), and the maximal reliability value R_{00} (usually $R_{00} = 1$). It can be well-recognized that any project values C, D and R satisfy

$$\begin{cases} C_{00} \leq C \leq C_0 \\ D_{00} \leq D \leq D_0 \\ R_0 \leq R \leq R_{00} . \end{cases} \qquad (14)$$

5. Harmonization Model for PERT-COST Projects

Using general relations (1-5) and (14) for the case of a PERT-COST project, we suggest to evaluate the project's utility by

$$U = \alpha_C \cdot [C_0 - C] + \alpha_D \cdot [D_0 - D] + \alpha_R \cdot [R - R_0], \qquad (15)$$

where C_0, D_0 and R_0 are the least permissible budget, due date and reliability values which can be implemented in a PERT-COST project, while values C, D and R are the corresponding current values for a project under consideration. Linear coefficients α_C, α_D and α_R define additional partial utilities which the project obtains by refining its corresponding parameter by a unit's value. Note that parameters C and D are independent parameters since they can be preset beforehand independently on each other, while parameter R is practically defined by values D and C and, thus, is a dependent parameter.

The multi-parametrical harmonization model is as follows: determine optimal non-contradictive project parameters $C^{(opt)}$, $D^{(opt)}$ and $R^{(opt)}$ resulting in the maximal project's utility

$$\underset{\{C,D,R\}}{Max}\ U(G) = \underset{\{C,D,R\}}{Max}\ \{U_0 + \alpha_C(C_0 - C) + \alpha_D(D_0 - D) + \alpha_R(R - R_0)\} \qquad (16)$$

subject to

$$C_{00} \le C^{(opt)} \le C_0, \qquad (17)$$

$$D_{00} \le D^{(opt)} \le D_0, \qquad (18)$$

$$R_{00} \ge R^{(opt)} \ge R_0. \qquad (19)$$

Note that since the basic utility U_0 is a constant value which remains unchanged, it may be canceled and, thus, the harmonization model satisfies

$$\underset{\{C,D,R\}}{Max}\ U(G) = \underset{\{C,D,R\}}{Max}\ \{\alpha_C(C_0 - C) + \alpha_D(D_0 - D) + \alpha_R(R - R_0)\} \qquad (20)$$

subject to (17-19). Values C, D and R are called <u>non-contradictive</u> if budget C can be reassigned among the project activities to satisfy

$$Pr\left\{T(G)_{c_{ij}} \le D\right\} = R \qquad (21)$$

subject to

$$\sum_{(i,j)} c_{ij} = C. \qquad (22)$$

6. The Model's Solution

Solving problem (17-20) can be carried out by solving two sequential problems: to determine an optimal budget value C and an optimal due date D (*Sub-Problem 1*) and to carry out the PHM (*Sub-Problem 2*).

Sub-Problem 1 centers on determining an optimal couple $\left(C^{(opt)}, D^{(opt)}\right)$ by means of a look-over algorithm that checks the feasibility of each possible combination (C, D). If the number of combinations is high enough and taking into account that:

- each combination requires a PHM solution, and
- *Sub-Problem 1* is a NP-complete one [8, 22],

- solving both problems on a look-over basis requires a lot of computational time. To avoid this obstacle, we suggest a two-level high-speed approximate heuristic algorithm. At the upper level a heuristic simplified search procedure, e.g. a cyclic coordinate search algorithm (CCSA) [1, 18], has to be carried out in the two-dimensional space in order to determine an optimal combination (C, D). At the bottom level, a heuristic high-speed procedure to optimize the partial harmonization model $PHM/_{C,D}$ with independent input values C and D, has to be implemented. Thus, we substitute objective (20) by

$$\underset{C,D}{Max}\left\{CCSA\{C,D\} \cup PHM/_{C,D} \Rightarrow U(C,D,R)\right\}, \tag{23}$$

where $\cup$ stands for a unification sign.

7. Partial Harmonization Model

As outlined above, parameters C and D are input values of *PHM Problem 2* as well as values $c_{ij\ min}$, $c_{ij\ max}$, A_{ij} and B_{ij}, $(i,j) \subset G(N,A)$. The problem is as follows: determine optimal reassigned budget values c_{ij} for each activity $(i,j) \subset G(N,A)$, to maximize the project's conditional reliability, i.e.,

$$\underset{\{c_{ij}\},\ \sum_{(i,j)} c_{ij}=C}{Max}\left[\ \Pr\left\{T\left(G_{c_{ij}}\right) \le D\right\}\right] \tag{24}$$

subject to

$$C_{ij\ min} \le C_{ij} \le C_{ij\ max}, \tag{25}$$

$$\sum_{(i,j)\subset G(N,A)} c_{ij} = C. \tag{26}$$

Model (24-26) is outlined in several previous publications [1, 9, 11, 19-20]. The corresponding algorithm can be easily programmed on PC. Thus, the system's model (see

Section 2) for the case of a PERT-COST network project together with the corresponding harmonization model can be represented on *Table 1*.

