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A MODEL FOR THE EX-ANTE U.K. STOCK MARKET RISK PREMIUM

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Abstract: We propose a model for the aggregate stock market together with its dividend yield and earnings yield so that the ex-ante risk premium could be extracted in an unobserved component modelling framework. We posit the model as a linked stochastic differential equation system and the linking variable is the ex-ante risk premium. By hypothesising a realistic dynamic structure for the ex-ante risk premium, we demonstrate how such a system could be estimated as a filtering problem. As a practical demonstration of the methodology we apply the model to the U.K. stock market data.
Key Words: Ex-Ante Risk Premium, Dividend Yield, Earnings Yield, Kalman Filter

1. Introduction:

Finance research has traditionally focussed on the relationship between stock price, dividends and earnings in order to find an acceptable explanatory model for the observed stock prices. The asset pricing theory predicts that these are related by a quantity that is essentially unobserved and it is referred to as the ex-ante risk premium. The estimation problem of this ex-ante risk premium has spawned a vast empirical literature.

In this letter we approach the problem of estimating the ex-ante risk premium by modelling the relationship between asset price, dividend yield, earnings yield and the ex-ante risk premium as a linked system of continuous time diffusion processes. In particular we assume that the dividend yield mean-reverts to some fraction of the earnings yield, and that the earnings yield itself mean reverts to a long run constant level. These (or related) assumptions can be found in earlier literature such as Davidson, Okunev and Tahir (1996), Campbell and Shiller (1988a, b), Campbell and Kyle (1993).

For the ex-ante risk premium there is less guidance in the literature as to an appropriate dynamic model. The simplest assumption (and that we employ here) is that it follows a mean-reverting process. This assumption was found to give good results in Bhar, Chiarella and Runggaldier (2004) that developed an approach to extract information about the index equity risk premium from index derivatives data. Here we assume that the volatility of the ex-ante risk premium is constant over the sample period. Later we would point out how this assumption could be relaxed and in fact, we have already started investigating other avenues.

The main contribution of this letter is to explain how such a model could be implemented in practice and we demonstrate the methodology using aggregate market data from the U.K. We also highlight the computational issues at the appropriate places in later sections.

In section 2 we discuss the stochastic dynamics for the index, the dividend yield, the earnings yield and the ex-ante risk premium. In section 3 we describe the filtering setup. Section 4 describes the data and the empirical results for the constant volatility case; Section 5 summarises the analysis and points out extensions to the model that are currently being pursued.

2. The Proposed Model:

Let us use $S(t)$, $r(t)$, $q(t)$ and $\pi(t)$ to denote respectively the index, the risk-free rate, the dividend yield rate and the ex-ante equity market risk premium at time t. We propose for the movements of the index the geometric Brownian motion process, (actual derivation of this process from the basic principle is described in a working paper available on request),

$$\frac{dS}{S} = \left[r(t) + \pi(t) - q(t)\right]dt + \sigma_S dZ_S \tag{1}$$

where Z_S is a Wiener process and σ_S is the instantaneous volatility of the index return. In this study we treats σ_S as constant, though we acknowledge that future research may need to consider the dynamics of σ_S also as a diffusion process, thus leading to a stochastic variance model. The intuition expressed by equation (1) is that the expected capital gain on the index equals the risk free rate plus the ex-ante equity market risk premium minus the continuous dividend yield.

Inspired by the approach of Chiang, Davidson and Okunev (1996), who model earnings and dividends as a linked diffusion system, we model earnings yield (e) and dividend yield according to,

$$dq = \beta_q\left(\gamma e - q\right)dt + \sigma_q dZ_q, \tag{2}$$

$$de = \beta_e\left(\bar{e} - e\right)dt + \sigma_e dZ_e. \tag{3}$$

Equation (2) states that the dividend yield is mean-reverting with speed β_q to some fraction of the earnings yield and expresses in yield form the original idea of Lintner (1956) that firms have in mind a target dividend that is some fixed proportion of earnings. When dealing with index returns it seems appropriate to express this relation in yield form.

Equation (3) expresses the notion that the earnings yield is mean-reverting to some long run value $\bar{e}$ with speed β_e. For both processes (2) and (3) we assume independent Wiener processes Z_q and Z_e and the associated volatilities σ_q and σ_e. It seems unlikely that the Wiener processes Z_S, Z_q, and Z_e would be completely independent, so we allow for correlation amongst all of them i.e. we assume the instantaneous correlations between the pairs of variables defined by, ρ_{Sq}, ρ_{Se}, and ρ_{qe}.

