BANKING SERVICES EVALUATION: A DYNAMIC ANALYSIS

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Abstract: Today, the most important asset for a bank is its customer and therefore, the main targets to achieve by management are: knowledge of his needs, anticipation of his concerns and to distinguish itself in his eyes. The awareness that a satisfied customer is a highly profitable asset effort to provide a satisfactory service to the customer by diversifying its services.  

This paper aims to analyze customer evaluation evolution of the main attributes of banking services to catch differences among the clusters and time lags through a dynamic factorial model.  

We propose an empirical study: the management of a national bank with a spread network throughout Italy wanted to analyze its reduced competitiveness in retail services, probably due to low customer satisfaction. The survey aims to analyze weaknesses in its retail services, propose possible recovery actions and measure their effectiveness across different “waves” (time lags).  

Key words: customer satisfaction; multiway factor analysis; trajectories analysis

1. Introduction and objectives

   Until the Nineties, the bank - customer relationship was heavily standardized and the low level of banking offers met the lack of demand from the customer. Management considered banking as very different from the entrepreneurial world.

   Since the Nineties, the increasing level of information and competence of the average retail customer has lead to a more complex needs architecture and a demand of diverse financial services. Therefore, today, the bank cannot act regardless of its customer
feelings. The customer can choose the bank offering him a tailored service therefore, the banks have to make strong innovation effort to retain their customers. [ABI, 2008].

However, the relationship between the bank and its customer differs from that of services and industry for two reasons [Mottura, 1982].

- First the peculiarity of financial products is that it remains within the supplier’s control after sale. In fact, the supplier can interfere with the product/service after purchase.
- Second, the relationship between the supplier and the user is strictly based on trust due to the information asymmetry and the mutually assumed risks/commitments. Hence, the bank’s image is the key to the quality of its offers.

Customer concerns and wishes change continuously, and necessitate therefore, non-stop improvement, quality enhancement, and their effectiveness by customer satisfaction testing.

Therefore, the diversifying of product attributes has been substituted by customer requests leading to homologated offers which render competition difficult. The bank’s ability in focusing on ‘how to serve the client’ and not on ‘what the client receives’ clearly creates customer satisfaction/dissatisfaction and consequently his loyalty. [ABI 2009].

The aim of the paper is to offer some ideas on the main trends in customer evaluation and expectation evolution relative to a set of service attributes. Using the Multiway technique we analyze variable dynamics over various time periods via a principal component analysis in order to get a common factor plan onto which drawing the dynamic satisfaction patterns.

We explore, in this way, the main aspects of the banking sector in terms of customer expectation and satisfaction and the identification of improvement areas. In particular, we focus our attention on the dynamic aspect of the customer satisfaction which represents a key asset for addressing banking recovery actions. The novelty of our work consisted in identifying customer trajectories and offering a useful tool to address management decisions on customer evolutions.

The paper is organized as follow: section 2 provides a brief outline of the Multiway Factor Analysis, section 3 contains a description of the case of study and the data, then main results are shown in the section 4. Finally results remarks (5) and conclusion (6) complete the paper.

2. Multiway Factor Analysis: the methodology

The Multiway technique is the best methodology to analyze variable dynamics over various time periods in a principal component analysis (PCA) [Coppi and Bolasco, 1989, Bolasco, 1999]. In our case, a sufficiently long term series is not available. More specifically, the need to repeat more than three times is due to the multiway technique rationale based on a particular decomposition of the total variability. The variability within groups on one side (where the groups are the different waves) and the variability between groups (due to time). This second part is modeled through a linear regression where the different times represent the observations of the covariate. It becomes clear that a time series with only three observations is inadequate as it neither allows the regression model to be estimated nor does it work properly.
Thus we propose a technique based on principal components which enables the definition of a “compromise plan” in order to perform a joint analysis of the three wave data.