8. Harmonization Model for Managing Stochastic Network Projects

In case when project $G(N,A)$ is represented in a formalized shape and activities $(i,j)\subset G(N,A)$ do not bear any engineering definitions and have an abstract meaning, we suggest to use harmonization modeling as the project's planning and control technique. Note that undertaking harmonization modeling for the project under consideration results in optimal budget reallocation among the project's activities. This basic assertion will be used later on, by implementing the project's on-line control.

We suggest a step-wise procedure to control the PERT-COST network project by means of harmonization as follows:

Table 1. System's model and PHM for project management systems

System's model	**Parameters**		**Partial harmonization models**
	Indep.	**Dep.**	
$G(N,A)$ - PERT-COST network; (i,j) - activity, $(i,j)\subset A\subset G(N,A)$; c_{ij} - budget assigned to (i,j); $c_{ij\ min}$, $c_{ij\ max}$ - lower and upper c_{ij} bounds; Total budget $C\ge\sum_{(i,j)} c_{ij\ min}$; Due date D; p.d.f. $t_{ij}(c_{ij})=\frac{12}{(b_{ij}-a_{ij})^4}(t-a_{ij})(b_{ij}-t)^2$; $a_{ij}=\frac{A_{ij}}{c_{ij}}$, $b_{ij}=\frac{B_{ij}}{c_{ij}}$, A_{ij}, B_{ij} - const.; $T\{G\mid(c_{ij})\}$ - random project duration with assigned c_{ij}.	B U D G E T C D U E D A T E D	R E L I A B I L I T Y *R*	Determine $c_{ij}^{(opt)}$ to $\underset{\{c_{ij}\}}{Max} R = \underset{\{c_{ij}\}}{Max}\left[Pr\{T\{G\mid(c_{ij})\}<\right.$ subject to $c_{ij\ min}\le c_{ij}\le c_{ij\ max}$; $C=\sum_{\{i,j\}} c_{ij}^{(opt)}$; $R^{(opt)}\mid c_{ij}^{(opt)} = R^{opt} = PHM(C$.

Step 0. Given the input information:

- PERT-COST project $G(N,A)$;
- pregiven values $c_{ij\ min}$, $c_{ij\ max}$, A_{ij} and B_{ij} for each activity $(i,j)\subset A\subset G(N,A)$;
- pregiven partial utilities α_C, α_D and α_R;
- pregiven admissible intervals $[C_{00},C_0]$, $[D_{00},D_0]$ and $[R_0,R_{00}]$.

Step 1. Undertake harmonization modeling for $G(N,A)$ beforehand, i.e., before the project actually starts to be carried out. Denote the corresponding optimized values which define the maximal project's utility, by C^*, D^* and R^*. Note that restrictions

$$\begin{cases} C_{00} \le C^* \le C_0 \\ D_{00} \le D^* \le D_0 \\ R_0 \le R^* \le R_{00} \end{cases} \tag{27}$$

hold, otherwise harmonization cannot be accomplished.

Step 2. If budget value C^* is accepted, reassign C^* among the project's activities according to values $c_{ij}^{(opt)}$ obtained in the course of undertaking harmonization at *Step 1*. Afterwards the project starts to be carried out.

Step 3. In [12], a control model for PERT-COST projects is outlined. The model determines planned trajectories, observes at each control point the progress of the project and its deviation from the planned trajectory, and establishes the next control point. This control model has to be implemented at *Step* 3, in order to determine the routine control point $t>0$.

Step 4. At each control point t the progress of the project is observed, i.e., network graph $G(N,A)$ has to be updated at point t, as well as the remaining budget C^*. Denote those values by $G_t^*(N,A)$ and C_t^*, correspondingly.

Step 5. At each routine control point $t>0$ solve harmonization problem in order to reallocate later on the remaining budget C_t^* among remaining activities $(i,j)\subset A_t\subset G_t(N,A)$. Denote the corresponding optimal budget values by $c_{ij\ t}^{(opt)}$.

Reallocate, if necessary, budget C_t^* among activities $(i,j)\subset A_t$ according to

Step 6. the results of *Step 5*. Note that implementing numerous budget reallocations is actually the only control action in the course of performing on-line control. Go to *Step 3*.

Step 7. The algorithm terminates after inspecting the project at the due date D, i.e., at the last control point.

