WWW.JAQM.RO

JOURNAL OF APPLIED QUANTITATIVE METHODS

Quantitative Methods Inquires

Vol. 7
No. 1
Spring
2012

ISSN 1842–4562

JAQM Editorial Board

Editors
Ion Ivan, University of Economics, Romania
Claudiu Herteliu, University of Economics, Romania
Gheorghe Nosca, Association for Development through Science and Education, Romania

Editorial Team
Cristian Amancei, University of Economics, Romania
Catalin Boja, University of Economics, Romania
Radu Chirvasuta, "Carol Davila" University of Medicine and Pharmacy, Romania
Irina Maria Dragan, University of Economics, Romania
Eugen Dumitrascu, Craiova University, Romania
Matthew Elbeck, Troy University, Dothan, USA
Nicu Enescu, Craiova University, Romania
Bogdan Vasile Ileanu, University of Economics, Romania
Miruna Mazurencu Marinescu, University of Economics, Romania
Daniel Traian Pele, University of Economics, Romania
Ciprian Costin Popescu, University of Economics, Romania
Aura Popa, University of Economics, Romania
Marius Popa, University of Economics, Romania
Mihai Sacala, University of Economics, Romania
Cristian Toma, University of Economics, Romania
Erika Tusa, University of Economics, Romania
Adrian Visoiu, University of Economics, Romania

Manuscript Editor
Lucian Naie, SDL Tridion

JAQM Advisory Board

Luigi D'Ambra, University of Naples "Federico II", Italy
Ioan Andone, Al. Ioan Cuza University, Romania
Kim Viborg Andersen, Copenhagen Business School, Denmark
Tudorel Andrei, University of Economics, Romania
Gabriel Badescu, Babes-Bolyai University, Romania
Catalin Balescu, National University of Arts, Romania
Avner Ben-Yair, SCE - Shamoon College of Engineering, Beer-Sheva, Israel
Constanta Bodea, University of Economics, Romania
Ion Bolun, Academy of Economic Studies of Moldova
Recep Boztemur, Middle East Technical University Ankara, Turkey
Constantin Bratianu, University of Economics, Romania
Irinel Burloiu, Intel Romania
Ilie Costas, Academy of Economic Studies of Moldova
Valentin Cristea, University Politehnica of Bucharest, Romania
Marian-Pompiliu Cristescu, Lucian Blaga University, Romania
Victor Croitoru, University Politehnica of Bucharest, Romania
Cristian Pop Eleches, Columbia University, USA
Michele Gallo, University of Naples L'Orientale, Italy
Angel Garrido, National University of Distance Learning (UNED), Spain
Bogdan Ghilic Micu, University of Economics, Romania
Anatol Godonoaga, Academy of Economic Studies of Moldova
Alexandru Isaic-Maniu, University of Economics, Romania
Ion Ivan, University of Economics, Romania
Radu Macovei, "Carol Davila" University of Medicine and Pharmacy, Romania
Dumitru Marin, University of Economics, Romania
Dumitru Matis, Babes-Bolyai University, Romania
Adrian Mihalache, University Politehnica of Bucharest, Romania
Constantin Mitrut, University of Economics, Romania
Mihaela Muntean, Western University Timisoara, Romania
Ioan Neacsu, University of Bucharest, Romania
Peter Nijkamp, Free University De Boelelaan, The Nederlands
Stefan Nitchi, Babes-Bolyai University, Romania
Gheorghe Nosca, Association for Development through Science and Education, Romania
Dumitru Oprea, Al. Ioan Cuza University, Romania
Adriean Parlog, National Defense University, Bucharest, Romania
Victor Valeriu Patriciu, Military Technical Academy, Romania
Perran Penrose, Independent, Connected with Harvard University, USA and London University, UK
Dan Petrovici, Kent University, UK
Victor Ploae, Ovidius University, Romania
Gabriel Popescu, University of Economics, Romania
Mihai Roman, University of Economics, Romania
Ion Gh. Rosca, University of Economics, Romania
Gheorghe Sabau, University of Economics, Romania
Radu Serban, University of Economics, Romania
Satish Chand Sharma, Janta Vedic College, Baraut, India
Ion Smeureanu, University of Economics, Romania
Ilie Tamas, University of Economics, Romania
Nicolae Tapus, University Politehnica of Bucharest, Romania
Timothy Kheng Guan Teo, University of Auckland, New Zeeland
Daniel Teodorescu, Emory University, USA
Dumitru Todoroi, Academy of Economic Studies of Moldova
Nicolae Tomai, Babes-Bolyai University, Romania
Victor Voicu, "Carol Davila" University of Medicine and Pharmacy, Romania
Vergil Voineagu, University of Economics, Romania

Contents

Ion IVAN
PhD, University Professor, Department of Economic Informatics
Faculty of Cybernetics, Statistics and Economic Informatics
University of Economics, Bucharest, Romania

E-mail: ionivan@ase.ro ; **Web Page:** http://www.ionivan.ro

Claudiu HERTELIU
PhD, Assistant Professor, Department of Statistics and Econometrics,
University of Economics, Bucharest, Romania

E-mail: claudiu.herteliu@gmail.com, **Web page:** http://www.hertz.ase.ro

Gheorghe NOSCA
PhD, Association for Development through Science and Education, Bucharest, Romania

E-mail: r_g-nosca@yahoo.com

We are happy to announce the winners of JAQM 2011 Awards.

After deliberations, the winners are:

1st Category

For his very important contribution in promoting and developing Quantitative Methods in Academia, as a recognized scholar, we grant to:

Gurjeet DHESI

from **London South Bank University, London, United Kingdom**

the ***2011 JAQM Medal of Honor***

2nd Category

For the most valuable, Quantitative Methods related, paper published in JAQM:
"Measuring Passenger Satisfaction: A Strategy Based on Rasch Analysis and the ANOM", JAQM Summer Issue, 2011, pp. 27-36
we granted to:

Michele GALLO

from **University of Naples L'Orientale, Naples, Italy**

the **2011 JAQM Best Paper Award**

3rd Category

For the most promising young researcher in Quantitative Methods area, we grant to:

Bogdan Vasile ILEANU

from **University of Economics, Bucharest, Romania**

the ***2011 JAQM Distinction***

DOES LOG-RANK TEST GIVE SATISFACTORY RESULTS?

Qamruz ZAMAN
PhD Associate Professor,
Department of Statistics, University of Peshawar, Pakistan

E-mail: ayanqamar@gmail.com

Karl Peter PFEIFFER
PhD Professor,
Department of Medical Statistics, Informatics and Health Economics,
Medical University Innsbruck, Austria

E-mail: peter.pfeiffer@i-med.ac.at

Abstract:

Comparison of effects of two treatments by log-rank test is a very common phenomenon in medical research. Researchers prefer to use log-rank test with out carrying about the assumptions of test, which sometimes not only destroy the effects of study but also misguides the readers. The idea of this article is to review some aspects of log-rank test and to provide some rules of thumb.

Key words: *Log-rank test, Proportional hazards assumption, Kaplan-Meier survival curve*

INTRODUCTION

Medical researchers are often interested in comparing the survival experience of two groups of individuals. They usually prefer to use simple and straight forward tests. For this purpose several methods are available for comparing survival distributions, out of which the most commonly used rank based test, is the log-rank test [1]. Log-rank test is the first choice of researchers due to its easy concept as well as easily availability of software. The test is based on different assumptions and performs better in a specific situation, but some researchers do not care about these. Like the Kaplan-Meier survival function [2], log-rank test is also based on the assumption of non-informative censoring. It is cited so many times in the literature that the log-rank test is more appropriate, powerful and reliable as compared to other tests in a situation where two or more survival curves do not cross i.e. whose hazard functions are proportional (Figure 1).

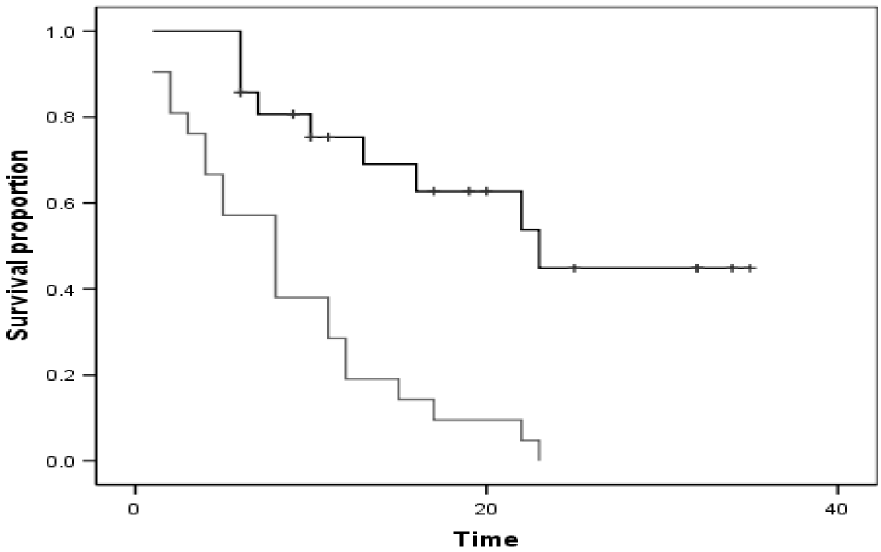

Figure 1. Survival curves of two groups

This assumption raises an important practical question that if any data set fulfills the proportional hazards assumption, log-rank test gives satisfactory results? We do not think so and we illustrate our point by considering two different published data sets which satisfied the proportional hazards assumption. Furthermore, we will try to provide some common rules of thumb on the use of log-rank test.

First data set consists of survival times of 30 cervical cancer patients, recruited to a randomized trial of the addition of a radiosensitiser to radiotherapy (Group-II) versus radiotherapy alone [3]. Group-I consists of 16 patients (5 censored and 11 events) and Group-II 14 patients (5 events and 9 censored). For further detail about the data concern the book. Table 1 summarized the data.

Table 1. Treatment group of 30 patients recruited to a cervical cancer trial.

Group-I	Group-II
1037	1476*
1429	827
680	519*
291	1100*
1577*	
90	1307
1090*	1360*
142	919*
1297	373
1113*	563*
1153	978*
150	650*
837	362
890*	383*
269	272
468*	

* Censored survival time

Survival comparison of two groups was made by using SPSS. This gave the value of log-rank test 1.682 with corresponding p-value 0.195, indicates the difference between groups is not statistically significant. This contradicts the fact which is shown in Figure 2. Except the log-rank test, we tried also different weighted tests, but every test gave the result in favour of null hypothesis that the two groups have the same survival probability. So log-rank test failed to detect difference.

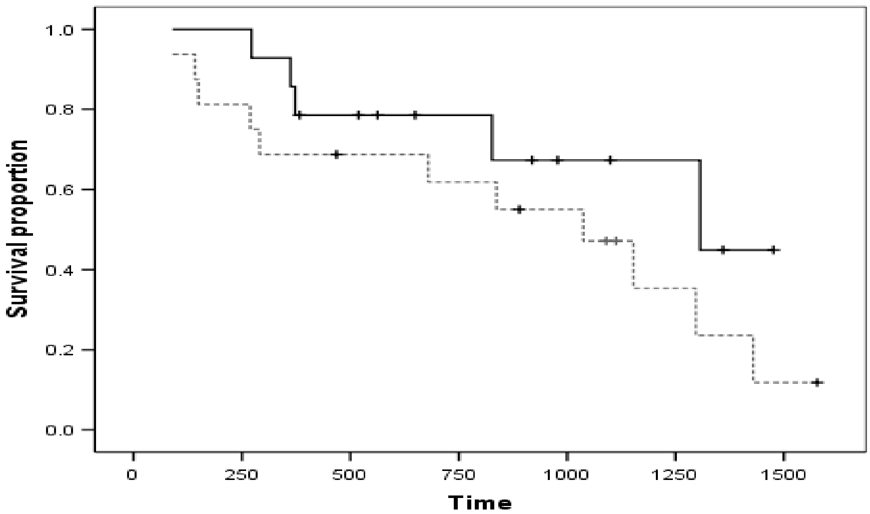

Figure 2. Kaplan-Meier survival curves of two treatments (radiosensitiser to radiotherapy denoted by dark line and radiotherapy denoted by dotted line).

For further verification, second data set is considered from Collet [4] (a brief introduction about the data set is given on page 7 of the book). The data set consists of survival times in months of women with tumours, which were classified negatively or positively stained with Helix pomatia agglutinin (HPA). There were 13 women in the negative stained group out of which 8 were censored. Positive group composed of 32 patients. Out of 32, 11 were censored. Data set is given in Table 2.The value of log-rank test along with some weighted tests values are summarized in the Table 3.

In comparison with $\alpha = 0.05$, p-value of the log-rank test does not support the fact (Figure 3), although two groups satisfied the proportional hazards assumption. While the two weighted tests, which are considered to be more appropriate for crossing curves, in this case give satisfactory results.