The main purpose of the multiway analysis is to draw a dynamic path [Kroonenberg P. M., 2007], through the waves of the main variables describing the banking service across the nine professional clusters highlighting the few summarizing factors. It summarizes the variability of a complex phenomenon by highlighting both similarities / dissimilarities among the “occasions” considered (the waves) and the main components of the average behavior in the time interval chosen.

The techniques used to analyze the data “volumes”, three way matrices, are based on multivariate dynamic analysis.

Dynamic multivariate techniques of analysis allow the management and the analysis of complex data structures in order to study a given instance phenomenon in both a structural (fixing basic relationships among interesting objects (variables)) and a dynamic way (to identify change and development of in accordance to the occasions referred to). We analyze three-way data matrices individuals x variables x occasions, therefore, $X_{ijk}$ type, with $i=1,..,N$ individuals, $j=1,..,M$ variables, $k=1,..,K$ occasions.

This exploratory analysis is possible if:

1. Three-way data matrices maintain the same variables and same individuals in each occasion;
2. Three-way data matrices maintain the same individuals but different variables for one or more occasions (all variables depend on a specific k occasion);
3. Three-way data matrices maintain the same variables but different individuals in one or more occasions (all individuals depend on a specific k occasion).

The examined case conforms to the first one, so we can operate on data matrix $X_{ijk}$ directly.

According to $k=3$, it is built a common factorial space, the compromise space, in which the elements are represented according to their average configuration relative to data volume as a whole. This space is obtained by means of a principal component analysis of the compromise matrix. This matrix is obtained by the weighted mean of the similarity/distance-matrices among individuals.

The weighting coefficients are the eigenvector corresponding to the first eigenvalue of the similarity/distance matrix. Being based on the first eigenvalue only, the compromise matrix is robust as it is not influenced by the small variations of the similarity matrices. Therefore the compromise matrix is the synthesis of all matrices considered through the most representative average one. Through the factorial plan centered on this matrix, we can examine the distance of the different matrices from the compromise one (that is excluding the common part of variability), and their relation with respect to the two principal axes.

The method [L’Hermier Des Plantes, H. 1976, Lavit et. al, 1994] has been performed in two steps:

1) Interstructure analysis
2) Intrastructure analysis

The the first step consists of performing PCA on $S_{1..M}$ matrices considered as variables. It searches a linear combination $F_1,..,F_r$ of the matrices $S_{1..M}$ that optimally account for the matrices $S_{1..M}$. These latter has been represented by a vector $Vec(S_j)$ which contains the elements of $S_j$ strung out rows-wise. Similarly $Vec(F_1),..,Vec(F_r)$ are the principal component of the variable $Vec(S_1),..,Vec(S_M)$. So the method minimize the following loss function:
over arbitrary matrices $F_1,...,F_r$ and the $(M \times r)$ matrix $C$ of loadings $c_{jl}$ of the variables on the components.

Then it defines the compromise matrix as the first principal component ($F_1$) of the matrices $S_1..S_M$. More formally $F_1 = \sum_{j=1}^{M} \alpha_j \cdot S_j$ where $\alpha_j$ is the first principal component weight of the matrix $S_j$ $j=1,..M$. The similarities among tables can be visualized in the space of principal components, after performing PCA, which is called interstructure analysis. The first eigenvector obtained after PCA is a global size variable representing the ‘agreement between tables’.

The second step consists of PCA of the compromise matrix that has been defined previously that is equivalent to minimizing the following equation

$$\sum_{j=1}^{M} \alpha_j S_j - \Lambda V V^T$$

over the diagonal matrix $\Lambda$ and the matrix $V$ ($N \times M$) to $V V^T = I$.

At the minimum of (2) matrix $V$ will contain the compromise component scores for the objects and $\Lambda$ will contain the correspondent eigenvalues.

It gives information about the similarity of objects in the first mode. Their distribution can be visualized in the space spanned by the principal components and the representation is called compromise plot.