It can be well-recognized that, besides undertaking on-line procedures, the suggested step-wise algorithm comprises both harmonization modeling and risk analysis models. Indeed, the latter are not similar to traditional risk management methods which involve technological risks, uncertainties in products' marketing, etc. However, optimal budget reallocation serves actually as a regulation model under random disturbances and can be regarded as a risk analysis element.

Note that in the course of the project's realization certain parameters entering the input information may undergo changes, e.g. restriction values C_0, C_{00}, R_0, R_{00}, D_0, D_{00}, as well as partial utility values α_C, α_D and α_R. New values have to be implemented in the harmonization model in order to facilitate optimal budget reallocation among the remaining project's activities at Step 5 of the algorithm. If problem (17-22) has no solution, the decision-making to be undertaken at the company level results either in obtaining additional budget value ΔC or in increasing the due date by ΔD. Both values can be determined by means of harmonization.

9. Practical Applications

This *Section* refers to considering practical achievement on the basis of implementing harmonization models for monitoring various PERT-COST network projects. The experimental design has been taken from a real design office [1].

We will henceforth consider a PERT-COST type project with random activity durations and p.d.f. satisfying (10), (11) or (12). The project's initial data is presented in *Table 2*. The basic project's parameters are as follows: project's budget C, due date D and reliability R. Partial utility coefficients are $\alpha_C = 1.0$, $\alpha_D = 0.5$ and $\alpha_R = 1.0$, while the initial search steps (first iteration) for $CCSA$ are $\Delta C = 4$ and $\Delta D = 2$. The number M of simulation runs for the PHM is taken $M = 2,000$. Computer program for the PHM algorithm is written in Borland C++ language on Pentium-IV PC. Other project's parameters are as follows: $R_0 = 0.7$, $R_{00} = 0.95$, $C_0 = 250$, $C_{00} = 230$, $D_0 = 95$, $D_{00} = 85$, $\delta C = 10$, $\delta D = 2.0$, $\delta R = 0.1$ and $\varepsilon = 0.001$.

The second iteration for the $CCSA$ is carried out with $\Delta C = 2.0$ and $\Delta D = 1.0$, while all further iterations, $v \geq 2$, are realized with $\Delta C = 1.0$ and $\Delta D = 1.0$.

The performance of the harmonization model's algorithm is illustrated on *Tables 3-5* (for the case of p.d.f. satisfying (10), (11) and (12), correspondingly). It can be well-recognized that:

1. The cyclic coordinate search algorithm for determining the optimal utility of a medium-size project requires only four iterations to carry out the optimization process. The increase of the project's utility parameter after completing the fourth iteration (14 search points), as compared with the initial search point, shows utility improvement of approximately 45%. Thus, the two-level heuristic algorithm to optimize the project's harmonization model performs well.

2. Using the beta-distribution function results in obtaining the highest values for the project's utility parameter. This stems from the obvious fact that the mean value $\mu = 0.6a + 0.4b$ for beta-distribution p.d.f. within the distribution range (a,b) is closer to the lower bound a, than in case of normal and uniform distributions with symmetrical mean values $\mu = 0.5(a+b)$. It goes without saying that lower mean activity – time values result in higher reliability estimates. Since values of the truncated normal distribution concentrate closer to the mean value, than uniformly distributed values, the corresponding project's utility estimates are slightly better for the normal distribution p.d.f. than for the uniform one.

3. Thus, the optimal project's utility can be determined for the following parametrical values:

$C = 245$, $D = 93$, $R = 0.914$, $U_G = 3.64$ *(beta-distribution)*,

$C = 245$, $D = 93$, $R = 0.893$, $U_G = 3.43$ *(normal distribution)* and

$C = 245$, $D = 93$, $R = 0.889$, $U_G = 3.39$ *(uniform distribution)*.

10. Conclusions and Application Areas

The following conclusions can be drawn from the study:

1. Problems of estimating the utility of complicated and usually multilevel management systems by means of establishing and solving harmonization problems, are very urgent, especially for organization systems with a variety of quality parameters. Applications of the utility theory in

Table 2. The project's initial data

N	i	j	$c_{ij\ min}$	$c_{ij\ max}$	A_{ij}	B_{ij}
1	1	2	2	8	25	81
2	1	3	1	6	22	60
3	1	4	1	8	75	105
4	1	5	2	15	80	132