Table 2. Survival times in month of tumours women

Negative staining		Positive staining	
23		5	68
47		8	71
69		1	76*
70*	0		105*
71*		1	107*

100*	3		109*
101*		1	113
148	8		116*
181		2	118
198*	4		143
208*		2	154*
212*	6		162*
224*		2	188*
	6		212*
		3	217*
	1		225*
		3	
	5		
		4	
	0		
		4	
	1		
		4	
	8		
		5	
	0		
		5	
	9		
		6	
	1		

* Censored survival time

Table 3. Chi-Square statistics and p-values from the application of the log-rank, Wilcoxon and Tarone-Ware tests for tumours data

Statistical test	Chi-Square	p-value
Log-rank	3.515	0.061
Wilcoxon	4.180	0.041
Tarone-Ware	4.050	0.044

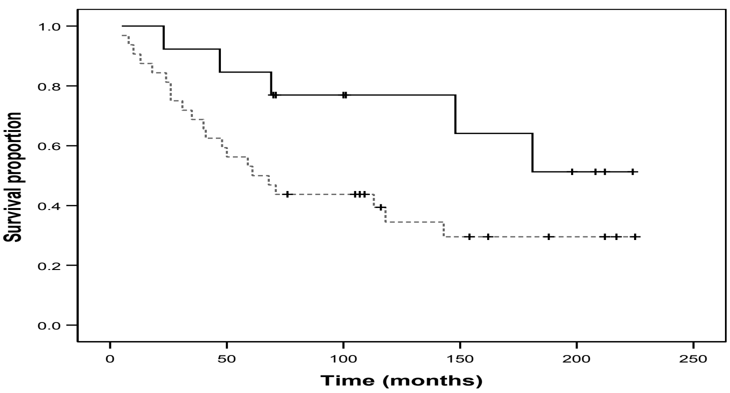

Figure 3. Kaplan-Meier survival curves of two treatments (positive stained by dark line and negative stained by dotted line).

Except these two data sets, there may be many more data sets which come across the same situation. In some cases weighted tests may be helpful but not always. It is also observed that in most cases in which log-rank test gives satisfactory results, weighted tests also and vice versa. Example is that of the famous leukaemia data set [5].

Therefore, the log-rank test which is considered to be the best choice, if the groups satisfied the proportional hazards assumption is not always true. Sometimes weighted tests are also helpful as in example 2. In some cases it may happen that no available test is able to detect the differences correctly as in example 1 and sometimes all tests give correct results.

Now the question is why the log-rank test fails in ideal situations?

The test is more suitable, if the risk of an event is considerably greater for one group [6]; although this proved in given examples still log-rank test fails. This means that the condition is not sufficient; there must be some other factors which influence the performance of log-rank test. The factor may be number of events ≤ 5, may be the range of data and may also be the difference between sizes of two groups.

We may face the same problem for crossing survival groups, on which no available weighted test fits well. This fact opens the door for new research and development. On the basis of these realities, one can not say any thing about an ideal test which is suitable in each and every situation; one test may be suitable in one situation and fails in other.

Conclusion

We conclude our discussion by mentioning the following rules

- The best way is to first check the proportionality assumption by plotting survival curves of groups or by hazard plotting of groups or by any other available method.
- It is not always true that if proportional hazards assumption satisfies, log-rank test gives a satisfactory answer.
- Do not restrict yourself to log-rank test, also apply the weighted tests. Sometimes weighted tests give more satisfactory results than log-rank test.
- If the existence tests are not able to produce appropriate results, develop a more powerful test.

References

1. Mantel N., **Evaluation of survival data and two new rank order statistics arising in its consideration**. Cancer Chemotherapy Report 1966; 50: 163-170.
2. Kaplan EL and Meier P., **Nonparametric estimation from incomplete observations. Journal of American Statistical Association** 1958; 53: 457-81.
3. Parmar MKB and Machin D., **Survival Analysis- A Practical Approach**. John Wiley & Sons Chichester 1995.
4. Collet D., **Modelling Survival Data in Medical Research**, (1st edn.) Chapman & Hall/CRC: Boca Raton 1994.

5. Freireich EJ, Gehan E, Frei III E, Schroeder LR, Wolman IJ, Anbari R, Burgert EO, Mills Sd, Pinkel D, Selwry OS, Moon JH, Gendel BR, Spurr, CL, Storrs R, Haurani F, Hoogstraten B, Lee S. **The effect of 6-mercaptopurine on the duration of steroid-induced remissions in acute leukaemia: a model for evaluation of other potentially useful therapy**. Blood, 1963; 21: 699-716.

6. Bland JM and Altman DG. **The log-rank test**. British Medical Journal 2004; 328: 1073.

BILEVEL PROGRAMMING PROBLEM WITH FUZZY PARAMETERS: A FUZZY GOAL PROGRAMING APPROACH

Surapati PRAMANIK[1]

PhD, Assistant Professor,
Nandalal Ghosh B. T. College, Panpur (India)

E-mail: sura_pati@yahoo.co.in

Abstract:

This paper describes how fuzzy goal programming can be efficiently used to solve bilevel programming problems with fuzzy parameters. In the model formulation of the problem, the tolerance membership functions for the fuzzily described objective functions of decision makers are defined by determining individual optimal solution of each of the level decision makers. Since the objectives are potentially conflicting in nature, a possible relaxation of the upper level and lower level decision are considered by providing preference bounds on the decision variables for avoiding decision deadlock. Then fuzzy goal programming approach is used for achieving highest degree of each of the membership goals by minimizing negative deviational variables. Three fuzzy goal programming models are presented. Distance function is used to identify which fuzzy goal programming models offers better optimal solution. A numerical example is presented to demonstrate the potential use of the proposed approach.

Keywords: bilevel programming problem, fuzzy goal programming, fuzzy parameters, deviational variables, distance functions

1. Introduction

Decision making within a hierarchical organization may be characterized by an attempt to satisfy a set of potentially conflicting objectives of different decision making units situated in hierarchical levels as completely as possible in an environment comprised of a set of finite resources, conflicting interest and a set of constraints in order to deal with the situation in which all objectives can not be completely and simultaneously satisfied. Constraints and objectives may be fuzzily described. Hierarchical optimization or multilevel programming (MLP) techniques are extensions of Stackleberg games for solving decentralized planning problems with multiple decision makers (DMs) in a hierarchical organization. Bilevel programming problem (BLPP) is a special case of Multilevel programming problems (MLPPs) of a large hierarchical decision system. Bilevel organization has following common characteristics: two decision makers namely, upper level decision maker (ULDM) and lower level decision maker (LLDM) are located at two different levels; the execution of decision is sequential, from upper level to lower level; each DM independently controls only a set of decision variables and is interested in optimizing his or her objective functions. Although ULDM independently optimizes his or her own objective functions, the decision may be affected by the reaction of the LLDM. Therefore, decision dead lock arises frequently in the decision making situation.

The formal formulation of the linear BLPP is defined by Candler and Towmsly [9] as well as Fortuny-Amat and McCarl [10]. During the last three decades, BLPP as well as MLPP in general for hierarchical decentralized planning problems have been deeply studied in [1-5, 7-11, 13, 17-22] and many methodologies have been proposed to solve them potentially such as economic systems, government policy, warfare, etc. The classical approaches developed so far, for BLPPs have been surveyed by Wen and Tsu [22]. Most of these methods are based on vertex enumeration [8] and transformation approach [9]. The former is to seek a compromise vertex by simplex algorithm based on adjusting upper level control variables. It is rather inefficient, especially for large size problems. The latter involves transforming the lower level programming problem to be constraints of the upper level by its Kuhn-Tucker (K-T) conditions or penalty function. Due to the presence of non-linear or Lagrangian terms appearing in the constraints, the auxiliary problems become complex and sometimes unmanageable. These methods are suitable for crisp environment. Sakawa et al. [17] presented an interactive fuzzy mathematical programming for linear MLPP with fuzzy parameters. Lai [11] at first developed an effective fuzzy approach by using the concept of tolerance membership functions for solving MLPPs in 1996. Shih et al. [18] extended Lai's concept by using non-compensatory max-min aggregation operator for solving MLPPs. Shih and Lee [19] further extended Lai's concept by introducing the compensatory fuzzy operator for solving MLPPs. Sinha [20] studied alternative MLP technique based on fuzzy mathematical programming (FMP). The basic concept of these fuzzy approaches is almost same and re-evaluation of the problem repeatedly by redefining the elicited membership values is essentially needed in the solution search process to obtain a satisfactory solution. So, computational load is also inherently involved in the fuzzy approaches developed so far. Pramanik and Roy [15] proposed fuzzy goal programming (FGP) approach to MLPPs using deviational variables.

Pramanik and Roy [16] proposed FGP approach for solving multi-objective transportation problem with fuzzy parameters. Pramanik and Roy [14] also developed priority based FGP approach for solving multi-objective transportation problem with fuzzy parameters. In this paper, FGP due to Pramanik and Roy [14, 16] is slightly modified and applied for solving BLPP with fuzzy parameters. Three FGP models are formulated for solving BLPPs. Distance function is used for identifying which FGP model offers better optimal solution.

2. Preliminaries

Some basic definitions are here given that will be used in the paper.

Definition 2.1 A fuzzy set $\tilde{A}$ in $\overline{X}$ is defined by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle \mid x \in \overline{X}\}$, where $\mu_{\tilde{A}}(x): \overline{X} \to [0, 1]$ is called the membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership to which $x \in \tilde{A}$.

Definition 2.2 Union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}$(x), $\mu_{\tilde{B}}$(x) is defined by a fuzzy set $\tilde{C}$ whose membership function is defined by $\mu_{\tilde{A}\cup\tilde{B}}$(x) = $\mu_{\tilde{C}}$(x) = max [$\mu_{\tilde{A}}$(x), $\mu_{\tilde{B}}$(x)], x∈ $\overline{X}$.

Definition 2.3 Intersection of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}$(x), $\mu_{\tilde{B}}$(x) is defined by a fuzzy set $\tilde{C}$ whose membership function is defined by $\mu_{\tilde{A}\cap\tilde{B}}$(x) = $\mu_{\tilde{C}}$(x) = min [$\mu_{\tilde{A}}$(x), $\mu_{\tilde{B}}$(x)], x∈ $\overline{X}$.

Definition 2.4 The α-cut of a fuzzy set $\tilde{A}$ of $\overline{X}$ is a non-fuzzy set denoted by ${}^{\alpha}A$ is defined by a subset of all elements x∈ $\overline{X}$ such that their membership functions exceed or equal to a real number $\alpha \in [0, 1)$, i.e. ${}^{\alpha}A = \left[x : \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1), \forall x \in \overline{X}\right]$.

3. Formulation of fuzzy goal programming having fuzzy parameters

Consider the following fuzzy optimization problem:

$$\text{Minimize } \tilde{Z}\left(\overline{X}\right) = \left(\tilde{C}_1\overline{X}, \tilde{C}_2\overline{X}, \ldots, \tilde{C}_K\overline{X}\right)^T \quad (1)$$

$$\text{subject to } \overline{X} \in S = \{\overline{X} \in \Re^n \mid \tilde{A}\overline{X} * \tilde{B}, \overline{X} \geq \overline{0}\}, \quad (2)$$

where $\tilde{C}_k\left(k = 1, 2, \ldots, K\right)$ are n-dimensional vector, $\tilde{B}$ is an m-dimensional vector, $\tilde{A}$ is an m × n matrix, and $\tilde{C}_k$, $\tilde{B}$, and $\tilde{A}$ are fuzzy numbers. Here, the symbol $*$ denotes respectively ≥, =, and ≤. $\overline{X} = (X_1, X_2, \ldots, X_n)^T$. Consider that the problem represented by (1) has fuzzy coefficients, which have possibilistic distributions. Assume that ${}^{\alpha}\overline{X}$ be a solution of (1) where $\alpha \in [0,1)$ represents the level of possibility at which all fuzzy coefficients is feasible.

Let ${}^{\alpha}\left(\tilde{R}\right)$ be the α-cut of a fuzzy number $\tilde{R}$ defined by

$${}^{\alpha}\left(\tilde{R}\right) = \left\{r \in \text{Supp}\left(\tilde{R}\right) \| \mu_{\tilde{R}}(r) \geq \alpha, \alpha \in [0,1)\right\} \quad (3)$$

where $\text{Supp}\left(\tilde{R}\right)$ is the support of $\tilde{R}$. Let ${}^{\alpha}\left(\tilde{R}\right)^L$ and ${}^{\alpha}\left(\tilde{R}\right)^U$ be the lower bound and upper bound of the α-cut of $\tilde{R}$ respectively such that ${}^{\alpha}\left(\tilde{R}\right)^L \leq {}^{\alpha}\left(\tilde{R}\right) \leq {}^{\alpha}\left(\tilde{R}\right)^U$ (4)

Then, for a prescribed value of α, for minimization-type objective function, $\tilde{Z}_k(\overline{X})$ (k = 1, 2, ..., K) can be replaced by the lower bound of its α-cut i.e.

$$^{\alpha}\left(\tilde{Z}_k(\overline{X})\right)^L = \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^L X_j \qquad (5)$$

Similarly, for maximization-type objective function, $\tilde{Z}_k(\overline{X})$ (k = 1, 2, ..., K) can be replaced by the upper bound of its α-cut i.e. $^{\alpha}\left(\tilde{Z}_k(\overline{X})\right)^U = \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^U X_j$ (6)

For inequality constraints

$$\sum_{j=1}^{n} \tilde{A}_{ij} X_j \geq \tilde{B}_i \text{ , } i = 1, 2, ..., m_1, \qquad (7)$$

and $\sum_{j=1}^{n} \tilde{A}_{ij} X_j \leq \tilde{B}_i$, $i = m_1+1, ..., m_2$, (8)

can be rewritten by the following constraints:

$$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^U X_j \geq {}^{\alpha}\left(\tilde{B}_i\right)^L \text{ , } i = 1, 2, ..., m_1 \qquad (9)$$

$$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^L X_j \leq {}^{\alpha}\left(\tilde{B}_i\right)^U \text{ , } i = m_1+1, ..., m_2 \qquad (10)$$

For fuzzy equality constraints

$$\sum_{j=1}^{n} \tilde{A}_{ij} X_j = \tilde{B}_i \quad i = m_2+1, ..., m, \qquad (11)$$

can be replaced by two equivalent constraints

$$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^L X_j \leq {}^{\alpha}\left(\tilde{B}_i\right)^U \qquad (12)$$

and $\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^U X_j \geq {}^{\alpha}\left(\tilde{B}_i\right)^L$ (13)

For proof of equivalency of (11) with (12) and (13), see Lee and Li [12].