3. The case and the data

Today every financial institution measures customer satisfaction with a high level of precision, with attention to client segmentation and his changing needs. This activity which includes competitive positioning and loyalty, although expensive, is a top priority for the bank given the obvious strict connection between customer satisfaction and bank profitability [Munari, 1999]. The competitive positioning is the bank’s capacity to attract new customers from competition which depends directly on satisfaction. Evidently satisfied customers tend to diffuse a positive image of the bank which reinforces competitive strength. A good competitive positioning has also positive internal reflections reinforcing employee sense of belonging. All these effects influence the bank’s profitability by increasing margins and recovering satisfaction costs. A strong market positioning maintains low acquisition and retention costs, low price elasticity, and high employee productivity [Munari, 2001].

Customer loyalty also contributes to an increase in the bank’s economic results. A customer that is loyal to a single supplier tends to intensify his purchases both in up selling and cross selling [Cosma, 2003]. Furthermore, the loyal customer can accept higher prices and temporary quality falls without requiring special recovery actions. This virtuous circle sustains good economic results in the years.

According to the most recognized concept, the SERVQUAL, satisfaction is the customer answer to the perceived discrepancies between pre-consumption expectations and product/service effective performance. In particular satisfaction derives from the positive confirmation of expectations on a certain product / services and it is as high as the distance expectation-performance [Oliver, 1977]. Satisfaction is a valuation output that influences
and is influenced by perceived quality. The value perceived by the customer is a summary of perceived quality and other personal elements of evaluation. The specific perceived quality components are: tangible factors, reliability, responsiveness, assurance and empathy [Parasuraman A., Zeithaml V. & Berry L., 1988].

Today SERVQUAL elements are combined and enriched to form different synthetic quality and satisfaction indicators [Churchill, 1982].

There are different indicators to measure satisfaction. In particular the banking sector privileges indicators to evaluate service quality and effectiveness to reach consumer desiderata. Furthermore, the reliability of the brand and customer promotion are considered a good proxy for success. Indicators like NPS (Net Promoter Score) and TRIM Index show a good historical correlation between economic profit and orientation towards the client through a few global items. The main limit of these satisfaction indicators is that they are not directly actionable by management as they do not focus on specific services / corporate aspects that could constitute strengths or weaknesses to be either pushed or improved. To have a clear picture of “what to do” further analysis is required.

For these reasons, the following analysis explores the main aspects of the banking sector in terms of customer expectation and satisfaction, the identification of improvement areas, and appropriate recovery actions. Through different survey waves, the management can monitor evaluation evolution and measure the impact of specific actions for opinion improvement.

After a progressive loss of some customer segments, the management decided to conduct a survey, choosing a sample of 27.000 retail customers who effected at least 5 retail requests, conjoint with other “contact points” of the bank (call center, e-banking,..) within a year. The same sample was monitored for three questionnaire waves to observe both customer preference evolution and management action impact on satisfaction.

The questionnaire was framed according to SERVQUAL model, therefore, with five dimensions to analyze perceived quality and expectation of the banking service. A variable number of items catches the quality dimensions (Tangibles, reliability, responsiveness, assurance, empathy). All the scores are on a Likert scale 1 to 10.

There are 16 questions (type A), measuring expectations / importance of items, and 16 questions (type B), catching evaluations on perceived quality of a particular item. One final question aims summarizing the entire banking service satisfaction.

The same questionnaire has been applied to the same sample for three waves: this is a perfect case for constructing a dynamic model to quantify variable changes across the three different occasions.

A descriptive analysis of the sample shows a homogeneous distribution across different ages, sex, instruction levels and profession segments. This reflects the Italian “banking population”: more than 60% is between 26 and 55 years old; the sample is equally distributed between the two sexes and shows a medium – low level of instruction.

The sample has been analyzed across 9 different professional clusters: entrepreneurs, managers, employees, workers, farmers, pensioners, housewives, student, others. The sample is well distributed across the different professional segments employees 24%, pensioners 22%, housewives 14%.