5	1	6	1	8	30	45
6	1	7	8	30	160	200
7	2	3	8	15	50	100
8	2	15	3	8	83	120
9	3	14	10	15	110	220
10	3	15	4	12	60	120
11	4	13	5	10	90	120
12	4	14	8	12	50	100
13	5	9	7	17	150	200
14	5	13	5	10	105	140
15	6	9	2	5	60	80
16	7	8	3	10	42	60
17	8	10	2	10	20	32
18	8	11	6	10	40	80
19	9	11	1	5	90	120
20	9	12	3	10	42	60
21	10	20	2	5	60	80
22	10	21	5	10	105	140
23	11	19	7	15	150	200
24	11	21	8	12	50	100
25	12	18	5	10	90	120
26	13	17	4	12	60	120
27	13	18	5	10	48	60
28	13	19	4	8	63	110
29	14	16	1	7	58	102
30	14	17	1	7	23	94
31	15	16	4	9	85	120
32	15	17	3	5	60	104
33	16	22	4	11	70	93
34	17	22	5	10	82	153
35	17	23	6	10	74	110
36	18	23	2	8	80	120
37	19	23	2	5	40	87
38	20	21	1	4	32	72
39	21	23	3	8	63	95
40	22	23	5	12	87	128

Table 3. Performance illustration of the harmonization algorithm (for a beta-distribution p.d.f.)

№ of search steps	C	D	R	№ v of iteration	*Feasibility*	*Utility* $U(C,D,R)$	*Value* $U^{(v)}$ *after the* v*-th iteration*
Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the fourth iteration							
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.996	1	Feasible	2.90	2.90
2	242	95	0.922	1	Feasible	3.02	3.02
3	238	95	0.793	1	Feasible	2.13	3.02
4	242	93	0.861	1	Feasible	3.41	3.41
5	242	91	0.723	1	Feasible	3.03	3.41
6	244	93	0.895	2	Feasible	3.55	3.55

7	246	93	0.912	2	Feasible	3.52	3.55
8	240	93	0.814	2	Feasible	3.14	3.55
9	244	94	0.936	2	Feasible	3.46	3.55
10	244	92	0.835	2	Feasible	3.45	3.55
11	**245**	**93**	**0.914**	**3**	**Optimal**	**3.64**	**3.64**
12	243	93	0.875	3	Feasible	3.45	3.64
13	245	94	0.951	4	Feasible	3.51	3.64
14	245	92	0.855	4	Feasible	3.55	3.64

Table 4. Performance illustration of the harmonization algorithm (for a normal distribution p.d.f.)

№ of search steps	C	D	R	№ ν of iteration	*Feasibility*	*Utility* $U(C,D,R)$	*Value* $U^{(\nu)}$ *after the* ν*-th iteration*
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.989	1	Feasible	2.90	2.90
2	242	95	0.915	1	Feasible	2.95	2.95
3	238	95	0.782	1	Feasible	2.02	2.95
4	242	93	0.829	1	Feasible	3.09	3.09
5	242	91	0.698	1	Non-feasible	-	3.09
6	244	93	0.868	2	Feasible	3.28	3.28
7	246	93	0.885	2	Feasible	3.25	3.28
8	240	93	0.802	2	Feasible	3.02	3.28
9	244	94	0.912	2	Feasible	3.22	3.28
10	244	92	0.811	2	Feasible	3.21	3.28
11	**245**	**93**	**0.893**	**3**	**Optimal**	**3.43**	**3.43**
12	243	93	0.847	3	Feasible	3.17	3.43
13	245	94	0.921	4	Feasible	3.21	3.43
14	245	92	0.839	4	Feasible	3.39	3.43
Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the fourth iteration							

Table 5. Performance illustration of the harmonization algorithm (for a uniform distribution p.d.f.)

№ of search steps	C	D	R	№ ν of iteration	*Feasibility*	*Utility* $U(C,D,R)$	*Value* $U^{(\nu)}$ *after the* ν*-th iteration*
0	250	95	1.000	1	Feasible	2.50	2.50
1	246	95	0.984	1	Feasible	2.90	2.90
2	242	95	0.912	1	Feasible	2.92	2.92
3	238	95	0.765	1	Feasible	1.85	2.92
4	242	93	0.821	1	Feasible	3.01	3.01
5	242	91	0.695	1	Non-feasible	-	3.01
6	244	93	0.864	2	Feasible	3.24	3.24
7	246	93	0.882	2	Feasible	3.22	3.24
8	240	93	0.795	2	Feasible	2.95	3.24

9	244	94	0.910	2	Feasible	3.20	3.24
10	244	92	0.807	2	Feasible	3.17	3.24
11	**245**	**93**	**0.889**	**3**	**Optimal**	**3.39**	**3.39**
12	243	93	0.844	3	Feasible	3.14	3.39
13	245	94	0.918	4	Feasible	3.18	3.39
14	245	92	0.835	4	Feasible	3.35	3.39
Since values $U^{(3)}$ and $U^{(4)}$ coincide, the algorithm terminates after the fourth iteration							

recent publications are restricted to market competitive models and do not deal as yet with complicated hierarchical systems' functioning. The nowadays existing multi-attribute utility theory can be applied only to the stage preceding the product's design and determining the objectives for future market competition.