Therefore, for a prescribed value of α, the problem represented by (1) can be transformed to the following problem:

Minimize $^{\alpha}\left(\tilde{Z}_k(\overline{X})\right)^L = \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^L X_j$, k = 1, 2, ..., K, (14)

subject to $\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^U X_j \geq {}^{\alpha}\left(\tilde{B}_i\right)^L$, $i = 1, 2, ..., m_1, m_2+1, ..., m$, (15)

$$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^L X_j \leq {}^{\alpha}\left(\tilde{B}_i\right)^U \text{ , } i = m_1+1, ..., m_2, m_2+1, ..., m, \qquad (16)$$

$X_j \geq 0$, j = 1, 2, ..., n. (17)

For simplicity, denote the system constraints (15), (16) and (17) as S`.

For a prescribed value of α, the problem (14) reduces to a deterministic linear programming problem with multiple objectives, which can be solved by applying the FGP proposed By Pramanik and Roy [14, 16].

The resulting membership functions for minimization-type objective functions are defined as:

$$\mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right) = \left[{}^{\alpha}(Z_k)^{-} - \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j\right] \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O}\right], \; k = 1, 2, \dots, K, \quad (18)$$

where the aspired level ${}^{\alpha}(Z_k)^{O}$ and highest acceptable level ${}^{\alpha}(Z_k)^{-}$ are ideal and anti-ideal solutions, respectively, which can be obtained by solving each of the following problem independently:

$${}^{\alpha}(Z_k)^{O} = \underset{X \in S`}{\text{Minimize}} \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{L} X_j, \quad (19)$$

$${}^{\alpha}(Z_k)^{-} = \underset{X \in S`}{\text{Maximize}} \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^{U} X_j, \; k = 1, 2, \dots, K. \quad (20)$$

For maximization-type objective function, ideal and anti-ideal solutions can be similarly obtained.

Assume that all of the fuzzy coefficients are trapezoidal fuzzy numbers. Trapezoidal fuzzy number $\tilde{R}$ can be defined as:

$\tilde{R} = (r^{(1)}, r^{(2)}, r^{(3)}, r^{(4)})$ and the membership function of the trapezoidal fuzzy number (see Figure1) will be interpreted as follows:

$$\mu_{\tilde{R}}(r) = \begin{cases} 0, & r \le r^{(1)}, \\ \dfrac{r - r^{(1)}}{r^{(2)} - r^{(1)}}, & r^{(1)} \le r \le r^{(2)}, \\ 1, & r^{(2)} \le r \le r^{(3)}, \\ \dfrac{r^{(4)} - r}{r^{(4)} - r^{(3)}}, & r^{(3)} \le r \le r^{(4)}, \\ 0, & r \ge r^{(4)} \end{cases} \quad (21)$$

So, an α-cut of $\tilde{R}$ [12] can be expressed by the following interval

$${}^{\alpha}\left(\tilde{R}\right) = \left[{}^{\alpha}\left(\tilde{R}\right)^{L}, {}^{\alpha}\left(\tilde{R}\right)^{U}\right] = \left[(r^{(2)} - r^{(1)})\alpha + r^{(1)}, -(r^{(4)} - r^{(3)})\alpha + r^{(4)}\right] \quad (22)$$

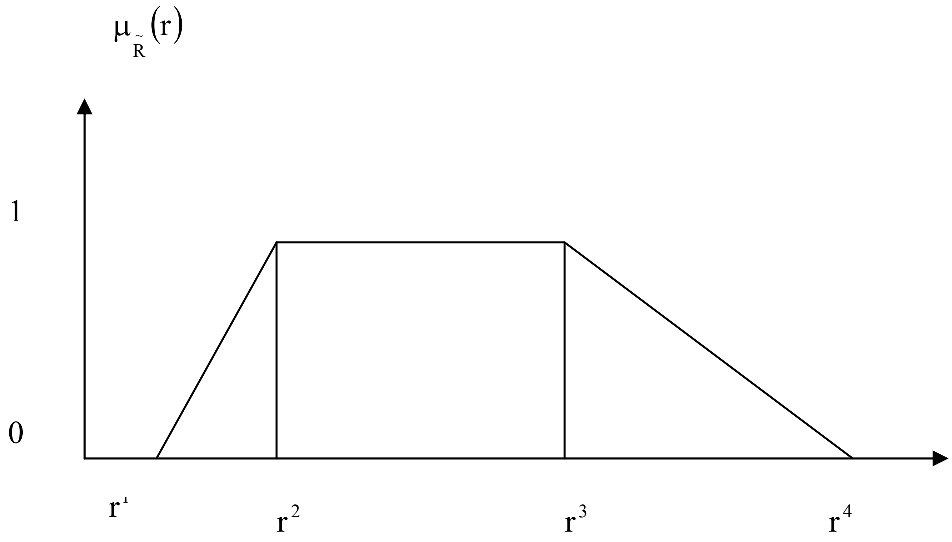

Figure 1. Trapezoidal fuzzy number $\tilde{R}$ = ($r^{(1)}$, $r^{(2)}$, $r^{(3)}$, $r^{(4)}$)

It is to be noted that when $r^{(2)} = r^{(3)}$, $\tilde{R}$ transforms into the triangular fuzzy number, specified by ($r^{(1)}$, $r^{(2)} = r^{(3)}$, $r^{(4)}$);

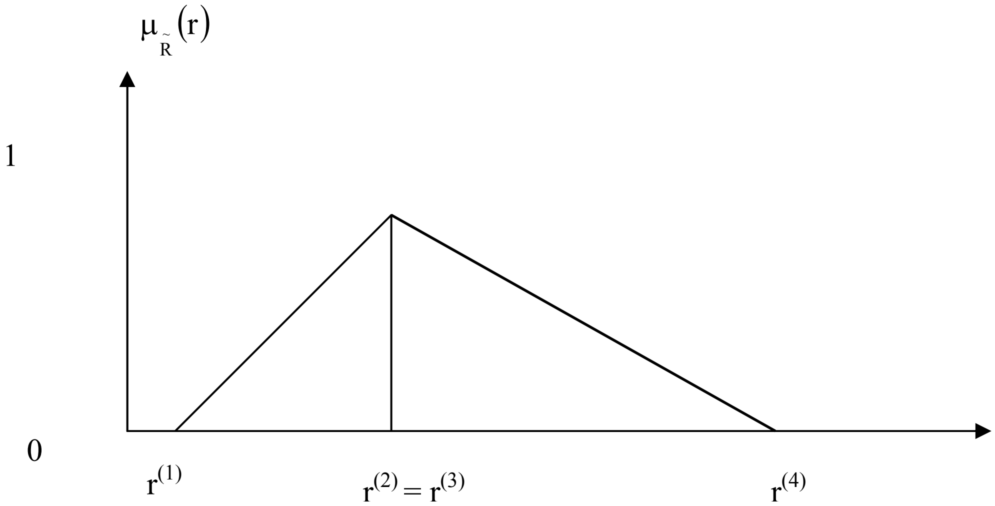

Figure 2. Triangular fuzzy number $\tilde{R}$ = ($r^{(1)}$, $r^{(2)}$= $r^{(3)}$, $r^{(4)}$)

For given value of α, the main interest of the decision maker is to maximize the degree of membership function of the objectives and constraints to the respective fuzzy goals i.e.

Maximize $\mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right)$ (23)

subject to $0 \le \mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right) \le 1$, (24)

$\overline{X} \in S`$ (25)

Here one can adopt Bellman- Zadeh's [6] fuzzy decision based on minimum operator.

$\mu_D(\overline{X}) = \overset{K}{\underset{k=1}{\wedge}} \mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right),\ \overline{X} \in S`$ (26)

The problem (23) can be transformed to the following problem:

Max γ (27)

$\gamma \le \mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right)$, k = 1, 2, ... , K, (28)

$\overline{X} \in S`$ (29)

$\mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right) \in [0, 1]$ (30)

where γ represents minimal acceptable degree of objectives.

The problem (27) can be transformed into linear goal program. The highest value of a membership function is 1. So for the defined membership functions in (27), the flexible membership goals having the aspired level unity can be represented as:

$\mu_k^{\alpha}\left(Z_k\left(\overline{X}\right)\right) + D_k^- - D_k^+ = 1$, k =1, 2, ..., K. (31)

Here D_k^-, D_k^+ are negative and positive deviational variables with $D_k^- \times D_k^+ = 0$. (32)

Any positive deviation from a fuzzy goal implies the full achievement of the membership value unity. Therefore, we assign only negative deviational variables in the achievement function. Therefore, (31) can be written as

${}^{\alpha}\mu_k\left(Z_k\left(\overline{X}\right)\right) + D_k^- = 1$ (32)

Pramanik and Roy [15] used inequality sign for FGP model that is ${}^{\alpha}\mu_k\left(Z_k\left(\overline{X}\right)\right) + D_k^- \ge 1$ for dealing with multilevel programming problem. Since $D_k^- \ge 0$ and there is no possibility of positive deviation, $D_k^- = 0$. Therefore, we omit the extra positive deviational variable and use equality sign as (32).

Under the framework of minsum goal programming, the FGP model of the problem can be explicitly formulated as:

Model (1):

Minimize λ (33)

subject to

$$\frac{\left[{}^{\alpha}(Z_k)^- - \sum_{j=1}^{n} {}^{\alpha}\left(\tilde{C}_{kj}\right)^L X_j\right]}{\left[{}^{\alpha}(Z_k)^- - {}^{\alpha}(Z_k)^O\right]} + D_k^- = 1,\ k = 1, 2, \ldots, K, \quad (34)$$

$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^U X_j \ge {}^{\alpha}\left(\tilde{B}_i\right)^L$, i = 1, 2, ..., m_1, m_2+1, ..., m, (35)

$\sum_{j=1}^{n} {}^{\alpha}\left(\tilde{A}_{ij}\right)^L X_j \le {}^{\alpha}\left(\tilde{B}_i\right)^U$, i = m_1+1, ..., m_2, m_2+1, ..., m, (36)

$\lambda \ge D_k^-$, k = 1, 2, ..., K, (37)

$X_j \ge 0$, j = 1, 2, ..., n, (38)

$D_k^- \geq 0$, k = 1, 2, …, K. (39)

Model (IIa): Minimize $\zeta = (\sum_{k=1}^{K} w_k^- D_k^-)$ (40)

and Model (IIb): Minimize $\xi = \sum_{k=1}^{K} D_k^-$ (41)

subject to the constraints given by (34), (35), (36), (38), and (39)

Using the interval expression (22), the problem (33) can be written as:

Minimize λ (42)

subject to

$$\left\{ {}^{\alpha}(Z_k)^{-} - \sum_{j=1}^{n} \left[C_{kj}^{(1)} + \left(C_{kj}^{(2)} - C_{kj}^{(1)} \right) \alpha \right] X_j \right\} \Big/ \left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O} \right] + D_k^- = 1, \; k = 1, 2, \ldots, K, \quad (43)$$

$$\left[A_{ij}^{(4)} - \left(A_{ij}^{(4)} - A_{ij}^{(3)} \right) \alpha \right] X_j \geq B_i^{(1)} + \left(B_i^{(2)} - B_i^{(1)} \right) \alpha, i = 1, \ldots, m_1, m_2 + 1, \ldots, m, \quad (44)$$

$$\left[A_{ij}^{(1)} + \left(A_{ij}^{(2)} - A_{ij}^{(1)} \right) \alpha \right] X_j \leq B_i^{(4)} - \left(B_i^{(4)} - B_i^{(3)} \right) \alpha, \quad i = m_1 + 1, 2, , \ldots, m_2, m_2 + 1, \ldots, m, \quad (45)$$

$\lambda \geq D_k^-$, k = 1, 2, …, K, (46)

$X_j \geq 0$, j = 1, 2, …, n, (47)

$D_k^- \geq 0$, k = 1, 2, …, K. (48)

Similarly, using the interval expression (22), the problem (40) and (41) can be written as

Minimize $\zeta = (\sum_{k=1}^{K} W_k^- D_k^-)$ (49)

and Minimize $\xi = \sum_{k=1}^{K} D_k^-$ (50)

subject to the constraints given by (43), (44), (45), (47), and (48).