The customer satisfaction was analyzed according to three different indicators to avoid dependency on the metrics used. For the three indicators, satisfaction scores are high (above 7/10 in the three cases) with the same trend across the waves. There is an increase in satisfaction from the first to the second wave and a decrease in the third wave. A gap analysis between questions A and B shows that expectations / item importance are always
higher than the perception of that item. A dynamic analysis shows evidence of a gap decrease between the first and third wave. The distance between perception and expectations decreases thanks to higher perceptions and stable expectation levels. This is a first effect of managerial actions taken after the results of the first wave. Service characteristics improved after the first wave provoked an increase in satisfaction in the second wave, with no more actions after the second wave, the satisfaction level decreases again in the last wave.

4. Main results

For the construction of the three-way matrix, $X_{ijk}$, 27,085 interviewed retail customers were considered grouped in $i = 1,\ldots,9$ professional clusters; $j = 1,24$ variables A and B (less than the original questionnaire because the tangible question block was linearly dependent on other variables and was eliminated); $k = 1,2,3$ waves.

Interstructure Results

In this step of the model, we evaluated the similarities among the three waves through a correlation analysis and a PCA on the three occasion-matrices. The correlation among the waves was very high and significant in every case (0.999; sig. (2 codes) = .000) hence, the internal structure of the occasion – matrices was similar and comparable among the waves. The interviewed customers maintain the same attitude toward he considered phenomena. The high similarity among the waves was also proven by plotting three point-matrices on the factorial axis resulting by PCA: which maintain the same position in the three occasions. This similarity justifies the search for a common “compromise” matrix by intrastructure analysis.

Intrastructure Results

The first step of the intrastructure consists in estimating, within the bidimensional space resulting from a second PCA, the “compromise” matrix in order to represent the average position of the professional clusters with respect to the selected variables (regardless of the different occasions). The PCA results are very sturdy: the first two components explain about 84% of the total variance and in particular the first one explains 54% alone. The “compromise” matrix based on the first eigenvalue is robust and can provide a realistic view of the evolution of variables and individual positioning in the time horizon considered. Also KMO index (0,8) and Bartlett test (1.312,7; sig. .000) show the quality of the factorial model created.

Representing the “compromise” matrix and the occasion-points of the k matrices on the factorial plan created by the first two components, a clear polarization of the variables on the two axes showed. The first component is characterized by the positive pole B variables (valuations on the different aspects of the service) and from the negative pole by A variables; the second component shows the contrary. In accordance to these results, the first axis is called “service evaluation” and the second “service expectations”.

Looking at the meaning of the four quadrants of the “compromise” plan according to the axis characterization, we can define the positioning of the individuals as follows:

- **Sustainable value** (first quadrant): the target customers for the bank who can be a real source of value. For these customers besides positive valuations of the service there are further improvement expectations for the future which are typically inelastic to service price.
Old Glories (second quadrant): clients who accumulated historically positive experience with the bank but who do not believe in future service improvement as they appear to perceive the bank as a traditional institution. The main risk for these clients is the churn event.

Lost cases (third quadrant): not satisfied at all and with no improvement expectation customers. These are either customers with no interest in the relationship with the bank or people that experienced dissatisfaction events on crucial service aspects in past occasions. It is very difficult for the bank to increase value for such clients.

Potentialities (fourth quadrant): usually newly acquired customers that have high future expectations of the bank but prudent satisfaction evaluation. In an optimal condition this quadrant should be a transition position versus the sustainable value quadrant.

