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ADDENDA TO WEIBULL DISTRIBUTION IN MATLAB (DEFINITIONS, CODE SOURCES FOR FUNCTIONS, APPLICATIONS)

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ABSTRACT

In this paper we make a new presentation of the Weibull distribution. We will make some add-ons for the Statistics Toolbox in MATLAB with our functions for the form with scale and displacement of the distribution. Finally we will use these new functions on applications of the Weibull distribution.

Key words: *probability density function (pdf), cumulative distribution function (cdf), survivor function (sf), reliability function (rf).*

INTRODUCTION

The Weibull distribution has numerous applications, but most of the programs that exist like MATLAB see [6], and others, have functions only for the scaled model of the distribution. In this paper we will present the Weibull distribution as probability function of the exponential distribution (for other types of generalizations of Weibull distribution see [7], [9]). We mention the classic application of the Weibull distribution which is using the scale parameter as an estimator of the wind speed in the wind turbines, see [1], [5], [12], [13], [14], [15], [16]. Additionally this estimation can be done in a mixt context where other distributions are used as well and even other techniques, see [2], [3], [4], [8], [10], [11]. The scope of this paper is to make a completion by using the Weibull distribution with its location parameter. So we will define the Weibull distribution the scaled model with displacement having the shape, scale and location parameter.

Like before mentioned, existing programs have functions only for the scaled form of the Weibull distribution. For this reason we come to complete the statistics toolbox in MATLAB with functions that compute the pdf, cdf, quantiles, mean and variance (dispersion) the Weibull distribution with 3 parameters. We also have functions that plot the cdf for each of the distribution. Through these functions we wish to encourage the usage of the 3-parameter Weibull distribution.

WEIBULL DISTRIBUTION

Definition 2.1. The Weibull distribution $T_{[W]} = W(\lambda, \beta)$ is defined through $T_E = T^\beta$ where $\beta \epsilon \mathbb{R}_+^*$ is the shape parameter and $T_E = E(\lambda)$ is the exponential distribution with $\lambda \epsilon \mathbb{R}_+^*$.

Observations 2.1.

i (scale) Formally $c = \lambda^{-\frac{1}{\beta}} \epsilon \mathbb{R}_+^*$ is called the scale parameter and the definition becomes $T_E = \left(\frac{T}{c}\right)^\beta$.

ii (positive defined) By definition the random variable *T* is positive defined.

iii (reliability function – rf, cumulative distribution function – cdf, probability density function – pdf) Specifically the Weibull distribution *T* is defined through:

- *Reliability function* (rf): $R_T : \mathbb{R}_+ \to [0,1]$, $R_T(t) = exp\left(-\lambda t^\beta\right)$, $\lambda, \beta \epsilon \mathbb{R}_+^*$ because $R_T(t) = P(T > t)(= P(T > t)) = P\left(T_E^{\frac{1}{\beta}} > t\right) = P\left(T_E > t^\beta\right) = R_{T_E}(t^\beta)$=exp$(-\lambda t^\beta)$;
- *Cumulative distribution function* (cdf): $F_T : \mathbb{R}_+ \to [0,1]$, $F_T = 1 - R_T$, $F_T(t) = P(T \le t) = \left(P(T < t)\right) = 1 - exp\left(-\lambda t^\beta\right)$;
- *Probability density function* (pdf): $f_T : \mathbb{R}_+^* \to \mathbb{R}_+^*$, $f_T = F_T'$, $f_T(t) = \beta \lambda t^{\beta - 1} \exp(-\lambda t^\beta)$ because F_T is right differentiable in 0 just for $\beta \ge 1$.