Numerical weight W_k^- associated with negative deviational variable is defined by

$W_k^- = 1/\left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O} \right]$, k = 1, 2, …, K (51)

In Model (IIa), numerical weight W_k^- (k = 1, 2, …, K)is the reciprocal of the admissible violation constant. The numerical weight associated with negative deviational variable represents the relative importance of achieving the aspired level of the fuzzy goal. The larger admissible violation of constants $\left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O} \right]$ indicates less important k-th fuzzy goal. i.e. the larger numerical weight $W_k^- = 1/\left[{}^{\alpha}(Z_k)^{-} - {}^{\alpha}(Z_k)^{O} \right]$, (k = 1, 2, …, K) indicates the more important of the k-th fuzzy goal. When the numerical weights associated with the negative deviational variables are all equal to unity, then Model (IIa) and Model (IIb) become identical. Therefore, Model (IIb) is a special case of Model (IIa).

4. Formulation of BLPP

A BLPP can be defined as a two- person game with perfect information in which each DM moves sequentially from upper level to lower level. This problem has a nested hierarchical structure with two levels of DMs. We consider a BLPP having maximizing type objective function at each level. Mathematically, the problem can be stated as:

$$\underset{\overline{X}_1}{\text{Maximize}}\, Z_1\left(\overline{X}\right) = \tilde{C}_{11}\,\overline{X}_1 + \tilde{C}_{12}\,\overline{X}_2 \quad \text{(Upper level DM's problem)} \qquad (52)$$

$$\underset{\overline{X}_2}{\text{Maximize}}\, Z_2\left(\overline{X}\right) = \tilde{C}_{21}\,\overline{X}_1 + \tilde{C}_{22}\,\overline{X}_2 \quad \text{(Lower level DM's problem)} \qquad (53)$$

subject to

$$\tilde{A}_1\,\overline{X}_1 + \tilde{A}_2\,\overline{X}_2 \le \tilde{B} \qquad (54)$$

$$\overline{X}_1 \ge \overline{0}\,, \overline{X}_2 \ge \overline{0}\,, \qquad (55)$$

$\overline{X}_1 = \{X_1^1, X_1^2, \ldots, X_1^{N_1}\}^T$: decision variables under the control of ULDM

$\overline{X}_2 = \{X_2^1, X_2^2, \ldots, X_2^{N_2}\}^T$: decision variables under the control LLDM

$\overline{Z} = (Z_1, Z_2)^T$, and T denotes transposition; $\tilde{C} = \begin{pmatrix} \tilde{C}_{11} & \tilde{C}_{12} \\ \tilde{C}_{21} & \tilde{C}_{22} \end{pmatrix}$ is the vector of coefficient vectors represented by fuzzy parameters.

Where $\tilde{A}_1$ is $M \times N_1$ and $\tilde{A}_2$ is $M \times N_2$ matrix, $\overline{B}$ is the M component column vector. $\overline{X} = \overline{X}_1 \cup \overline{X}_2$ is the set of decision vector, $N = N_1 + N_2$, total number of decision variables in the system and M is the total number of the constraints of the problem. Z_1, Z_2 are linear and bounded.

4.1. Characterization of Membership Function of BLPP

In the decision making situation, each DM is interested in maximizing his or her own objective function. So, the optimal solution of each DM when calculated in isolation would be considered as the aspiration level of each of the respective fuzzy objective goals. For a prescribed value of α, to construct membership function for maximization-type objective function, $\tilde{Z}_i\left(\overline{X}\right)$ (i = 1, 2) can be replaced by the upper bound of its α-cut i.e.

$$^{\alpha}\left(\tilde{Z}_i\left(\overline{X}\right)\right)^U = \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{ij}\right)^U X_j \,, \; i = 1, 2 \qquad (56)$$

For inequality constraints, $\tilde{A}_1\,\overline{X}_1 + \tilde{A}_2\,\overline{X}_2 \le \tilde{B}$, we write

$$^{\alpha}\left(\tilde{A}_1\right)^L \overline{X}_1 + {}^{\alpha}\left(\tilde{A}_2\right)^L \overline{X}_2 \le {}^{\alpha}\left(\tilde{B}\right)^U \qquad (57)$$

Therefore, for a prescribed value of α, the problem reduces to the following problem:

$$\underset{\overline{X}_1}{\text{Maximize}}\ {}^{\alpha}\left(\tilde{Z}_1(\overline{X})\right)^{U} = \underset{\overline{X}_1}{\text{Maximize}}\ \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{1j}\right)^{U} \overline{X}_j, \quad (58)$$

$$\underset{\overline{X}_2}{\text{Maximize}}\ {}^{\alpha}\left(\tilde{Z}_2(\overline{X})\right)^{U} = \underset{\overline{X}_2}{\text{Maximize}}\ \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{2j}\right)^{U} \overline{X}_j, \quad (59)$$

subject to

$${}^{\alpha}\left(\tilde{A}_1\right)^{L} \overline{X}_1 + {}^{\alpha}\left(\tilde{A}_2\right)^{L} \overline{X}_2 \le {}^{\alpha}\left(\tilde{B}\right)^{U} \quad (60)$$

$$\overline{X}_1 \ge \overline{0}, \overline{X}_2 \ge \overline{0}, \quad (61)$$

For simplicity, denote the system constraints (60) and (61) as S.

For a prescribed value of α, the fuzzy BLPP reduces to deterministic BLPP, which can be solved by using FGP models discussed in section 3.

Let ($\overline{X}_1^1, \overline{X}_2^1;\ {}^{\alpha}\left(Z_1^B\right)^U$) and ($\overline{X}_1^2, \overline{X}_2^2;\ {}^{\alpha}\left(Z_2^B\right)^U$) be the individual optimal decision of the DMU and DML respectively when calculated in isolation,

where $${}^{\alpha}\left(Z_1^B\right)^U = \underset{\overline{X}\in S}{\text{Maximize}}\ {}^{\alpha}\left(\tilde{Z}_1(\overline{X})\right)^{U} = \underset{\overline{X}\in S}{\text{Maximize}} \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{1j}\right)^{U} \overline{X}_j \quad (62)$$

and $${}^{\alpha}\left(Z_2^B\right)^U = \underset{\overline{X}\in S}{\text{Maximize}}\ {}^{\alpha}\left(\tilde{Z}_2(\overline{X})\right)^{U} = \underset{\overline{X}\in S}{\text{Maximize}} \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{2j}\right)^{U} \overline{X}_j \quad (63)$$

If the individual optimal solutions are identical, then optimal compromise solution is automatically reached. However, this rarely happens due to the conflicting objectives. Then the fuzzy objective goals of the ULDM and LLDM appear as:

$${}^{\alpha}\left(\tilde{Z}_1(\overline{X})\right)^{U} \underset{\sim}{\ge} {}^{\alpha}\left(Z_1^B\right)^U,\quad {}^{\alpha}\left(\tilde{Z}_2(\overline{X})\right)^{U} \underset{\sim}{\ge} {}^{\alpha}\left(Z_2^B\right)^U;$$

To formulate membership functions for the maximization type objective functions, we define:

$${}^{\alpha}\left(Z_1^W\right)^L = \underset{\overline{X}\in S}{\text{Minimize}}\ {}^{\alpha}\left(\tilde{Z}_1(\overline{X})\right)^{L} = \underset{\overline{X}\in S}{\text{Minimize}} \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{1j}\right)^{L} \overline{X}_j \quad (64)$$

$${}^{\alpha}\left(Z_2^W\right)^L = \underset{\overline{X}\in S}{\text{Minimize}}\ {}^{\alpha}\left(\tilde{Z}_2(\overline{X})\right)^{L} = \underset{\overline{X}\in S}{\text{Minimize}} \sum_{j=1}^{2} {}^{\alpha}\left(\tilde{C}_{2j}\right)^{L} \overline{X}_j \quad (65)$$

where ${}^{\alpha}\left(Z_i^B\right)^U$, ${}^{\alpha}\left(Z_i^W\right)^L$, (i = 1, 2) are best and worst or ideal and anti-ideal solutions respectively.

Then the resulting membership functions can be defined as:

$$\mu_i\ {}^{\alpha}\left(Z_i(\overline{X})^U\right) = \begin{cases} 1, & \text{if } {}^{\alpha}\left(Z_i(\overline{X})\right)^U \ge {}^{\alpha}\left(Z_i^B\right)^U \\ \dfrac{{}^{\alpha}\left(Z_i(\overline{X})\right)^U - {}^{\alpha}\left(Z_i^W\right)^L}{{}^{\alpha}\left(Z_i^B\right)^U - {}^{\alpha}\left(Z_i^W\right)^L}, & \text{if } {}^{\alpha}\left(Z_i^W\right)^L \le {}^{\alpha}\left(Z_i(\overline{X})\right)^U \le {}^{\alpha}\left(Z_i^B\right)^U \\ 0, & \text{if } {}^{\alpha}\left(Z_i(\overline{X})\right)^U \le {}^{\alpha}\left(Z_i^W\right)^L \end{cases}, \text{ (i= 1, 2)} \quad (66)$$

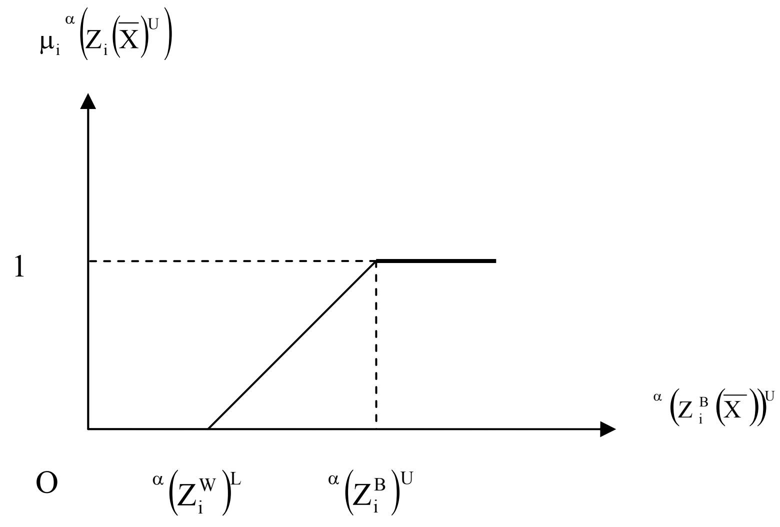

Figure3. Membership function for objective function $^{\alpha}\left(\tilde{Z}_i(\overline{X})\right)^U$ (i = 1, 2).

In BLPP, Mishra [13] considered arbitrary relaxations on decision variables provided by DMs by providing preference bounds on the decision variables. In the proposed approach, DMs provide the upper and lower bounds on the decision variables under their control. Suppose $\left(\overline{X}_i^B - \overline{R}_i^-\right)$, $\left(\overline{X}_i^B + \overline{R}_i^+\right)$ (i = 1, 2) are the upper and lower bounds of decision vector provided by the i-th level DM where $\overline{X}_i^B$ is the individual best solution when calculated in isolation. Here, $\overline{R}_i^-$ and $\overline{R}_i^+$ are the negative and positive tolerance vectors, which are not necessarily same. Generally $\overline{X}_i$ lies between $\left(\overline{X}_i^B - \overline{R}_i^-\right)$ and $\left(\overline{X}_i^B + \overline{R}_i^+\right)$. DMs may prefer to shift the range of $\overline{X}_i^B$ which may be left of $\overline{X}_i^B$ or right of $\overline{X}_i^B$ only depending on the needs and desires of the level DMs in the decision making situation. For example, if $\overline{X}_i = \overline{0}$, then $\overline{X}_i$ should lie on the right of $\overline{0}$. Then DM should assign $\overline{R}_i^- \leq \overline{0}$, $\overline{R}_i^+ \geq \overline{0}$ and $|\overline{R}_i^-| \leq |\overline{R}_i^+|$.If the DM wants the shift towards left of $\overline{X}_i^B$, then $\overline{R}_i^-$ should be assigned positive value while $\overline{R}_i^+$ should be assigned a negative value i.e. $\overline{R}_i^- \geq \overline{0}$, $\overline{R}_i^+ \leq \overline{0}$ and $|\overline{R}_i^-| \geq |\overline{R}_i^+|$. Similarly, if the shift is required to right of $\overline{X}_i^B$, then DM should assign $\overline{R}_i^- \leq \overline{0}$, $\overline{R}_i^+ \geq \overline{0}$ and $|\overline{R}_i^-| \leq |\overline{R}_i^+|$.