Finally, we model the ex-ante risk premium as a mean reverting stochastic system with its own Wiener process. In other words, we specify,

$$d\pi = \beta_\pi\left(\bar{\pi} - \pi\right)dt + \sigma_\pi dZ_\pi. \tag{4}$$

Also, in this version we allow the Wiener process driving the risk premium to be independent of other Wiener processes in the model. Equations (1), (2), (3) and (4) form a linked stochastic dynamic system. In the next section we show how this linked system could be expressed in the state space framework and the Kalman filter may be applied to estimate the unknown parameters of the system. This estimation process would also allow us to infer the conditional mean and the variance of the ex-ante equity risk premium.

3. State Space Setup for the Proposed Model:

It will be convenient to express (1) in terms of $s = \ln\left(S\right)$, i.e.

$$ds = \left(r + \pi - q - 0.5\sigma_S^2\right)dt + \sigma_S dZ_S \tag{5}$$

With this change of variable equations (2), (3), (4) and (5) form a system of linear stochastic differential equations, to which it is appropriate to apply the Kalman filter. We treat s, q and e as observed quantities and the ex-ante risk premium as the unobserved state variable of the system. For the Kalman filtering application it is convenient to discretise the system (2), (3), (4) and (5) using the Euler-Maruyama scheme. The discretisation of the state equation (4) yields,

$$\pi_t = \beta_\pi\bar{\pi}\Delta t + \left(1 - \beta_\pi\Delta t\right)\pi_{t-1} + \sigma_\pi\varepsilon_{\pi,t}\sqrt{\Delta t} \tag{6}$$

where, $\varepsilon_{\pi,t} \sim N\left(0,1\right)$.

Next, we discretise the measurement equations (2), (3) and (5) and write these in vector notation as,

$$\begin{bmatrix}\Delta s_t\\ \Delta q_t\\ \Delta e_t\end{bmatrix}=\begin{bmatrix}r_{t-1}-q_{t-1}-0.5\sigma_s^2\\ \beta_q\gamma e_{t-1}-\beta_q q_{t-1}\\ \beta_e\bar{e}-\beta_e e_{t-1}\end{bmatrix}\Delta t+\begin{bmatrix}\Delta t\\ 0\\ 0\end{bmatrix}\pi_t+\begin{bmatrix}\varepsilon_{s,t}\\ \varepsilon_{q,t}\\ \varepsilon_{e,t}\end{bmatrix} \tag{7}$$

where, $\varepsilon_S, \varepsilon_q,$ and ε_e are standard normal variates. The variance-covariance (H) structure of the noise terms in the observation equation (7) implied by equations (2), (3), and (5) is given by,

$$H \equiv \begin{bmatrix}\sigma_s^2 & \rho_{s,q}\sigma_s\sigma_q & \rho_{s,e}\sigma_s\sigma_e\\ \rho_{s,q}\sigma_s\sigma_q & \sigma_q^2 & \rho_{q,e}\sigma_q\sigma_e\\ \rho_{s,e}\sigma_s\sigma_e & \rho_{q,e}\sigma_q\sigma_e & \sigma_e^2\end{bmatrix}\Delta t. \tag{8}$$

The model parameter vector is, therefore, $\Theta \equiv \left(\sigma_s, \beta_\pi, \bar{\pi}, \sigma_\pi, \beta_q, \gamma, \sigma_q, \beta_e, \bar{e}, \sigma_e, \rho_{s,q}, \rho_{s,e}, \rho_{q,e}\right)$. These parameters are to be estimated by maximising the prediction error form of the likelihood function. The important observation to be made is that the measurement equation (7) is linear in the state variable (π). The details of the algorithm are available from the author on request. This is also discussed in standard books e.g. Bhar and Hamori (2004, page 11).

4. U.K. Market Application:

We have taken the monthly earnings and dividend yields and index price data for the FTSI for the period February 1973 to February 2003, from Data Stream. For the risk-free rate we the UK 3-month Treasury bill rates.

The parameter estimates are displayed in Table 1. The results of various model diagnostic tests are displayed in Tables 2. The filtered equity risk premium is displayed in Figures 1. We see from Table 1 that most estimates are significant. However, β_π, the speed of mean reversion of the risk-premium seems an important exception and the estimate for β_e also has relatively large standard error. Table 2 indicates a reasonably good model fit using a number of diagnostics. As for the risk-premium, Figure 1 tends to indicate that for the U.K. market index there is a structural break, the first around mid 1979 and the second around mid 1998. Over all periods the two standard deviations band is around ±1.5%.