The expressions with scaling are:

- rf: $R_T(t) = \exp\left(-\left(\frac{t}{c}\right)^\beta\right)$;
- cdf: $F_T(t) = 1 - \exp\left(-\left(\frac{t}{c}\right)^\beta\right)$;
- pdf: $f_T(t) = \frac{\beta}{c}\left(\frac{t}{c}\right)^{\beta - 1} \exp\left(-\left(\frac{t}{c}\right)^\beta\right)$;

iv (Weibull distribution with displacement) Weibull distribution with displacement (with $\alpha \epsilon \mathbb{R}_-$.location parameter and *a=0*) $T_{[W][a]} = W(\lambda, \beta, \alpha)$ is defined through $T_E = (T_\alpha - \alpha)^\beta$ where $\beta \epsilon \mathbb{R}_+^*$ is the shape parameter and $T_E = E(\lambda, 0)$ is the exponential distribution with displacement equal to 0 and $\lambda \epsilon \mathbb{R}_+^*$.

The Weibull distribution with displacement T_α is defined through:

- *Reliability function* (rf): $R_{T_\alpha} : \mathbb{R} \to [0,1]$, $R_{T_\alpha}(t) = \begin{cases} \exp(-\lambda(t - \alpha)^\beta, t > \alpha \ge 0 \\ 1, \ otherwise \end{cases}$; $\lambda, \beta \epsilon \mathbb{R}_+^*$, $\alpha \epsilon \mathbb{R}_+$

Because $R_{T_\alpha}(t) = P(T_\alpha > t) = P\left(T_E^{\frac{1}{\beta}} + \alpha > t\right) = P\left(T_E^{\frac{1}{\beta}} > t - \alpha\right)$ =

$$P(\tilde{T}_E > (t-a)^\beta) = R_{\tilde{T}_E}((t-a)^\beta) = \begin{cases} \exp(-\lambda(t-a)^\beta, t > a \geq 0 \\ 1, \quad otherwise \end{cases}$$

- *Cumulative distribution function* (cdf): $F_{\tilde{T}_a}: \mathbb{R} \to [0,1], F_{\tilde{T}_a} = 1 - R_{\tilde{T}_a}$,

$$F_{\tilde{T}_a}(t) = \begin{cases} 1 - \exp(-\lambda(t-a)^\beta, t > a \geq 0 \\ 0, \ otherwise \end{cases};$$

- *Probability density function* (pdf): $f_{\tilde{T}_a}: \mathbb{R} \to \mathbb{R}_+$, $f_{\tilde{T}_a} = F'_{\tilde{T}_a}$, $\bar{\mathbb{R}} = \mathbb{R} \setminus \{a\}$ with the exception of $a \epsilon \mathbb{R}_+$ because $F_{\tilde{T}_a}$ is not differantiable in *a* just for $\beta \geq 1$, meaning:

$$f_{\tilde{T}_a}(t) = \begin{cases} \beta\lambda(t-a)^{\beta-1}\exp\left(-\lambda(t-a)\right)^\beta, t > a \geq 0 \\ 0, otherwise \ (t \neq a) \end{cases}$$

By definition the Weibull distribution with displacement is positive defined.

Similar to the expressions with scaling we have:

- rf: $R_{\tilde{T}_a}(t) = \begin{cases} \exp\left(-\left(\frac{t-a}{c}\right)^\beta\right), t > a \geq 0 \\ 1, \ otherwise \end{cases};$

- cdf: $F_{\tilde{T}_a}(t) = \begin{cases} 1 - \exp\left(-\left(\frac{t-a}{c}\right)^\beta\right), t > a \geq 0 \\ 0, otherwise \end{cases};$

- pdf: $f_{\tilde{T}_a}(t) = \begin{cases} \frac{\beta}{c}\left(\frac{t-a}{c}\right)^{\beta-1} \exp\left(-\left(\frac{t-a}{c}\right)^\beta\right), t > a \geq 0; \\ 0, otherwise \ (t \neq a) \end{cases}$