Therefore, $\left(\overline{X}_i^B - \overline{R}_i^-\right) \leq \overline{Xi} \leq \left(\overline{X}_i^B + \overline{R}_i^+\right)$, (i=1, 2) (67)

4.2. Formulation of FGP Model

The proposed FGP formulation can be presented as:

Model (I):

Minimize λ (68)

subject to

$$\mu_i^{\alpha}\left(Z_i(\overline{X})^U\right) + D_i^- = 1, (i = 1, 2) \quad (69)$$

$$\left(\overline{X}_i^B - \overline{R}_i^-\right) \le \overline{X}i \le \left(\overline{X}_i^B + \overline{R}_i^+\right), (i = 1, 2) \quad (70)$$

$$^{\alpha}\left(\tilde{A}_1\right)^L \overline{X}_1 + {}^{\alpha}\left(\tilde{A}_2\right)^L \overline{X}_2 \le {}^{\alpha}\left(\tilde{B}\right)^U, \quad (71)$$

$$\lambda \ge D_i^-, (i = 1, 2) \quad (72)$$

$$D_i^- \ge 0, \overline{X}_1 \ge \overline{0}, \overline{X}_2 \ge \overline{0} \quad (73)$$

Model (IIa): Minimize $\zeta = \sum_{i=1}^{2} W_i^- D_i^-$ (74)

and Model (IIb): Minimize $\xi = \sum_{i=1}^{2} D_i^-$ (75)

subject to the constraints (69), (70), (71), (73)

The numerical weight $W_i^- = 1/[{}^{\alpha}\left(Z_i^B\right)^U - {}^{\alpha}\left(Z_i^W\right)^L]$ associated with negative deviational variable is determined as discussed in section 3. By solving FGP formulations (68), if the DMs are satisfied with this solution, then a satisficing solution is reached. Otherwise, the DMs should provide new tolerance limits for the control variables until a satisficing solution is reached. In general, considering a set of possible relaxation offered by DMs, the solution becomes satisficing for both level DMs. Similarly, other two FGP formulations are solved.

5. Selection of compromise solution

The concept of utopia point (the ideal-point) and the use the distance function for group decision analysis has been studied by Yu [23]. Since the aspired level of each of the membership goals is unity, the point consisting of the highest membership value of each of the goals would represent the ideal point. The distance function can be defined as:

$$L_P = \left[\sum_{k=1}^{K}\left[1 - \mu_k^{\alpha}\left(Z_k(\overline{X})^U\right)\right]^p\right]^{1/p}, p \ge 1; (k = 1, 2, \ldots, K). \quad (76)$$

Where $\mu_k^{\alpha}\left(Z_k(\overline{X})^U\right)$ is the membership value for the solution $\overline{X}$. Here, we consider p =1, 2, ∞ only. Now, it can be easily realized that the solution for which L_p is minimum would be the most satisfactory solution. Here, distance function is used only to identify which FGP model (Model I, Model IIa, Model IIb) gives better optimal solution.

6. FGP algorithm for BLPP with fuzzy parameters

Step1. For specified value of α, to construct membership functions for the objective functions of the DMs, the upper and lower bounds of their α-cuts are defined. Similarly, for inequality constraints, upper and lower bounds of their α-cuts are defined.

Step2. Calculate the individual maximum and minimum values for lower and upper α-cuts of the objective functions subject to constraints (60) and (61).

Step3. Determine the weight $W_i^- = 1/[{}^{\alpha}\left(Z_i^B\right)^U - {}^{\alpha}\left(Z_i^W\right)^L]$, (i = 1, 2).

Step4. Construct the membership function $\mu_i^{\alpha}\left(Z_i\left(\overline{X}\right)^U\right)$, (i = 1, 2).

Step5. Consider the preference bounds on the decision vectors provided by the decision makers under their control such that $\left(\overline{X}_i^B - \overline{R}_i^-\right) \le \overline{X}i \le \left(\overline{X}_i^B + \overline{R}_i^+\right)$, (i =1, 2).

Step6. Formulate the three FGP models.

Step7. Solve the three FGP models.

Step8. Compute the distance function for optimal solution obtained from three models.

Step9. Find the optimal solution for which the distance function is minimal. This optimal solution will be the compromise solution for the BLPP.

7. Numerical Example

$$\underset{X_1, X_2}{\text{Maximize}}\,\overline{Z}_1 = \tilde{5}X_1 + \tilde{6}X_2 + \tilde{4}X_3 + \tilde{2}X_4 \qquad \text{(Upper-level)}$$

$$\underset{X_3, X_4}{\text{Maximize}}\,\overline{Z}_2 = \tilde{8}X_1 + \tilde{9}X_2 + \tilde{2}X_3 + \tilde{4}X_4 \qquad \text{(Lower-level)}$$

subject to

$$\begin{aligned} &\tilde{3}X_1 + \tilde{2}X_2 + X_3 + \tilde{3}X_4 \le \tilde{40} \\ &\tilde{2}X_1 + \tilde{4}X_2 + X_3 + \tilde{2}X_4 \le \tilde{35} \\ &X_1 + \tilde{2}X_2 + X_3 + \tilde{2}X_4 \le \tilde{30} \\ &X_1, X_2, X_3, X_4 \ge 0 \end{aligned} \qquad (77)$$

where all the fuzzy numbers are assumed as triangular fuzzy numbers and are given as follows:

$$\tilde{2} = (0,2,3), \tilde{3} = (2,3,4), \tilde{4} = (3,4,5),$$
$$\tilde{5} = (4,5,6), \tilde{6} = (5,6,7), \tilde{8} = (6, 8, 10),$$
$$\tilde{9} = (8, 9, 10), 3\tilde{0} = (28,30,32),$$
$$3\tilde{5} = (33,35,37), 4\tilde{0} = (35,40,45)$$

By replacing the fuzzy coefficients by their α-cuts, problem (77) can be written as

$$\underset{X_1, X_2}{\text{Maximize}}\ {}^{\alpha}(Z_1)^U = (6-\alpha)X_1 + (7-\alpha)X_2 + (5-\alpha)X_3 + (3-\alpha)X_4$$
$$\underset{X_3, X_4}{\text{Maximize}}\ {}^{\alpha}(Z_2)^U = (10-2\alpha)X_1 + (10-\alpha)X_2 + (3-\alpha)X_3 + (5-\alpha)X_4$$

subject to

$(2+\alpha)\,X_1 + 2\alpha X_2 + X_3 + ((2+\alpha)\,X_4 \le 45\text{-}5\alpha$

$2\alpha\ X_1 + (3+\alpha)X_2 + X_3 + 2\alpha X_4 \le 37\text{-}2\alpha$

$X_1 + 2\alpha X_2 + X_3 + 2\alpha X_4 \le 32\text{-}2\alpha$

$X_1, X_2, X_3, X_4 \ge 0$

For, α = .5, let the decision makers provide the preference bounds to the decision variables

$0 \leq X_1 \leq 15$,

$0 \leq X_2 \leq 4$,

$0 \leq X_3 \leq 10$,

$0 \leq X_2 \leq 4$

Then FGP Model (1) offers the solution

λ^* =0.1220651, X_1^* =11.57139, X_2^* = 4, X_3^* = 9.571533, X_4^* = 0 with Z_1^* = 132.7145, Z_2^* =166.0713, μ_{Z_1} = 0.8779349, μ_{Z_2} = 0.8779349, L_1= 0.2441301, L_2 = 0.1726261, L_∞= 0.1220651

FGP Model (IIa) offers the solution

ζ^* = 0.1422829E-02, X_1^* =11.4, X_2^* = 4, X_3^* = 10, X_4^* = 0;

Z_1^* = 133.7, Z_2^* =165.6, μ_{Z_1} = 0.884454, μ_{Z_2} = 0.8754433, L_1= 0.2401026, L_2 = 0.1698977, L_∞=0.1245567.

FGP Model (IIb) offers the solution

ξ^* = 0.2401026, X_1^* =11.4, X_2^* = 4, X_3^* = 10, X_4^* = 0;

Z_1^* = 133.7, Z_2^* =165.6, μ_{Z_1} = 0.884454, μ_{Z_2} = 0.8754433, L_1= 0.2401026, L_2 = 0.1698977, L_∞=0.1245567.

Table1. Comparison of distances for the optimal solutions of example 1 based on FGP Models

Method	Z_1^*, Z_2^*	μ_{Z_1}, μ_{Z_2}	L_1	L_2	L_∞
FGP Model (I)	132.7145, 166.0713	.8779349, .8779349	.2441301	.1726261	.1220651
FGP Model (IIa)	133.7, 165.6	.884454, .8754433	.2401026	.1698977	.1245567
FGP Model (IIb)	133.7, 165.6	.884454, .8754433	.2401026	.1698977	.1245567

On comparing L_1, and L_2 we see that FGP model (IIa) and (IIb) offer better optimal solution. On comparing, L_∞ we see that FGP model (I) offers better optimal solution.

8. Conclusions

In this paper, FGP due to Pramanik and Roy [14, 16] is slightly modified and applied for solving BLPPs with fuzzy parameters. It is an alternative way to solve BLPPs with fuzzy parameters. Distance function is used to identify which FGP model offer better compromise optimal solution. The proposed approach can be extended to optimization problems in different areas, such as decentralized planning problems, agricultural planning problems and other real world multi-objective programming problems involving fuzzily described different parameters. The proposed approach can also be extended for multilevel multi-objective programming problem with fuzzy parameters.

References

1. Anandalingam, G. **A mathematical programming model of decentralized multi-level systems**. *Journal of the Operational Research Society*, 39 (11), 1988, pp.1021–1033.
2. Arora, S. R., and Gupta, R. **Interactive fuzzy goal programming approach for bilevel programming problem**, *European Journal of Operational Research*, 194, 2009, pp. 368-376.
3. Baky, I. A. **Fuzzy goal programming algorithm for solving decentralized bi-level multi-objective programming problems,** *Fuzzy Sets and Systems*, 160, 2009, pp.2701-2713.
4. Bard, J. F. **An efficient point algorithm for a linear two-stage optimization problem,** *Operations Research*, 31, 1983, pp.670– 684.
5. Bard, J. F. **Coordination of a multidivisional organization through two levels of management**, *Omega*, 11, 1983, pp.457– 468.
6. Bellmann, R. F., and Zadeh, L. A. **Decision-making in a fuzzy environment**, *Management Sciences*, 17, 1970, pp.141–164.
7. Bialas, W. F., and Karwan, M. H. **Two level linear programming**, *Management Science*, 30, 1984, pp.1004–1020.
8. Bialas, W. F., and Karwan, M. H. **On two-level linear optimization**, *IEEE Transaction Automatic Control*, 27, 1982, pp. 221 – 214.
9. Candler, W., and Townsley, R. **A linear bilevel programming problems**, *Computers & Operations Research*, 9, 1982, pp.59-76.
10. Fortuny-Amat, J., and McCarl, B. **A representation and economic interpretation of a two-level programming problem**, *Journal of the Operational Research Society*, 32, 1981, pp.783-792.
11. Lai, Y. J. **Hierarchical Optimization: A satisfactory solution**, *Fuzzy Sets and Systems*, 77, 1996, pp. 321 – 335.
12. Lee, E. S., and Li, R. J. **Fuzzy multiple objective programming with Pareto optimum**, *Fuzzy Sets and Systems*, 53, 1993, pp.275 – 288.

13. Mishra, S. **Weighting method for bi-level linear fractional programming problems**, *European Journal of Operational Research*, 183, 2007, pp.296-302.

14. Pramanik, S., and Roy, T. K. **Multiobjective transportation model based on priority based fuzzy goal programming**, *Journal of Transportation Systems Engineering and Information Technology*, 7 (3), 2008, pp.40-48.

15. Pramanik, S., and Roy, T. K. **Fuzzy goal programming approach to multilevel programming problems**, *European Journal of Operational Research*, 176 (2), 2007, pp. 1151-1166.

16. Pramanik, S., and Roy, T. K**. A fuzzy goal programming technique for solving multi-objective transportation problem**, *Tamsui Oxford Journal of Management Sciences* 22 (1), 2006, pp.67-89.

17. Sakawa, M., Nishizaki, I. and Uemura, Y. **Interactive fuzzy programming for multi-level linear programming problems with fuzzy parameters**, *Fuzzy Sets and Systems*, 109 (1), 2000, pp.03– 19.

18. Shih, H. S., Lai, Y. J. and Lee, E. S. **Fuzzy approach for multi-level programming problems**, *Computers & Operations Research*, 23, 1996, pp.73–91.

19. Shih, H. S., and Lee, E. S. **Compensatory fuzzy multiple level decision making**, *Fuzzy Sets and Systems*, 14, 2000, pp.71–87.

20. Sinha, S. **Fuzzy programming approach to multi-level programming problems**, *Fuzzy Sets and* Systems, 136, 2003, pp.189 – 202

21. Wang, G., Wang, X., and Wan, Z. **A fuzzy interactive decision making algorithm for bilevel multi-followers programming with partial shared variables among followers,** *Expert Systems with Applications*, 36 (7), 2009, pp.10471-10474.

22. Wen, U.P., and Hsu, S.T. **Linear bilevel programming problems-a review**, *Journal of the Operational Research Society*, 42, 1991, pp. 125–133.