2. We suggest to implement the utility concept as a generalized system's quality estimate which takes into account several essential parameters. The latter usually define the quality of the system as a whole. We have developed a generalized harmonization problem in order to maximize the system's utility. The corresponding model is optimized by means of a two-level heuristic algorithm. At the upper level (the level of independent parameters) a relatively simple search procedure, e.g. the cyclic coordinate algorithm, has to be implemented. At the lower level partial harmonization problems to optimize the dependent parameters, have to be used. Note, that nowadays there is no formalized linkage between the system's parameters and attributes and, thus, no optimization problem can be put and solved in order to maximize the product's utility within its specific life cycle. The developed research enables implementing such a linkage, in future, on the stages of both designing and creating new products and, later on, on the stage of marketing the product.

3. For stochastic PERT-COST network projects three parameters are implemented in the model: the budget assigned to the project, the due date and the project's reliability to meet the due date on time. The harmonization model's solution is achieved by means of implementing a two-level heuristic algorithm. At the upper level a cyclic coordinate search algorithm to determine the quasi-optimal couple (budget – due date) is suggested. At the bottom level a high-speed heuristic procedure serving as a partial harmonization sub-model, is implemented: on the basis of input values (the assigned budget and the set due date) to maximize the probability of meeting the deadline on time by undertaking optimal budget reallocation among the project's activities.

4. Harmonization models can be applied directly to all kinds of PERT-COST network projects with uncertainties associated with activities' durations but without either technological risks or uncertainties on the stage of marketing the project's products. Such projects usually refer to the public service area, like constructing new hospitals, schools, stadiums, theatres, bridges and tunnels, new urban areas, factories, etc. In our opinion, those projects represent an overwhelming majority of existing projects and, thus, require good quality monitoring. For such projects we suggest to use the newly developed harmonization techniques both for estimating the project's utility and for

introducing regulating control actions at inspection points to enhance the progress of the project in the desired direction. Thus, harmonization modeling enables certain on-line control procedures for projects under random disturbances.

5. Being a regulation model, harmonization can be implemented (in a random disturbances environment) as a risk assessment tool as well. Thus, for this class of projects, harmonization, controlling and risk assessment actually meet.

References

1. Ben-Yair, A. **Harmonization models in strategic management and safety engineering,** Ph.D. Thesis, Ben-Gurion University of the Negev, Beer-Sheva, Israel, 2004
2. Ben-Yair, A., Golenko-Ginzburg, D. and Laslo, Z. **Harmonized decision-making in managing reliability and safety,** Journal of Business Economics and Management, 6(1), 2005, pp. 43-51
3. Berny, J. **A new distribution function for risk analysis,** Journal of the Operational Research Society, 40 (12), 1989, pp. 1121-1127
4. Clark, C.E. **The PERT model for the distribution of an activity,** Operations Research, 10, 1962, pp. 405-406
5. Donaldson, W.A. **The estimation of the mean and variance of PERT activity time,** Operations Research, 13, 1965, pp. 382-385
6. Elmaghraby, S.E. **Activity Networks: Project Planning and Control by Network Models,** Wiley, New York, 1977
7. Farnum, N.R. and Stanton, L.W. **Some results concerning the estimation of beta distribution parameters in PERT,** Journal of the Operational Research Society, 38, 1987, pp. 287-290
8. Garey, U.R. and Johnson, D.C. **Computer and Intractability: A Guide to the Theory of NP-Completeness,** Freeman and Co., San-Francisco, 1979
9. Golenko, D. **Statistical Methods in Network Planning and Control,** Nauka, Moscow, 1968, (in Russian)
10. Golenko-Ginzburg, D. **On the distribution of activity time in PERT,** Journal of the Operational Research Society, 39 (8), 1988, pp. 767-771
11. Golenko-Ginzburg, D. **A two-level decision-making model for controlling stochastic projects,** International Journal of Production Economics, 32, 1993, pp. 117-127
12. Golenko-Ginzburg, D. and Gonik, A. **On-line control model for cost-simulation projects,** Journal of the Operational Research Society, 47, 1996, pp. 266-283
13. Keeney, R.L. **The art of assessing multiattribute utility functions,** Organizational Behavior and Human Performance, 19, 1977, pp. 267-310
14. Keeney, R.L. and Raiffa, H. **Decisions with Multiple Objectives: Preferences and Value Tradeoffs,** Wiley, New York, 1976
15. Keeney, R. and Winterfeldt, D. van **Eliciting probabilities from experts in complex technical problems,** IEEE Transactions on Engineering Management, 38 (3), 1991, pp. 191-201
16. Laslo, Z. and Gurevich, G. **Minimal budget for activities chain with chance constrained lead-time,** International Journal of Production Economics, 171(1), 2007, pp. 164-172
17. Laslo, Z. **Activity time-cost tradeoffs under time and cost chance constraints,** Computers & Industrial Engineering, 44(3), 2003, pp. 365-384
18. Luenberger, D.G. **Linear and Non-Linear Programming, 2nd ed.**, Addison Wesley Publishing Co., Massachusetts, 1989