Moments of the Weibull distribution. The mean and variance (dispersion) of the random variable *T* are:

$$m = M(T) = \lambda^{-\frac{1}{\beta}}\Gamma\left(\frac{1}{\beta}+1\right)$$

$$\sigma^2 = \mu_2(T) = M_2(T) = var(X) = D(X) = \lambda^{-\frac{2}{\beta}}\left(\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right)$$

Moments of the Weibull distribution with scaling.

$$m = M(T) = c\Gamma\left(\frac{1}{\beta}+1\right)$$

$$\sigma^2 = \mu_2(T) = M_2(T) = var(X) = D(X) = c^2\left(\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right)\right)$$

To generalize the moments of *k* order of random variable *T* is:

$$M_k(T) = c^k\Gamma\left(\frac{k}{\beta}+1\right), k \epsilon \mathbb{N}^*$$

$$\mu_k(T) = M_k(T) = (-1)^k c^k \sum_{i=0}^{k}(-1)^i C_k^i \Gamma^{k-i}\left(\frac{1}{\beta}+1\right)\Gamma\left(\frac{i}{\beta}+1\right), k \epsilon \mathbb{N}^* \setminus \{1\}$$

Moments of the Weibull distribution with scaling and displacement. We have (by definition) the mean $M(\tilde{T}) = M(T)$, variance (dispersion) $D(\tilde{T}) = D(T)$ similar to the moments of superior order, we have:

$$M(\tilde{T}_a) = M(\tilde{T}) + M(a) = c\Gamma\left(\frac{1}{\beta}+1\right) + a$$

$$D(\tilde{T}_a) = D(\tilde{T}) + D(a) = D(\tilde{T}) = c^2(\Gamma\left(\frac{2}{\beta}+1\right) - \Gamma^2\left(\frac{1}{\beta}+1\right))$$

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MATLAB – STATISTICS TOOLBOX – WEIBULL DISTRIBUTION

Waloddi Weibull offered the distribution that bears his name as an appropriate analytical tool for modeling the breaking strength of materials. Current usage also includes reliability and lifetime modeling. The Weibull distribution is more flexible than the exponential for these purposes. [1]

Matlab Statistics Toolbox offers functions for the Weibull distribution with two parameters $y = abx^{b-1}e^{-ax^b}$, where **a** is the scale parameter and **b** is the shape parameter.

We will briefly go through the available functions in Matlab for the Weibull distribution.

i) wblpdf - Weibull probability density function

Syntax

`Y = wblpdf(X,A,B)`

The pdf of the Weibul distribution is:

$$y = f(x|a,b) = ba^{-b}x^{b-1}e^{-\left(\frac{x}{a}\right)^b} I_{(0,\infty)}(x)$$

For the pdf of the Weibull distribution with one parameter we will use A = 1.

ii) wblcdf - Weibull cumulative distribution function

Syntax

```
P = wblcdf(X,A,B)
[P,PLO,PUP] = wblcdf(X,A,B,PCOV,alpha)
```

The cdf of the Weibull distribution is:

$$p = F(x|a,b) = \int_0^x ba^{-b}t^{b-1}e^{-\left(\frac{t}{a}\right)^b} I_{(0,\infty)}(x)$$

iii) wblinv - Weibull inverse cumulative distribution function

Syntax

```
X = wblinv(P,A,B)
[X,XLO,XUP] = wblinv(P,A,B,PCOV,alpha)
```

Inverse cumulative distribution function:

$$x = F^{-1}(p|a,b) = -a[\ln(1-p)]^{1/b} I_{[0,1]}(p)$$

iv) wblrnd – Weibull random numbers

Syntax

```
R = wblrnd(A,B)
R = wblrnd(A,B,m,n,...)
R = wblrnd(A,B,[m,n,...])
```

v) wblplot - Weibull probability plot

Syntax

```
wblplot(X)
h = wblplot(X)
```

vi) wblstat - Weibull mean and variance (dispersion)