23. P.L. Yu. **Multiple criteria decision making: concepts, techniques, and extensions**, Plenum Press, 1985, New York.

---

[1]Dr. Pramanik is an Assistant Professor of Mathematics, Nandalal Ghosh B. T. College, Panpur (India). He received Ph. D. in Mathematics from BESU, Shibpur, India. He is a senior life member of Operation Research Society of India, ISI (Kolkata) and Calcutta Mathematical Society. His current research interest includes fuzzy optimization, grey system theory, game theory, and neutrosophy. He received Golakpati Roy Memorial Award. His paper together Partha Pratim Dey was awarded best paper in West Bengal State Science and Technology Congress (WBSSTC)-2011 in mathematics. His paper together with Manjira Saha was awarded best paper in WBSSTC-2010 in Social Sciences. His paper together with Sourendranath Chakrabarti and T. K. Roy was awarded best paper in WBSSTC-2008 in mathematics. His publications have appeared in International Journal of Computer Applications, European Journal of Operational Research, Journal of Transportation Systems Engineering and Information Technology, Tamsui Oxford Journal of Management Sciences, Notes on Intuitionistic Fuzzy Sets.

THE SPENDING EXPLOSION: POSITIONAL EXTERNALITIES AND EXPONENTIAL CONSUMPTION GROWTH[1]

Michael A. LEWIS[2,3]
Hunter College School of Social Work, New York, USA

E-mail: michael.a.lewis@hunter.cuny.edu

Abstract:

Since the 1960s, The United States has experienced increased income inequality. Economist Robert Frank has argued that this increase in inequality has resulted in an expenditure cascade as people have tried to maintain their relative socioeconomic statuses by imitating the spending patterns of those in their reference groups. Although some researchers have tried to empirically determine the extent to which Frank's assessment is correct, none have focused on the implications of Frank's argument for the dynamics of consumption. That is, none have focused on how time series of measures of consumption should look, assuming Frank is right. This paper does exactly that. Drawing on a mathematical model from population biology, it is argued that if Frank is correct, measures of consumption should exhibit exponential growth. It was found that the exponential growth models provided excellent fits to available data on United States consumption measures.

Key words: Spending Explosion; indicators; Positional Externalities; Exponential Consumption Growth

In 1967, the lowest quintile of United States (U.S.) households had 4.0% of household income, the next fifth 10.8%, the fifth above that one 17.3%, the next fifth 24.2%, and the highest fifth 43.6% of household income. By 2009, these percentages had changed to 3.4%, 8.6% 14.6%, 23.2%, and 50.3%. That is, the top fifth of households went from having about 11 times more income to 15 times more than the bottom fifth (U.S. Census Bureau, 2010). U.S. economist Robert Frank has argued that this growth in income inequality has led to what he calls expenditure cascades resulting from positional externalities. Positional externalities result when the consumption patterns of some people influences what others desire to consume. That is, A may consume something and, because B judges how well off he[4] is by comparing what he has to what A has, B decides to consume it too. As will be seen below, this can lead to a cascade of "imitative spending" where A buys something which leads B to buy it because B compares herself to A. But because B buys it, this lads C to buy it because C compares their status to B's, etc. (Frank, 2010a and Frank 2010b).

A number of researchers have taken an interest in Frank's work on positional externalities and conducted empirical work in an effort to assess the degree to which it they exist (Vendrik and Hirata, 2010; Fischer and Torgler, 2010; Torhler, Schmidt, and Frey, 2010, Rablem, 2008; Brown, Bulte, and Zhang, 2010; Schaffner and Torgler, 2010; Solnick and Hemenway, 1998). There has been little work, however, on the implications of the existence of such externalities, and associated expenditure cascades, for trajectories of consumption. This paper focuses on such implications. It will argue that expenditure cascades can result in spending patterns similar to what is found in "runaway" population growth, and that, therefore, a mathematical model often used in population biology is applicable to modeling the dynamics of several measures of consumption in the U.S.

Positional Externalities and Expenditure Cascades

Frank (2010a and Frank 2010b) provides an extensive discussion of the concepts of positional externalities and expenditure cascades. Externalities, in general, exist when market transactions affect others whom are not parties to these exchanges. For example, a firm may buy someone's labor to use in production of some consumption good. The firm benefits from having access to labor, the worker benefits from having access to a job, and consumers benefit from having access to the good. But suppose a by-product of production of the good is some kind of pollution. Those who are not owners of the firm, employees at the firm, or consumers of the product, but in the vicinity of the firm, suffer from this pollution, the resulting externality.

Frank relates this notion of externality to the increase in income inequality referred to above. Similar to many sociologists, he tells us that people often judge how well they are doing by comparing their state to those in their reference group. Members of one's reference group may be in the same socioeconomic class one is in or in a class slightly above it. Given that this is the case, if a member of one's reference group, say A, consumes a good, this may lead others in that group to consume the good as well. This is because if A consumes the good and B does not, A may now be relatively better off than B, with, of course, B being relatively worse off than A. In other words, A's consumption has altered the relative standing of B, and this is the sense in which there is a positional externality. As suggested above, Frank's key insight comes from considering how increasing inequality can affect this process. As the rich come to possess more of a nation's total household income, their expenditures tend to increase. This may lead others, even those of slightly less socioeconomic means, to increase their spending in order to regain a loss in their relative position. This may, in turn, lead others, even lower on the socioeconomic ladder, to increase their spending for the same reason. This process may cascade all the way down the socioeconomic ladder, at least well into the middle class, as people try to maintain their relative position by comparing what they have to what others have.

Frank's focus seems to be on how this process gets started by a change in spending on the part of the rich. But if people judge how well they're doing by comparing what they have to what those slightly above them have, it's also the case that the better off may judge how they're doing by comparing themselves to what others slightly below them have. If all of these upward and downward comparisons obtain, there may end up a situation where we have a perpetual expenditure cascade. Increased spending generates more spending, which generates more spending, etc. This description of an expenditure cascade has the "markings" of a "runaway" or "out of control" growth process. One of the more common of such processes, discussed in the scientific literature, is exponential population growth, such as that associated with bacteria and other organisms when there are no constraints on such growth or those constraints are not yet binding. In the biological literature (Gotelli, 2001) such growth is modeled with the following differential equation:

$$dN/dt = rN \quad (1)$$

Here N is the number of individuals in the population at time t, t is time, and r is a constant called the instantaneous rate of increase. In more detail, r = b – d, where b is the instantaneous birth rate and d the instantaneous death rate. When b = d, it can be seen from equation 1 that the population is constant over time, when d exceeds b the population is declining, and when b exceeds d, it is growing. Thus, runaway growth implies that b must exceed d.

Equation 1 is an ordinary differential equation that can be solved analytically by the separation of variables method. If we divide both sides of the equation by dt and N and integrate, we get:

$$N_t = N_0e^{rt} \quad (2)$$

where N_0 is the population size at time 0 or the initial population size. Equation 2 is the mathematical representation of exponential growth. This equation is used to model growth in many areas of science but the use that's most relevant to this paper is its use in modeling population dynamics.

According to biologists, the exponential model of population growth applies when there are no density-independent or density-dependent checks on growth or when these checks are not yet binding. Examples of density-independent checks would be droughts, hurricanes, and other weather related and climatic phenomena. Examples of density-dependent checks would be the presence of other organisms whether members or one's own species or not.

The expenditure cascade described above can lead to dynamics represented by equations 1 and 2. If we replace N with C we end up with:

$$dC/dt = rC \quad (3)$$

and

$$C_t = C_0e^{rt} \quad (4)$$

Here C is a measure of real (inflation adjusted) consumption and C_0 is real consumption at time 0 or initial consumption.

Many people, of course, have incomes that are too constrained for them to be able to engage in the expenditure cascade without access to borrowed money. Thus, if the exponential model holds, we should see exponential growth in measures of debt as consumers try to maintain their relative socioeconomic statuses. That is we would expect:

$$dC_{debt}/dt = rC_{debt} \quad (5)$$

$$C_{tdebt} = C_{0debt}e^{rt} \quad (6)$$

$$dM/dt = rM \quad (7)$$

$$M_t = M_0e^{rt} \quad (8)$$

Here C_{debt} is a measure of real consumer debt, C_0 is real consumer debt at time 0, M is a measure of residential mortgage debt, and M_0 is residential mortgage debt at time 0.

From a qualitative point of view, equations 3-8 represent a system of runaway spending financed by runaway debt all in an effort to "keep up with" and "stay ahead of the joneses." The question to ask, of course, is whether available data are consistent with such a representation.

Data

The Economic Report of the President (2010) is a yearly document written by the Chairman of the Council of Economic Advisers in an effort to highlight the nation's economic progress. The report contains tables of all kinds of data that purports to measure important economic variables (Government Printing Office, 2010). Four of these tables (Tables B-16, B-60, B-76, and B-77) provided the data for this paper. B-16 contains data on personal consumption expenditures ($billion/year), B77-on consumer debt ($million/year), B-76 on mortgage debt ($billion/year), B-60 on the consumer price index (a pure number). The time periods covered by these data differed so in order to conduct the analysis, I used the years 1965 to 2008 because these were the years for which data were available for all four variables.

The first three tables mentioned above contained nominal data. If I had simply based my analysis on these nominal time series, it would have been misleading. I suspect that Frank's concern regarding expenditure cascades is about real expenditures. That is, he's not just interested in spending increases that result merely from higher prices or inflation. He's concerned about how people are really increasing their spending not because they face higher prices but because they want to maintain their relative statuses by keeping up with and (I added) staying ahead of the joneses. An analysis based on an adjustment to these nominal variables seemed to be the better way to proceed.

To adjust personal consumption, consumer debt, and mortgage debt for inflation, I followed the standard method of dividing them by the decimal form of the consumer price index. These adjusted variables are the focus of the analysis discussed below.

Analysis and Results

With equations 3-8 in mind, I fit three exponential models to the Economic Report of the President data displayed in Figures 1, 2, and 3.