Trajectories analysis

By projecting the unities on the factorial axis of the “compromise” phase, movements with respect to the different waves can be analyzed (Graph 1). The graph shows a non continuous trend. On the evaluation axis, the professional clusters show increased satisfaction from the first to the second wave moving right, which however, are reduced in the third wave. The most relevant movements in terms of distance are those of Farmers/Craftsmen/Retailers and employees. Service evaluation is essentially made up from aspects such as timing evaluation and contact personnel relationship and showing increased satisfaction. This could be considered an optimal result for the banking group as the employees count for 25% of total retail clients which is the cluster in which the bank was losing its market share. The evaluations improvements resulting in the first to the second wave are attributable to management action to fill the satisfaction gap emerging from the survey results. Firstly employees were trained on key competences and above all, on relational improvement. Secondly an incentive system on incremental satisfaction was launched. Other action was focused on queuing time optimization and service rapidity by retail space rationalization avoiding high capital expenditure.

This action besides higher satisfaction in some customer segments, lead to higher retail productivity thanks to faster service.

After this positive return, management stopped acting and relied on the first improvement in satisfaction, but in the third wave, as visible from the graph, satisfaction decreased in almost every cluster. Only the management cluster showed continual improvement. Time is apparently the most important attribute and improvements in this area increase satisfaction more than other aspects. These trends, with respect to the x-axis, are also visible in terms of covered distances in table 1: in the third wave all the clusters, except for the managers, move backwards reducing service evaluation. It covers a limited distance and is, in some cases, negative.

The expectation analysis shows restricted a non homogeneous movement among the clusters from the first to the second wave. It appears that management action had no significant effects on cluster expectations. The third wave shows a particularly strong decrease in expectation level which renders the entire path negative. Time, the most important factor in determining expectation, decreases in the third occasion apart from managers as in the evaluation axis. A slight decrease in satisfaction corresponds to a relevant fall in expectations of future improvements in service for workers, housewives,
pensioners, farmers, craftsmen, retailers. This could be interpreted as a signal of possible future decreases in satisfaction level confirming lower expectations.

These results are typical in a “traditional” context with customer needs evolving in time. Today, in fact, service aspects such as time optimization and personnel competencies are considered more and more as “hygienic factors” their presence is embedded in the service level without providing a significant increase in satisfaction. However, their absence represents strong dissatisfaction. Today service differentiation is mainly linked to multi channel distribution and, in particular, online services. In this case such aspects are not. The existing information shows that continuity in service aspect improvement would maintain satisfaction along the waves. Management awareness of reduced competitiveness in such areas and in some customer segments permits rapid and economic action, without promoting a permanent and consistent change to modernize the service. A clear change signal from clients that goes unanswered could stop future growth opportunities and have an impact on profitability.

Graph 1. Clusters trajectories

Table 1. Covered Distances

<table>
<thead>
<tr>
<th>Professional Cluster</th>
<th>X-axis movements</th>
<th>Y-axis movements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1w-2w 2w-3w Net Route</td>
<td>1w-2w 2w-3w Net Route</td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>0.423 -0.140 0.283</td>
<td>0.056 -0.740 -0.683</td>
</tr>
<tr>
<td>Managers</td>
<td>0.174 0.849 0.823</td>
<td>-0.146 0.250 0.104</td>
</tr>
<tr>
<td>Employees</td>
<td>0.537 -0.286 0.251</td>
<td>-0.116 -0.954 -1.070</td>
</tr>
<tr>
<td>Workers</td>
<td>0.284 -0.516 -0.252</td>
<td>-0.046 -1.501 -1.547</td>
</tr>
<tr>
<td>Farmers/Craftsmen/Retailers</td>
<td>-0.570 -0.005</td>
<td>0.111 -1.608 -1.494</td>
</tr>
<tr>
<td>Retired</td>
<td>0.414 -0.480 -0.066</td>
<td>0.162 -1.552 -1.390</td>
</tr>
<tr>
<td>Housewives</td>
<td>0.416 -0.724 -0.309</td>
<td>-0.213 -1.225 -1.438</td>
</tr>
<tr>
<td>Students</td>
<td>-0.112 0.464 0.352</td>
<td>-0.005 -0.400 -0.405</td>
</tr>
<tr>
<td>Other</td>
<td>0.528 -0.194 0.334</td>
<td>-0.231 -0.929 -1.160</td>
</tr>
</tbody>
</table>
5. Results Remarks