Syntax

```
[M,V] = wblstat(A,B)
```

The mean of Weibull distribution with parameters a and b is:

$$a\left[\Gamma\left(1+b^{-1}\right)\right]$$

the variance is:

$$a^2\left[\Gamma\left(1+2b^{-1}\right)-\Gamma\left(1+b^{-1}\right)^2\right]$$

vii) wblfit - Weibull parameter estimates

Syntax

```
parmhat = wblfit(data)
[parmhat,parmci] = wblfit(data)
[parmhat,parmci] = wblfit(data,alpha)
[...] = wblfit(data,alpha,censoring)
[...] = wblfit(data,alpha,censoring,freq)
[...] = wblfit(...,options)
```

Estimates the Weibull distribution parameters in the probability density: $y = f\left(x|a,b\right) = ba^{-b}x^{b-1}e^{-\left(\frac{x}{a}\right)^b} I_{(0,\infty)}(x)$

viii) wbllike - Weibull negative log-likelihood

Syntax

```
nlogL = wbllike(params,data)
[logL,AVAR] = wbllike(params,data)
[...] = wbllike(params,data,censoring)
[...] = wbllike(params,data,censoring,freq)
```

The Weibull negative log-likelihood for uncensored data is:

$$(-logL) = -log\prod_{i=1}^{n} f(a,b|x_i) = -\sum_{i=1}^{n} logf(a,b|x_i)$$

where *f* is the Weibull pdf.

wbllike is a utility function for maximum likelihood estimation.

4. FUNCTIONS FOR THE THREE PARAMETER WEIBULL DISTRIBUTION

The Statistics Toolbox does not have functions for the Weibull distribution with three parameters. For this reason we come with our own functions.

i) The probability density function

The pdf of 3 parameter Weibull distribution is:

$$f(t,\beta,\theta,\gamma) = \begin{cases} \frac{\beta}{\theta}\left(\frac{t-\gamma}{\theta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\theta}\right)^{\beta}}, & t > \gamma \geq 0 \\ 0, & otherwise \end{cases}$$

```
function [P] = wbl3pdf(x, theta, beta, gamma)
%theta scale parameter
%beta shape parameter
```

```
%gamma location parameter
n = length(x);
f = zeros(1,n);
for i=1:n
    if((x(i) > gamma) && (x(i)>0) && (gamma>=0))
        f(i) = (beta/theta)*( ( (x(i)-gamma)/theta )^(beta-1) )*exp( -
( (x(i)-gamma)/theta )^beta );
    else
        f(i) = 0;
    end
end
P = f;
end
```

ii) The cumulative distribution function

The cdf of 3 parameter Weibull distribution is:

$$F(t,\beta,\theta,\gamma) = \begin{cases} 1 - e^{-\left(\frac{t-\gamma}{\theta}\right)^{\beta}}, & t > \gamma \geq 0 \\ 0, & otherwise \end{cases}$$

where $\theta = \frac{1}{\sqrt[\beta]{\lambda}}, \;\; \lambda = \frac{1}{\theta^{\beta}};$

```
function [P] = wbl3cdf(x, theta, beta, gamma)
%theta scale parameter
%beta shape parameter
%gamma location parameter
n = length(x);
F = zeros(1,n);
for i=1:n
    if( (x(i) > gamma) && (x(i)>0) && (gamma>=0))
        F(i) = 1 - exp(-( (x(i)-gamma)/theta)^beta);
    else
         F(i) = 0;
    end
end
P = F;
end
```

iii) The inverse cumulative distribution function

The inverse cdf of 3 parameter Weibull distribution is:

$$F(p,\beta,\theta,\gamma) = \begin{cases} -\theta[\ln(1-p)]^{1/\beta} + \gamma, & 1p \geq 0 \\ 0, & otherwise \end{cases}$$

```
function [P] = wbl3inv(x, theta, beta, gamma)
%x probability that is calculated
%theta scale parameter
%beta location parameter
%gamma displacement parameter
n = length(x);
F = zeros(1,n);
```

```
for i=1:n
    if((x(i)>0) && (x(i)<1) && (gamma>=0))
        F(i) = (-theta*((log(1-(x(i))))^(1/beta)))*sqrt(-1)-gamma;
    else
        F(i) = NaN;
    end
end
P = real(F);
end
```

iv) The mean and the variance (dispersion)