19. Menipaz, E. and Ben-Yair, A. **Three-parametrical harmonization model in project management by means of simulation,** Mathematics and Computers in Simulation, 59 (5), 2002, pp. 431-436
20. Menipaz, E. and Ben-Yair, A. **Harmonization simulation model for managing several stochastic projects,** Mathematics and Computers in Simulation, 61, 2002, pp. 61-66
21. Shtub, A., Bard, J. and Globerson, Sh. **Project Management: Engineering, Technology and Implementation,** Prentice Hall International, Inc., New York, 1994
22. Taha, H.A. **Operations Research: An Introduction**, MacMillan, New York, 1987
23. Welsh, D. **Errors introduced by a PERT assumption,** Operations Research, 13, 1965, pp. 141-143
24. Williams, T.M. **Practical use of distributions in network analysis,** Journal of the Operational Research Society, 43 (3), 1992, pp. 265-270
25. Williams, T.M. **What are PERT estimates?**, Journal of the Operational Research Society, 46 (12), 1995, pp. 1498-1504

APPENDIX

Nomenclature

$G(N,A)$ finite, connected, oriented activity – on – arc network of PERT-COST type;

-

$(i,j) \subset G(N,A)$ - activity leaving node i and entering node j;

t_{ij} - random time duration of activity (i,j);

c_{ij} - budget assigned to activity (i,j);

$c_{ij\ min}$ - minimal budget capable of operating activity (i,j) (pregiven);

$c_{ij\ max}$ - maximal budget required to operate activity (i,j) (pregiven); in case $c_{ij} > c_{ij\ max}$ additional value $c_{ij} - c_{ij\ max}$ is redundant;

C - budget assigned to carry out project $G(N,A)$**;**

D - the due date for the project $G(N,A)$**;**

R - the project's reliability value, i.e., its probability of meeting the deadline D on time;

$G_t^*(N,A)$ - PERT-COST graph updated at point $t>0$;

C_t^* - the project's budget updated at point $t>0$;

$T(G)_{c_{ij}}$ - the project's random duration on condition that budget values c_{ij} are assigned to activities (i,j);

$R(G)_{c_{ij}}$ - the project's local reliability, i.e., the probability of meeting its deadline on time on condition that values c_{ij} are assigned to $(i,j) \subset G(N,A)$, $R(G)_{c_{ij}}$ = $Pr\left\{T(G)_{c_{ij}} \le D\right\}$;

$R(G)_{C,D} = \underset{\{c_{ij}\},\sum_{(i,j)} c_{ij}=C}{Max} R(G)_{c_{ij}}$ - the project's conditional reliability (on condition that values C and D are preset beforehand; to be calculated);

$PHM/_{C,D}$ - the partial harmonization model to optimize reliability R with independent input values C and D;

C_0 - the maximal possible budget to be assigned to project $G(N,A)$ (pregiven);

D_0 - the maximal permissible due date for the project $G(N,A)$ to be accomplished (pregiven);

R_0 - the minimal permissible reliability value for project $G(N,A)$ (pregiven);

ΔC - budget search step (pregiven);

ΔD - due date's search step (pregiven);

C_{00}, D_{00}, R_{00}, P_{00} - the best possible values of parameters C, D, R and P (pregiven);

δ_C - budget unit value (pregiven);

δ_D - time unit value (pregiven);

δ_R - reliability unit value (pregiven);

α_C - partial utility value for parameter C (pregiven);

α_D - partial utility value for parameter D (pregiven);

α_R - partial utility value for parameter R (pregiven);

$U(G) = U(C,D,R)$ - the project's utility;

$U_0 = U(C_0,D_0,R_0)$ - the project's basic utility;

$U/_{C,D}$ - conditional project's utility on the basis of unification $\left\{C,\ D,\ PHM/_{C,D}\right\}$;

$U/_{C,D}$ - conditional project's utility on the basis of unification $\left\{C,\ D,\ PHM/_{C,D},\ P_{hf}/_{C,D}\right\}$;

$CCSA\{G\}$ - the cyclic coordinate search algorithm which undertakes a search in the E^2 space of budget values C and due dates D;

$v \ge 1$ - ordinal number of a current iteration in $CCSA\{G\}$;

$CCSA^{(v)}$ - the results of the v-th current iteration in the course of carrying out $CCSA\{G\}$;

$\varepsilon > 0$ - pregiven search tolerance (accuracy) in the course of optimizing the project's utility.