Beyond a shadow of doubt results illustrated are really interesting from both perspectives: customer management and strategic decision of the bank service. At the same time we need to highlight that deriving a dissimilarity measure for time trajectories is not a new research topic. Works of Carlier [1986], D’urso e Vichi [1998], D’Urso [2000] are well known in literature but they face such problem from a different perspective. Carlier defining a measure which is a compromise between distances between couple of different points for the same time lag and the distance between position of the same point across different time lags without any suggestion about how estimate the linear combination of the components. D’urso and Vichi, instead, defined a dissimilarities between track patterns as combination of trends velocities and accelerations of the pair trajectories. This approach could have been applied also in our case of study but we find not a few interpretation issues, especially related to velocity and acceleration components of the distances measure: they seem to loose their explicative power for customer satisfaction data.

Such remarks will head our study towards a definition of a new dissimilarities measure with exploits a dimension reduction obtained with PCA. Before doing that we focused our attention on some limitations of our approach related with the geometrical properties of the PCA applied. As it is well known the coordinates of a point in the space tell us where the point is located respect to a particular set of a coordinate axes. When we map data in a lower dimensional space (Tab. 1) we have to consider that such projection might cause original scatter point distortion. That is due to the fact that distances of the points from the factors effect the representation of the points itself. Quality of representation of each point onto each factor axe depends on the closeness of a point to the axe itself: goodness of representation improves how more data lay close to the factor.

Therefore in a quantitative approach of customer satisfaction analysis it has to take into account, which means factor coordinates obtained with the PCA can not have the same weight. In order to set our analytical strategy according with such geometrical issue we need to estimate a new coordinates set where each factor is re-weighted in terms of goodness of point representation. We know such data transformation might generate a new patterns modifying configuration of customer satisfactions illustrated in Figure 1. We are already studying an integrated approach to fix such aspect that will be part of a further work on this topic.

6. Conclusions

The empirical study analyzed in this paper aims to offer some ideas on the main trends in customer evaluation and expectation evolution relative to a set of service attributes. One limit of the model is the restricted number of historical occasions. Three waves at about a yearly distance are too short a time to observe trend evolution. On the other hand, this limit reinforced the statistical robustness of the PCA and the theoretical soundness of the whole model.

Multiway analysis particularly suits the study continuously evolving social phenomena. The bank-retail customer relationship changes incessantly due to changes in the actors and the external environment. By dynamic monitoring of satisfaction resulting from the gap between expectations and experience, management can plan specific action to reduce weaknesses in some customer segments and in some geographic areas. A systematic analysis of management action-customers reaction provides the effectiveness of every
decision to increase satisfaction, loyalty and consequently bank profitability. By such continual analysis management can check the correct direction of action, and change it according to customer desiderata evolution.

The results show that management action, in response to customer requests has an effective impact on satisfaction. Discontinuity in the latter provokes primarily a fall in future expectations and in the medium term a consistent fall in satisfaction, too.

By such analysis management can count on high satisfaction level in most customer segments and can be considered forward looking. It acts to search for weaknesses in its services and to improve them before witnessing a progressive loss of clients. It is a mistake to think that “una tantum” improvements can grant a sustainable growth in satisfaction.

To complete the above analysis a study of competitor strengths and service model are necessary to find a differentiation key for the analyzed bank and to understand the relative positioning of each attribute.

The client cost-benefit relationship also remains to be analyzed. Service improvements following the first wave might increase costs for the customer which could cause a drop in service appreciation.

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5 The quantification matrix (that can often be considered as a similarity matrix), $S_j$ ($N \times N$), reflects the similarities between $N$ objects within this data table, is computed:

$$S_j = X_k^r X_k^t$$

where $X_k$ is a matrix of dimension $N \times J$ relatives to the occasion $k$ and $X_k^t$ is its transpose matrix.