The formulas respectively for mean and **variance** (dispersion) are:

mean: $m = \theta\Gamma\left(\frac{1}{\beta}+1\right)+\gamma$

variance (dispersion): $\sigma = \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$

```
function [M V] = wbl3stat(theta, beta, gamma)
%theta scale parameter
%beta shape parameter
%gamma location parameter
mean = theta*gamma( (1/beta) + 1) + gamma;
var = (theta^2)*( gamma( (2/beta) +1 ) - ( gamma( (1/beta)+1 ) )^2 );
M= mean;
V = var;
end
```

v) Random numbers generation

$y = [-\theta^\beta \ln(1-x)]^{1/\beta} + \gamma$ where x comes from a uniform distribution with values between 0 and 1.

```
function [X] = wbl3rnd(theta, beta, gamma, n)
%theta scale parameter
%beta shape parameter
%gamma location parameter
y = zeros(1,n);
for i =1:n
    y(i) = (( -(theta^beta) * log( (1- (rand(1,1)) ) ) )^(1/beta)) +
gamma;
end
X = y;
end
```

vi) Plotting of the cumulative distribution function

```
function wbl3plot(x, theta, beta, gamma,)
%theta scale parameter
%beta shape parameter
%gamma location parameter
```

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```
n = length(x);
lnx = zeros(1,n);
Fecdf = wbl3cdf(x, theta, beta, gamma);
for i=1:n
    lnx(i) = log(x(i));
end
plot(lnx, Fecdf, 'bo');
end
```

APPLICATIONS

1) Data were gathered regarding the behavior of a truck gearbox, for a length of 15.000 km. During the period of analysis, 141 breaks of the gearbox were recorded.

The results, as frequency distribution function of the breaks, are in Table I:

Table I

t_i	k_i	$\hat{F}_{(xi)}$ (%)	$\hat{R}(t_i)$
1	2	3	4
3000	26	18.6	0.82
6000	49	53.2	0.46
9000	39	81.1	0.18
12000	22	96.6	0.04
15000	4	99.7	0.008
Total	Σ = 141	-	-

By constructing and studying the probabilistic networks a 3-parameter Weibull model could be used, with the shape parameter β=2, scale parameter θ=6900 and the location parameter γ=0.

The survivor function will have the form:

$$R(t;\gamma,\beta,\theta) = e^{-\left(\frac{t-0}{6900}\right)^2}$$

We will use our functions for the 3-Parameter Weibull Distribution.

Since the location parameter is 0 we have a 2-Parameter Weibull Distribution, so we can also use the Mathlab functions to verify the results of our functions.

Command history

To shorten the command history we will not always show the results of the commands.

First we initialize variables with the values of the scale, shape and locations parameters, and a vector with the number of kilometers traveled.

theta = 6900

beta = 2

gamma = 0

t = [3000 6000 9000 12000 15000]

We calculate the density distribution, first with the Mathlab function, then with our function:

wblpdf(t, theta, beta)
wbl3pdf(t, theta, beta, gamma)
The answer was identical for the both queries. The answer is:
ans = 1.0e-003 *
0.1043 0.1183 0.0690 0.0245 0.0056

We calculate the distribution function:
wblcdf(t, theta, beta)
wbl3cdf(t, theta, beta, gamma)
The answer is:
ans = 0.1722 0.5305 0.8176 0.9514 0.9911

We calculate the quantiles:
wblinv(0.25, theta, beta)
wbl3inv(0.25, theta, beta, gamma)
The answer is:
ans = 3.7009e+003

wblinv(0.5, theta, beta)
wbl3inv(0.5, theta, beta, gamma)
The answer is:
ans = 5.7446e+003

wblinv(0.75, theta, beta)
wbl3inv(0.75, theta, beta, gamma)
The answer is:
ans = 8.1241e+003

We calculate the mean and the **variance** (dispersion):
[m v] = wblstat(theta, beta)
[m v] = wbl3stat(theta, beta, gamma)
The answer is:
mean: *m* = 6.1150e+003
variance (dispersion): *v* = 1.0217e+007

2) Trials have been made over five elements of a technical system. The cycles which the breaks have followed (ascending reordered) were: 1.2, 2.0, 2.5, 2.9, 3.6. The estimator of the distribution function:$\hat{F} = [100(i-0.5)/n]$ lead to (in %): 10, 30, 50, 70, 90, which, represented on a probabilistic Nelson Thompson network along with the values of the working time allows us to state that the variable „work time" (in cycles) follows a Weibull law with$\hat{\beta}$ = 2.65 and$\hat{\theta}$ = 2.7 cycles.(A.9)We will add the location parameter γ= 1 cycle, which helps us to state that a component will work at least 1 cycle until it will break.

***Command history*:**

theta = 2.7

beta = 2.65

gamma = 1

t = [1.2 2.0 2.5 2.9 3.6]

wbl3pdf(t, theta, beta, gamma)

The result is:

ans = 0.0134 0.1774 0.3014 0.3706 0.3731

wbl3cdf(t, theta, beta, gamma)

The result is:

ans = 0.0010 0.0694 0.1899 0.3257 0.5954

wbl3inv(0.25, theta, beta, gamma)

The result is:

ans = 0.5636

wbl3inv(0.50, theta, beta, gamma)

The result is:

ans = 1.1789

wbl3inv(0.75, theta, beta, gamma)

The result is:

ans = 1.8303

[m v] = wbl3stat(theta, beta, gamma)

The result is:

mean: *m =* 3.3996

variance (dispersion):*v =* 0.9499

We will now draw the dispersion function:

wbl3plot(t, theta, beta, gamma)

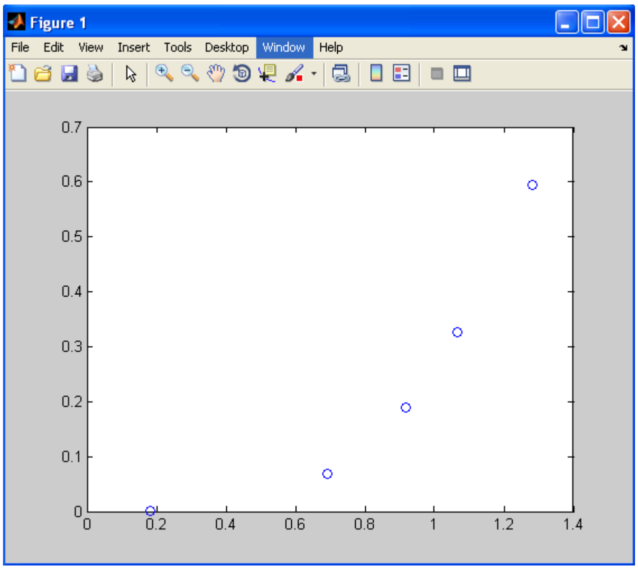

Fig. 1. Plotting of pdf from application 2)

Observation. We could have used as an estimator the *survival function instead of the* cumulative *distribution function*. We then would have had the survival function estimator, which would lead to the values (in %): 90, 70, 50, 30, 10.

6. CONCLUSIONS

This article treats more the probabilistic side of the Weibull distribution, for this reason we don't have functions that estimate the parameters for the scaled model with displacement of the Weibull distribution.

The functions presented in this paper can be used in MATLAB also for the case when the location parameter is 0, but their purpose is to be used on applications of the Weibull distribution when the location parameter has a value that influences the calculation of the pdf, cdf or quantile.

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