---

[1] Avner Ben-Yair is lecturer in the Industrial Engineering and Management Department, Sami Shamoon College of Engineering, Israel. He was born in Moscow in 1961. He received his B.Sc. in Mechanical Engineering from the Moscow Polygraphic Institute, Russia, and his M.Sc. degree in Health and Safety Engineering and Management (Summa Cum Laude) from the Ben Gurion University of the Negev, Israel. He also received his Ph.D. degree in Industrial Engineering and Management from the Ben Gurion University of the Negev, Israel. His professional experience includes 13 years of engineering and management positions in Israeli chemical, pharmaceutical and high-tech industries. His current research interests are in economic aspects of safety, reliability and failure analysis, trade-off optimization models for organization systems, production planning, scheduling and control, cost optimization and PERT-COST models, and strategic management. He has published 40 articles in various scientific sources. His recent publications have appeared in Mathematics and Computers in Simulation, International Journal of Production Economics, Communications in Dependability and Quality Management, and Computer Modelling and New Technologies.

[2] Corresponding author

[3] Dimitri Golenko-Ginzburg is Professor-Emeritus and member of the Paul Ivanier Center on Robotics Research and Production Management, in the Industrial Engineering and Management Department of the Ben Gurion University of the Negev, Israel, as well as professor in Department of Industrial Engineering and Management, Ariel University Center in Samaria, Israel. He was born in Moscow in 1932. He received his M.A. degree in Mathematics from the

Moscow State University in 1958 and his Ph.D. degree in Applied Mathematics from the Moscow Physico-Technical Institute (Russia) in 1962. In Russia during the period 1958÷1985 he wrote 14 books and more than 200 articles. In 2007 Prof. Dimitri Golenko-Ginzburg was nominated Honorary Member of the Russian Project Management Association (SOVNET) and elected Foreign Member of the Russian Academy of Natural Sciences, the Branch of Computer Science and Cybernetics. His professional experience includes 50 years of scientific research and academic teaching in leading academic institutions in the former USSR and, after repatriating in 1985, in Israel. His current research interests are in managing and planning under uncertainty, planning and control of network projects, and industrial scheduling. During the period 1985÷2007 he published more than 170 articles. His recent publications have appeared in Mathematics and Computers in Simulation, International Journal of Production Economics, Automation and Remote Control, Communications in Dependability and Quality Management, and Computer Modelling and New Technologies.

[4] Dr. Doron Greenberg earned his doctoral degree in Economics from The University of Houston in 1992; his MBA specialized in Operation Management from the Recanati School of Management in Tel Aviv (1985); and his B.Sc. in Industrial Engineering from the Technion in Haifa (1980).
He serves as the Head of the financial branch of the
Department of Economics and Business Administration in the Ariel University Center (AUC). In 2008 he has initiated and was appointed to be the 1st director of the 1st college chapter of Global Association of Risk Professionals (GARP) in Israel which is affiliated with AUC.
Dr. Doron Greenberg published about 10 articles in various scientific source including: The Journal of Real Estate Finance and Economics, Communications in Dependability and Quality Management-An International Journal, Technological and Economic Development of Economy, Risk Management, Computer Modelling and New Technologies, Communications in Dependability and Quality Management,
He also served as a lecturer in the Recanati School for management in The Tel Aviv University, in The Negev College on behalf of Pinchas Sapir, and in the Tel Aviv College of Engineering. Dr. Greenberg gave many courses in Economics and Finance and wrote dozens articles in the local newspapers regarding public wealth and finance. His main research areas are Economics and Finance.
Dr. Greenberg serves currently as director and chair of the Investment Committee of the Technical Engineers & Technicians Professional Study fund in Israel.
Dr. Greenberg has reached a profound experience in the Israeli capital market: as a director and chair of the Investment Committee of Pension fund of agricultural in Israel, as a financial analyst for institutional investors in Israel, as chairman of Gahelet, a municipal firm which deals with savings for high education, and as a member of the board of directors of PEKAN, one of the top Israeli managing firms of mutual funds.
Dr. Greenberg devotes major part of his time and energy to the community. As such he established with other the Israeli chapter of The Transparency International Organization (TI), and serves as the CFO of the organization. He also serves as a director in several municipal firms, including: the Economic firm of Ariel and the Water & Sewage firm of Ariel.