Figure 1
Ex-Ante Equity Risk Premium: U.K. (Filtered Estimate)

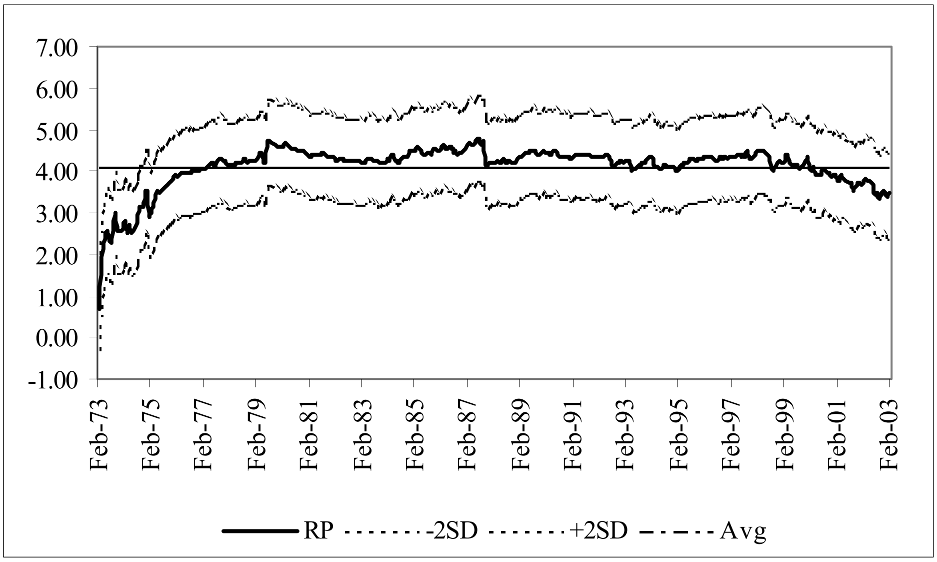

Table 1
Ex-Ante Equity Risk Premium Model for the U.K. Stock Market

Parameters	U.K.
σ_S	2.45369
	(0.08353)
β_π	0.55231
	(0.33519)
$\bar{\pi}$	4.23528
	(0.45011)
σ_π	0.59899
	(0.27487)
β_q	0.14295
	(0.07284)
γ	0.43442
	(0.12988)
σ_q	0.01279
	(0.00043)
β_e	0.02240
	(0.06499)
$\bar{e}$	0.00005
	(0.00140)
σ_e	0.03484
	(0.00116)
ρ_{Sq}	-0.91965
	(0.00735)
ρ_{Se}	-0.82044
	(0.01541)
ρ_{qe}	0.92970
	(0.00649)

Standard errors are in parentheses below the parameter estimates.

Table 2
Residual Diagnostics and Model Adequacy Tests (U.K.)

Equations	Portmanteau	ARCH	KS Test	MNR
ΔS	0.176	0.682	0.176	0.980
Δ Div Yield	0.020	0.068	0.141	0.996
Δ Earn Yield	0.001	0.035	0.193	0.999

Entries are p-values for the respective statistics except for the KS statistic. These diagnostics are computed from the recursive residual of the measurement equation, which corresponds to the spot index process. The null hypothesis in portmanteau test is that the residuals are serially uncorrelated. The ARCH test checks for no serial correlations in the squared residual up to lag 26. Both these test are applicable to recursive residuals as explained in Wells (1996, page 27). MNR is the modified Von Neumann ratio test using recursive residual for model adequacy (see Harvey (1990, chapter 5). KS statistic represents the Kolmogorov-Smirnov test statistic for normality. 95% significance level in this test is 0.072. When KS statistic is less than 0.072 the null hypothesis of normality cannot be rejected at the indicated level of significance.

5. Summary and Concluding Remarks:

We have set up the relationship between the stock market index level, the dividend yield, the earnings yield and the ex-ante risk premium as a system of stochastic differential equations. We have used unobserved component modelling approach and Kalman filtering methodology to estimate the model and obtain filtered estimates of the ex-ante risk premium. The long run levels for the risk premium are consistent with the point estimates obtained from various ex-post regression based studies. The empirical results also suggest that there are at least two different regimes that classify the behaviour of the ex-ante risk premium in the U.K over the sample period.

The methodology developed here shows promise and may be extended in different directions further. First, the indication of the regime changes in the unobserved component could be captured in a hidden Markov framework. In fact we have already started exploring this avenue. Second, some practical application of the one-step ahead prediction available from the filter may be explored. For example, the predicted risk premium may be used to restructure stock portfolio to take advantage of possible excess return.

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