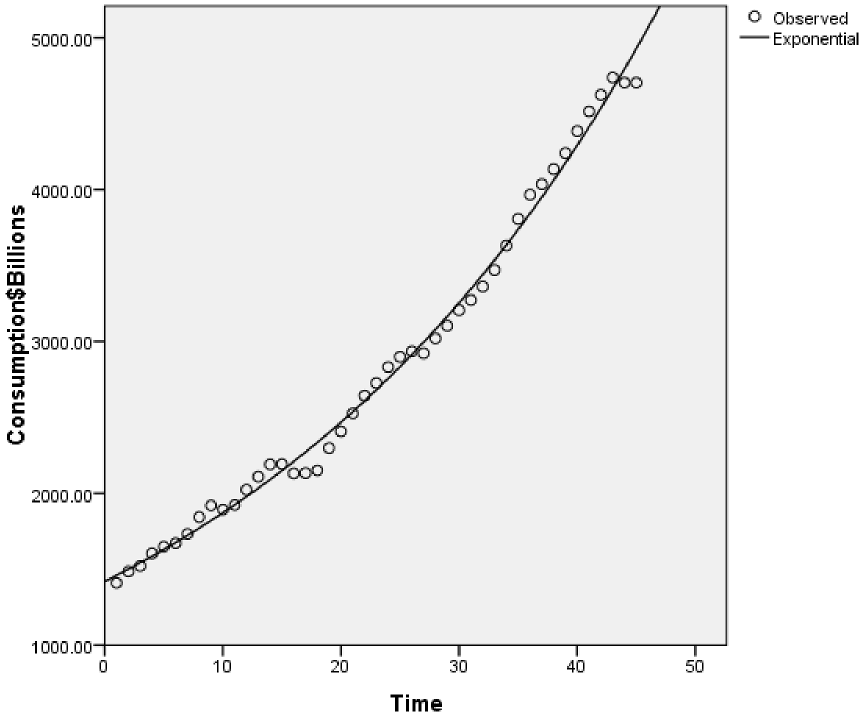

Figure 1 Time Series of Inflation Adjusted Personal Consumption Spending

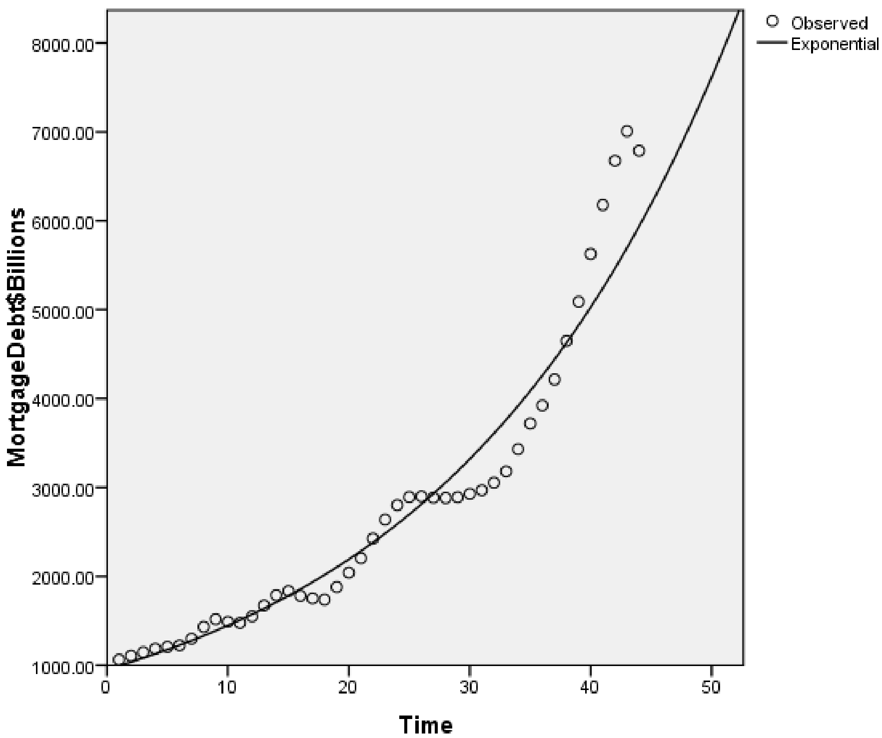

Figure 2 Time Series of Inflation Adjusted Mortgage Debt

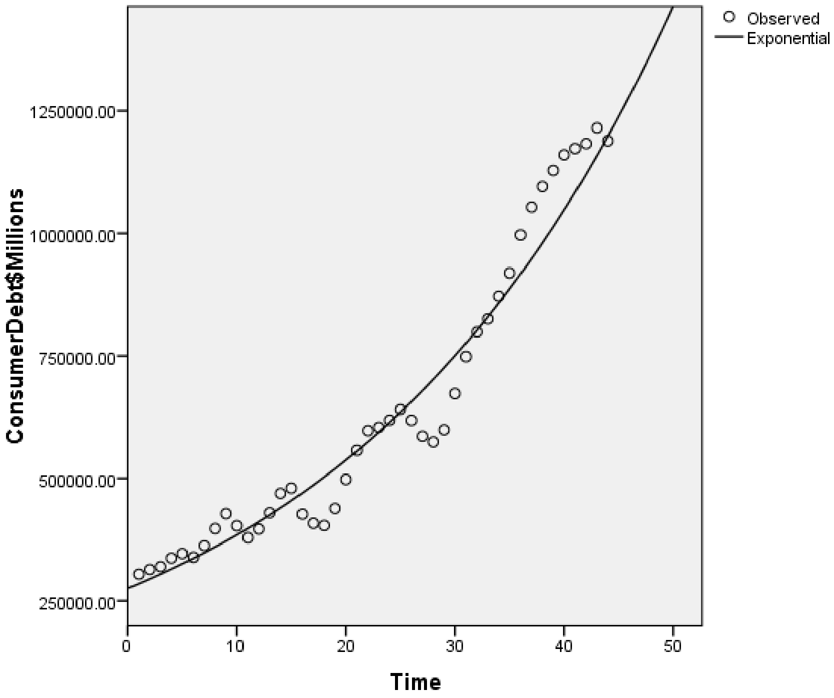

Figure 3 Time Series of Inflation Adjusted Consumer Debt

Figure 1 displays the time series data on inflation adjusted personal consumption and the exponential model fitted to these data. The equation for this model is:

$$C_t = 1419e^{.03t} \tag{9}$$

The adjusted r^2, a measure of goodness of fit, for this model was .99, about as good as it gets for models of social processes.

Figure 2 shows the time series for inflation adjusted mortgage debt. The equation fitted to these data was:

$$M_t = 955e^{.04t} \tag{10}$$

The adjusted r^2 for this model was .97, also an excellent fit.

Lastly, Figure 3 displays the series for inflation adjusted consumer debt with the following fitted model:

$$C_{tdebt} = 275681e^{.03t} \tag{11}$$

The fit of this model was .96, a slightly worse fit than the first two models but still excellent by social science standards.

All of these models were fitted to relatively short available time series. Ideally, one would want to use them to make predictions and then test these against real data that were not used to construct the model. Such data are not yet available, however, so this use of the models will have to await future work.

Discussion

Economist Robert Frank has insightfully connected increasing income inequality to a cascade of competitive spending, as people try to maintain their relative standing or status. Although there has been a good deal of work to assess Frank's idea, there has been little on what should be expected of the dynamics of consumption measures, assuming he is right. It was argued that the expenditure cascade he describes sounds a lot like descriptions of runaway population growth and that the same exponential model used to describe population explosion might also be relevant to what might be called expenditure explosion. Time series data on inflation adjusted personal consumption, consumer debt, and mortgage debt seemed to be well fit by exponential models, although the fit for consumer debt was slightly worse than for the other two models.

I stated earlier in this paper that the exponential model of population growth applies when there are no constraints on such growth or they are not yet binding. Frank (2010a) and Frank (2010b) propose a policy induced constraint on exponential consumption growth. He argues that an increased marginal tax rate on the rich may constrain their consumption enough that their role as the engine of the expenditure cascade will be disrupted. Of course, other economic changes may curtail exponential expenditure growth too. In fact, this may already be happening. The U.S. (and much of the rest of the world) has just gone through and are still feeling the effects of the worse economic downturn since the Great Depression. High unemployment, high mortgage foreclosure rates, and increased savings have resulted from this increase (Mui, 2010; Nance, 2010; and U. S. Department of Labor, 2010). It remains to be seen whether these will check exponential consumption growth, as increased population density or bad weather might check such growth in a biological population.

References

1. Alpizar, Francisco; Carlsson, Fredrik; Johansson-Stenman, Olof., **How Much do we Care About Absolute Versus Relative Income and Consumption**. Journal of Economic Behavior and Organization, 56, 2005, 405-421.

2. Brown, Philip H.; Bulte, Erwin; and Zhang, Xiaobo, **Positional Spending and Status Seeking in rural China** www.colby.edu/economics/faculty/phbrown/papers/BrownBulteZhangJDE2010.pdf (retrieved on September 20, 2010)

3. Fischer, Justina A. V. and Torgler, Benno, **Does Envy Destroy Social Fundamentals? The Impact of Relative Position on Social Capital.** www.yak.edu/leitner/resources/docs/ycias_relative_income_position_social_capital_rev.pdf (retrieved on May 20. 2010).

4. Frank Robert, **Are Positional Externalities Different from Other Externalities?** www.nd.edu/~adult/activities/documents/FrankArePositionalExternalitiesDifferent.pdf (retrieved on May 20, 2010).

5. Frank, Robert, **Luxury Fever: Money and Happiness in an Era of Excess.** New Jersey: Princeton University Press, 2010

6. Gotelli, Nicholas J., **A Primer of Ecology.** Massechusetts: Sinauer Associates, Inc., 2010

7. Government Printing Office, **Economic Report of the President Table B-16**. http://www.gpoaccess.gov/eop/tables10.html (retrieved on September 20, 2010).

8. Government Printing Office, **Economic Report of the President Table B-60.** http://www.gpoaccess.gov/eop/tables10.html (retrieved on September 20, 2010).

9. Government Printing Office, **Economic Report of the President Table B-76.** http://www.gpoaccess.gov/eop/tables10.html (retrieved on September 20, 2010).

10. Government Printing Office, **Economic Report of the President Table B-76.** http://www.gpoaccess.gov/eop/tables10.html (retrieved on September 20, 2010).

11. Mui, Ylan Q., **U.S. Savings Rate at Highest Level inYear Data Show.** Washington Post Online Edition, http://www.washingtonpost.com/wp-dyn/content/article/2010/08/03/AR2010080305317.html (retrieved on September 1, 1010).

12. Nance, Scott, **The Crisis Continues: U.S. Foreclosure Rates to Double This Year Analyst Predicts.** The Democratic Daily, http://thedemocraticdaily.com/2010/02/17/the-crisis-continues-u-s-foreclosure-rates-to-double-this-year-analyst-predicts/ (retrieved on September 1, 2010).

13. Rablen, Matthew D. Rablen. **Relativity, Rank, and the Utility of Income.** The Economic Journal, 118, 528, 2008, 801-821.

14. Schaffner, Markus and Torgler, Benno, **Meet the Joneses: An Empirical Investigation of Reference Groups** in Relative Income Position Comparisons. www.crema-research.ch/paper/2009_13.pdr (retrieved on May 1, 2010).

15. Solnick, Sara J., **Is More Always Better? A Survey of Positional Concerns**. Journal of Behavior and Organization, 37, 3, 1998, 373-383.

16. Tongler, Benno, Schmidt, Sascha L.; and Frey Brunno S., **Relative Income Position and Performance: An Empirical Panel Analysis**. http://time.dufe.edu.cn/wencong/freyliewwp268.pdf (retrieved on May 20, 2010).

17. U.S. Census Bureau, **Table A-2.** www.census.gov/hhhes/www?income?data/historical/inequality/taba2.pdf (retrieved on May 20, 2010).

18. U.S. Department of Labor, **Unemployment Rate**. http://www.bls.gov/ (retrieved on September 10, 2010).

19. Vendrik, Maarten and Hirata, **Experienced Versus Decision Utility of Income: Relative or Absolute Happiness**. http://arno.unimass.nl/show.cgi?fid=396 (retrieved on May 20, 2010).

---

[1] **Acknowledgement**
Thanks to Steve Strogatz for his helpful comments on this paper.

[2] **Correspondence:** Michael A. Lewis, 129 East 79th Street, New York, NY 10075

[3]Michael Lewis holds a masters degree in social work From Columbia University and a doctorate in Sociology from the City University of New York Graduate Center. He teaches courses in social welfare policy and political economy at the Hunter College School of Social Work. Lewis' research interests are in poverty, social welfare policy, and quantitative methods. His work, some co-authored with Eri Noguchi, has appeared in a number of peer-review journals.

[4] In an effort to be gender equitable, I'll sometimes use "he," sometimes "she," and sometimes "their"(out of respect for transgendered persons).

NATURAL RESOURCE MANAGEMENT IN THE CONTEXT OF SUSTAINABLE GROWTH

Gheorghe VLASCEANU
PhD, University Professor, Bucharest University of Economic Studies

E-mail: gheorghevlasceanu@rei.ase.ro

Abstract:

Present realities show that the 21st century is the period of the greatest discoveries and transformations of the human civilization but, at the same time, of the most complex and, quite often, unsuspected effects on life. The demographic explosion and the unprecedented progress of all activity branches were naturally followed by an increase in the demand for raw materials and energy, and this in turn showed the need for thoughtful exploitation of natural resources so that the development of society may be a sustainable one. Moderate exploitation of resources has become essential given the fact that the sustainable development of human society directly depends on the sustainable exploitation of natural resources. A sustainable exploitation of resources leads to the preservation of biodiversity and atmospheric stability as well as to a steady base of resources.

Key words: Natural resources, sustainable development, sustainable exploitation, biodiversity, demographic explosion, the Brundtland Report

Motto:

To waste, to destroy our natural resources, to skin and exhaust
the land instead of using it so as to increase its usefulness
will result in undermining in the days of our children the very prosperity
which we ought by right to hand down to them amplified and developed.
(Theodore Roosevelt[*])

In a world of constant changes and witnessing an unprecedented demographic explosion, natural resources, both regenerating and non-regenerating, and their efficient exploitation often come to the society's attention.

The present paper aims to analyze the way in which natural resource exploitation affects the environment and to come with viable solutions leading to an efficient exploitation, following the requirements of sustainable development.

Natural resources represent the total mineral and ore deposits, cultivable lands, forests and waters that a country possesses,[1] which means that natural resources are substances that appear naturally and which are considered valuable in their natural form, without being modified. Natural resources are considered to be land, forests/wood, minerals and other natural goods which, when extracted from their natural environment, can be transformed in goods the use of which implies their direct consumption. A country's natural resources determine not only its wealth, but also its status in the world economic system, by determining its political influence. Developed states are less dependent on natural resources for wealth because they have a base in the infrastructure capital for production.

Resources represent physical or abstract elements which, from a dimensional standpoint, are characterized by four attributes: quantity, quality, time and space.[2] Resources are used in order to meet human necessities. In other words, natural resources are environmental goods drawn into the economical circuit in order to produce goods and services necessary to man.

Throughout history, natural resources have had an important influence over the evolution of the human society, over the development of national economies and that of the world economy. Today, the increasing need for physical resources represents a concern for scientists (most of the resources are non-regenerating, thus being "*the factor that limits the lifespan of the human species*"). [3] Even resources that are, in theory, regenerating can become, almost totally and irreversibly, non-regenerating when one considers pollution,[4] and in the contemporary society's development pollution is more and more present.

As a whole, resources can be grouped in three big categories:

- human resources (the population);
- natural resources (solar radiation, water, air, plant and animal life, soils, mineral substances in the soils, rocks, fossil fuels and ores);
- capital resources;[5]

There are several factors that determine this division of natural resources. They concern both the duration of the use of the resource as well as the degree of knowledge about them or their origin. A classification of natural resources according to the above-mentioned factors is presented in the following table.