[5]Zohar Laslo is Associate Professor and Department Chair of the Industrial Engineering and Management Department, Sami Shamoon College of Engineering, Israel. Zohar was born in 1945. He received his B.Sc. and M.Sc. in Industrial Engineering and Management from the Technion - Israel Institute of Technology, Haifa, and his Ph.D. degree in Industrial Engineering and Management (Summa Cum Laude) from the Ben Gurion University of the Negev, Israel. His professional experience includes 24 years of engineering and management positions in Israeli aeronautical, electronic, pharmaceutical and high-tech industries. His current research interests are in matrix structure performance, network design, planning scheduling and control, planning under time-cost uncertainty, quality of planning, maintainability and hazard analysis techniques, and technological education. His recent publications have appeared in Computers Modelling and New Technologies, Mathematics and Computers in Simulation, Computers and Industrial Engineering,, European Journal of Engineering Education, IEEE Transactions on Engineering Management and Communications in Dependability and Quality Management.

[6] Codification of references:

[1]	Ben-Yair, A. **Harmonization models in strategic management and safety engineering,** Ph.D. Thesis, Ben-Gurion University of the Negev, Beer-Sheva, Israel, 2004
[2]	Ben-Yair, A., Golenko-Ginzburg, D. and Laslo, Z. **Harmonized decision-making in managing reliability and safety,** Journal of Business Economics and Management, 6(1), 2005, pp. 43-51
[3]	Berny, J. **A new distribution function for risk analysis,** Journal of the Operational Research Society, 40 (12), 1989, pp. 1121-1127
[4]	Clark, C.E. **The PERT model for the distribution of an activity,** Operations Research, 10, 1962, pp. 405-406
[5]	Donaldson, W.A. **The estimation of the mean and variance of PERT activity time,** Operations Research, 13, 1965, pp. 382-385
[6]	Elmaghraby, S.E. **Activity Networks: Project Planning and Control by Network Models,** Wiley, New York, 1977
[7]	Farnum, N.R. and Stanton, L.W. **Some results concerning the estimation of beta distribution parameters in PERT,** Journal of the Operational Research Society, 38, 1987, pp. 287-290
[8]	Garey, U.R. and Johnson, D.C. **Computer and Intractability: A Guide to the Theory of NP-Completeness,** Freeman and Co., San-Francisco, 1979

[9]	Golenko, D. **Statistical Methods in Network Planning and Control,** Nauka, Moscow, 1968, (in Russian)
[10]	Golenko-Ginzburg, D. **On the distribution of activity time in PERT,** Journal of the Operational Research Society, 39 (8), 1988, pp. 767-771
[11]	Golenko-Ginzburg, D. **A two-level decision-making model for controlling stochastic projects,** International Journal of Production Economics, 32, 1993, pp. 117-127
[12]	Golenko-Ginzburg, D. and Gonik, A. **On-line control model for cost-simulation projects,** Journal of the Operational Research Society, 47, 1996, pp. 266-283
[13]	Keeney, R.L. **The art of assessing multiattribute utility functions,** Organizational Behavior and Human Performance, 19, 1977, pp. 267-310
[14]	Keeney, R.L. and Raiffa, H. **Decisions with Multiple Objectives: Preferences and Value Tradeoffs,** Wiley, New York, 1976
[15]	Keeney, R. and Winterfeldt, D. van **Eliciting probabilities from experts in complex technical problems,** IEEE Transactions on Engineering Management, 38 (3), 1991, pp. 191-201
[16]	Laslo, Z. and Gurevich, G. **Minimal budget for activities chain with chance constrained lead-time,** International Journal of Production Economics, 171(1), 2007, pp. 164-172
[17]	Laslo, Z. **Activity time-cost tradeoffs under time and cost chance constraints,** Computers & Industrial Engineering, 44(3), 2003, pp. 365-384
[18]	Luenberger, D.G. **Linear and Non-Linear Programming, 2nd ed.**, Addison Wesley Publishing Co., Massachusetts, 1989
[19]	Menipaz, E. and Ben-Yair, A. **Three-parametrical harmonization model in project management by means of simulation,** Mathematics and Computers in Simulation, 59 (5), 2002, pp. 431-436
[20]	Menipaz, E. and Ben-Yair, A. **Harmonization simulation model for managing several stochastic projects,** Mathematics and Computers in Simulation, 61, 2002, pp. 61-66
[21]	Shtub, A., Bard, J. and Globerson, Sh. **Project Management: Engineering, Technology and Implementation,** Prentice Hall International, Inc., New York, 1994
[22]	Taha, H.A. **Operations Research: An Introduction**, MacMillan, New York, 1987
[23]	Welsh, D. **Errors introduced by a PERT assumption,** Operations Research, 13, 1965, pp. 141- 143
[24]	Williams, T.M. **Practical use of distributions in network analysis,** Journal of the Operational Research Society, 43 (3), 1992, pp. 265-270
[25]	Williams, T.M. **What are PERT estimates?**, Journal of the Operational Research Society, 46 (12), 1995, pp. 1498-1504