Table 1. Natural resource classification

After the duration of their use	➢ **non-regenerating resources –** limited natural resources presenting the risk of exhaustion (oil, methane gas, coal) and ores;
	➢ **regenerating resources –** resources that replenish in time; normally, this category is represented by living things that can grow back (fish, forests etc.) and can, through rational use, be used on an unlimited basis; soil, water, wind, tides and solar radiation are also included here, even if they are not alive;
After the degree of knowledge	➢ **set resources** – resources known in detail;
	➢ **estimated resources** – resources only partially known, on the basis of analogy with other know resources or of research made in isolated places;
	➢ **potential resources** – resources not yet identified but with the chance of being discovered in a more or less near future as a result of research;
After the type of management	➢ **resources in private property –** resources managed by economical agents in a decentralized way;
	➢ **resources in public property –** resources managed by the state or leased to economical agents;
After their origin	➢ **biotic resources**, derived from animals and plants;
	➢ **abiotic resources**, derived from earth, air, water etc. (this category also includes mineral and energetic resources);
After their effects	➢ **polluting resources** (oil, coal etc.);
	➢ **non-polluting resources** (solar energy);

Source: replica after Vlăsceanu, G. **Economical World Geography,** course notes, 2012

Today the problems regarding the sustainable use[6] of natural resources has become essential to the ends of sustainable development of the human society, and the careful management of natural resources represents one of the fundamental objectives of sustainable development. The two concepts, sustainable development of the human society and the exploitation of resources, influence each other in the sense that the very essence of sustainable development derives from the present and future ways of managing natural, energetic, material and information resources. The theory of sustainable development[7] is relatively new, having appeared almost 40 years ago[8] as a response to environmental problems and the natural resource crisis, and is growing. An attempt at defining the concept of sustainable development came in 1987 when the Brundtland Commission, approaching themes such as man's interdependence with the environment, the need for a global vision and for common principles, the connections between economic and social growth and environmental protection, put together the Brundtland Report, called *"Our Common Future"*. By trying to find a reconciliation between the environment and economy, the report tries to find a development path that sustains human progress not only in several places and for several years, but for the entire planet and for a distant future.[9] In the vision of the report, sustainable development is seen as a process of change, in which resources are exploited, the direction for investments is chosen, development technologies are oriented and the institutions act in synchronization, improving a potential future for human needs and desires or, in other words, sustainable development means meeting the needs of the current generation without compromising the ability of future generations to meet their needs.

The concept of sustainable growth is a holistic one, combining social, economical and natural aspects and involving two of humanity's fundamental problems: the ability to create and to destroy.[10] This concept designates all forms and methods of social and economic development whose fundamental principle is, above all else, assuring a balance between social and economical elements and the natural capital. The roots of sustainable development can be found in promoting the sustainable use of natural resources. The concept of sustainability has major implications in the case of non-regenerating resources which have to be exploited in such a manner as to avoid the danger of their future exhaustion and to assure that the benefits of this type of exploitation are shared with all mankind.[11] The definitions of sustainable use are many, but they reflect the concept of equality between generations. Though weak at first, the concept of sustainable development started to gain in influence when the subject of environment started appearing as a main point in political debates. In the context of sustainable development it becomes more and more clear that the environment must be adapted and organized to meet the needs of individuals, which implies the exploitation of natural resources in order to serve the populace. The main areas which characterize sustainable development are: economy (efficient resource management), nature (maintaining a natural life base by reducing waste build-up) and society (its needs).

The problem of the environment started being a point of interest for the European Community along with the acknowledgment that the decrease in natural resources and the effects of pollution could not be efficiently fought just between national borders. A more extended approach was needed. As such, in time, regional approaches gave way to strategies of sustainable development and the problem of the environment was integrated in all the components of Community politics. Environmental problems were approached by the international community through global scale collective measures, which it tried to define and apply based on an adequate international framework. In time, this international framework for action has suffered a dynamic evolution, comprising legal measures with a compulsory character, in the form of treatises or conventions, or with an optional character, in the form of declarations, resolutions or sets of guidelines and political orientations, institutional measures and viable financing mechanisms.

Environment protection has become, especially in the last decade, a major problem, discussed worldwide. This has led, in time, to a series of disputes between developed and developing countries, which in turn led to the forming of organizations whose main objectives were to find ways to reduce pollution and improve environment quality as a whole.

Research in ways to reduce pollution and improve environment quality has led to the adoption of a set of actions and measures which mainly address:

- thorough knowledge of the environment, of the interactions between the economic system and natural systems, as well as their consequences;
- rational and economical use of natural resources;
- preventing and fighting against damage done to the environment, whether by man or natural causes;
- harmonizing the immediate and future interests of the society or the economical agents with regards to environment factors.

The first reunion of the international community that discussed and analyzed global environment problems as well as development necessities took place in Stockholm between the 5th and the 16th of June 1972[12]. As a result of this reunion, a series of environmental programs were adopted,[13] programs considered the founding stone of the first international framework regarding environmental problems. Although it admitted that the environmental problems of industrialized countries (the damage suffered by natural habitats, toxicity, acid rains) are not important problems for all countries, the Conference held as its main subject environmental problems, leading to an increase in public awareness of this field.

Since its foundation on the basis of recommendations from the Stockholm Conference, the United Nations Environment Program has conducted activities[14] in order to fulfill its role as catalyst and coordinator in environment issues in the United Nations.

Gradually, global environment problems became predominant, which required the start of additional actions for the growth of public awareness and for the taking, in a timely fashion, of functional measures both on a national and international level. Thus, in 1983 the United Nations founded the World Commission on Environment and Development known as the Brundtland Commission, which in 1987 elaborated and published the document *"Our Common Future"*[15] which was to be the foundation of the 40 chapters of Agenda 21 and of the 27 principles of the Rio Declaration and which, to summarize, defined sustainable development as the development that meets the necessities of the present generation without compromising the ability of future generations to meet their own necessities.

Before the 1992 Earth Summit, at the conference "Mining and the Environment" of June 1991, organized by the United Nations together with the German International Development Foundation, the Berlin Guidelines were formulated. Their formulation was possible through the cooperation of representatives from industry, governments and non-governmental organizations. On this occasion the problem of transforming sustainable development rhetoric intro operational standards was raised and some requirements were identified both by the governments and by the mining companies and companies in the extraction industry.

The next step towards sustainable development was the United Nations Conference on Environment and Development which took place in Rio de Janeiro between the 3rd and the 13th of June 1992 and saw 115 of the world leaders take part. On this occasion, the need to integrate economical development and environment protection in the sustainable development objective was officially recognized, acknowledging the growing importance of international environment law as a mechanism for coding and promoting sustainable development. After the conference, a series of documents[16] on environmental protection and sustainable development were adopted, even reaching an agreement on conventions regarding biodiversity and climate changes.

Reactions after the Rio Summit were positive and led to the start of implementation initiatives of Agenda 21 on a regional level and to the reorientation of environment protection policies. As a result, commissions for sustainable development were founded in many states and strategies for such a development were prepared.

In 1997 in New York, heads of state and government met to evaluate the progress shown after the Rio Conference. Conclusions showed a series of shortcomings regarding social equity and poverty, which led to an appeal for a firmer implementation of accords and international conventions on environment and development. In these conditions, between the 22nd and the 26th of November 1999, the second "Mining and the Environment" (or Berlin II) conference took place. There, the evolution of mining since the Berlin I conference was analyzed, from the sustainable development point of view.

Between August 26th and September 6th 2002 the United Nations Summit on Sustainable Development, bringing together 104 heads of state, took place in Johannesburg. At the conference, sustainable development reasserted itself as a central element of international agenda.

In June 2012, the United Nations Conference on Sustainable Development – Rio+ took place. In 9 days there were over 500 debates on subjects such as green energy, ecological transportation, sustainable economy, disaster reduction, desertification and others.

The event ended with a symposium where deciding authorities from all around the world took part. The commitments made during the conference aimed at gathering funds of over $500 billion for the enactment of the sustainable development concept.

The final document, "The Future We Want", comprises the commitments made by the representatives of the 193 governments, state members of the United Nations.

"The Future We Want" contains a series of measures meant to ensure a sustainable development for the human society, including:

- establishing sustainability objectives;
- planning a new forum;
- developing a new financing strategy for the specific projects;
- encouraging the civil society to engage in environment protection etc.

The image presented shows a constant degradation of the environment which, on the background of society's exponential growth of needs, determines the necessity of applying special measures that should stop the decline of environmental factors.

The increasingly articulate context at the level of both the United Nations and the regional administrations offers at this time a more certain perspective for future generations in comparison to the situation two decades ago (Rio 1992).

Bibliography

Bran, F., Rojanschi, V. and Diaconu, G. **Politici ecologice,** Ed. ASE, Bucharest, 1997

Crocker, D. **Criteria for Sustainable Development,** Tucson, University of Arizona Press, 2002

Csorba, L. **Dezvoltare regionala durabila**, Revista de comert, no. 7, July 2006

Durigu, A. **Managementul resurselor naturale in dezvoltarea durabila,** PhD thesis, ASE, Bucharest, 2005

Duțu, M. **Ecologie. Filosofia naturala a vietii,** Ed. Economica, Bucharest, 1999

Georgescu-Roegen, N. **Legea entropiei și procesul economic,** in „Idei contemporane" Ed. Politica, Bucharest, 1979

WCED, **Our Common Future,** Oxford University Press, New York, 1987

---

[*] Theodore Roosevelt (1858-1919) was the 25th President of the United States of America (1901-1909).

[1] www.dexonline.ro

[2] F. Bran, V. Rojanschi, G. Diaconu, *Politici Ecologice,* Editura ASE, București, 1997, p. 13.

[3] N. Georgescu-Roegen, *Legea entropiei și procesul economic* in Idei contemporane, Editura Politică, București, 1979, pp. 76-77.

[4] Pollution represents the modification of natural components in the presence of foreign components, called pollutants, as a result of human activities, and which through their nature, concentration or time of action cause harmful effects to health, create discomfort or prevent the use of environmental components essential to life. (The U.N. World Conference, Stockholm, 1972). Pollution is a real, sustainable and, mostly, measurable effect. The Romanian environmental legislation does not directly refer to pollution but defines the notion of environmental damage seen as an *"alteration of physical, chemical and structural features of the natural environmental components, the reduction of biological diversity and productivity of natural and man-made eco-systems, the damage of the ecologic balance and the quality of life mainly caused by water, atmospheric and soil pollution, over-exploitation of resources, their poor management and capitalization, as well as poor land administration."* (Environmental protection Law nr. 137/1995, published in Romania's Official Gazette nr. 304 from December 30 1995). Most of the existing definitions and regulations see pollution as a corruption of the environment, even up to actions which cause physical damage with demonstrable results. (Mircea Duțu, *Ecologie. Filosogia naturală a vieții,* Editura Economică, București, 1999, p. 61).

[5] N. Georgescu-Roegen, *Legea entropiei și procesul economic* in Idei contemporane, Editura Politică, București, 1979, pp. 76-77.

[6] The Convention regarding diversity states that *the sustainable use of resources consists in using the components of biological diversity in a manner and at a speed that do not lead to the long-term decline of biological resources, as such maintaining their potential of fulfilling the needs and aspirations of present and future generation.*

[7] The term "sustainable development" was created and introduced in the middle of the 1970's by Barbara Word, the founder of the International Institute for Environment and Development. In 1997, after the Maastricht Treaty, sustainable development becomes one of the European Union's main objectives. In 2001, during the Göteborg summit, the EU's Sustainable Development Strategy is adopted. In 2002, an external dimension was added to the strategy in Barcelona. The EU's Sustainable Development Strategy, revised in 2006, includes a series of key-objectives, political principles and challenges which are reference fields for all activities and policies in the EU, with the purpose of keeping them on the way of sustainable development.

[8] The Human Environment Conference in Stockholm in 1972 was the moment when the fact that human activities harm the environment (and, as such, threaten the Planet) was recognized.

[9] *Our Common Future,* WCED, Oxford University Press, New York, 1987, p.4.

[10] D. Crocker, *Criteria for Sustainable Development,* University of Arizona Press, Tucson, 2002.

[11] Stockholm Conference, Principle 5.

[12] Stockholm Conference on Human Environment.

[13] Under the motto **One Earth**, the Stockholm Conference in 1972 proclaimed the 5th of June as World Environment Day and adopted a series of documents:

- The Stockholm Declaration represents the main document of the conference and contains 26 principles that are meant to underline the major importance and necessity of protecting the environment and state, among others, every man's duty to protect and improve the environment;

- The Action Plan for the Human Environment, having three parts: the program for evaluating the global environment, activities for managing the environment and support measures;
- The United Nations Environment Program – UNEP, whose Board and Secretariat were founded in December 1972 by the United Nations General Assembly;
- The Voluntary Environment Fund, founded in January 1973 in accordance with financial procedures of the United Nations.

[14] The activities of the United Nations Environment Program can be classified in two big groups oriented towards different problems:

- regional problems of environment factors: water, air and soil pollution (especially damaged terrains);
- global problems: acid rains, exhaustion of the ozone layer, climatic changes, deforestation and desertification, conservation of biodiversity, the international transportation of toxic and dangerous products or waste, protecting the environment in times of armed conflict.

[15] The Brundtland Report.

[16] The documents adopted at the Rio Conference were:

- The Rio Declaration, containing 27 principles;
- Agenda 21, a plan of action for the sustainable development starting from the 21st century, spanning over 40 chapters destined to specific program fields, structured in the terms of the base of action, of objectives to accomplish, of activities that need to be conducted and of implementation methods;
- A document with no compulsory power, containing the principles for the management of conservation and sustainable development of all types of forest (Statement of Principles on Forests);
- The institutional organization of the World Commission for Sustainable Development;
- The financing mechanism for the implementation of Agenda 21.