

## **UPON SCHEDULING AND CONTROLLING LARGE-SCALE STOCHASTIC NETWORK PROJECTS**

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**Abstract:** *The problem of controlling large-size stochastic network projects of PERT type is considered. A conclusion is drawn that the need of proper control models for PERT projects is very important. The authors suggest aggregating the initial model in order to modify the latter to an equivalent one, but of medium or small-size.*

*For those network models effective on-line control algorithms are already developed. After observing the project's output at a routine control point and introducing proper control actions the aggregated network is transformed to the initial one, and the project's realization proceeds.*

*The developed control techniques are especially effective for those R&D projects, when an on-line control has to be undertaken under a chance constraint. The suggested control model can be regarded as an additional tool to help the project manager to realize the project in time.*

**Key words:** *project management; on-line control; scheduling; network project; generalized network models*

### **1. Introduction**

In recent years the problems associated with controlling projects by means of network analysis have not been discussed extensively in the literature. Scanty publications refer mostly to network modelling and to the calculation of activity network parameters. However, little investigation has been undertaken in the area of decision-making and determining control actions while controlling stochastic network projects. The main questions: "How a PERT project should be controlled?" and "What are the main stages of controlling PERT projects?" have not previously received satisfactory answers, especially for

highly complicated long-term projects under random disturbances. It can be well-recognized from studying the literature on planning and control techniques in project management that an overwhelming majority of modern management systems use only PERT techniques to plan and control projects with uncertainty [6]. This occurs because PERT is simpler than other more complicated techniques [2-4] with a high level of uncertainty. However, the PERT conception deals with random disturbances since a PERT network comprises activities of random duration.

It can be clearly recognized that in the last two decades, various control problems in project management, especially for PERT projects, have been the subject of lengthy debate and very sharp criticism [1, 5-7]. In our opinion, the main reason that, in practice, those projects are all usually completed late and remain uncontrolled is that PERT projects are carried out under random disturbances (new estimates of a random nature without any previous experience, random activities' durations, periodical revisions of networks over time due to random emergency situations, etc.). However, project managers usually [6] avoid probabilistic terms since they are not sufficiently trained. They are trying to control highly complicated projects with uncertainty by using deterministic techniques. This leads to biased estimates that usually underestimate the actual time needed to accomplish the project. Therefore the project's due date can rarely be met. *Thus the need of proper control models for PERT projects is very important.*

In our opinion, there is another important reason for numerous failures of PERT techniques in project management. This is because the models are too complicated to be effective. They are not flexible. Usually, they incorporate both scheduling and control techniques. But since it is practically impossible to develop a *proper deterministic schedule for a project under random disturbances*, such models are not adequate to the real life. Therefore the control procedures are also non-effective.

We suggest using a control model only at several control (inspection) points in order to determine the next routine inspection point and the project's speed to proceed with until that next control point. Such control techniques can be applied only to a network model with a medium amount of activities (up to 50-100 activities). Thus, the problem is to modify the initial network model (which for some projects may comprise a tremendous amount of activities) to an equivalent one, but of medium or small size.

For such a model an activity is equivalent to a subnetwork (a fragment) of the initial network. Such aggregated, small-size networks for construction projects of deterministic type have been developed in [9].

In the next section we will describe the general idea of an aggregation for PERT type projects with activities of random durations.

## **2. Developing Enlarged Aggregated Networks with Random Activity Durations**

According to the project's Work Breakdown Structure (WBS) [10] an initial network is presented in the form of a group of lists of initial activities. The name of the activity is taken from the WBS.

We will henceforth call a fragment a list of activities together with all the links both entering and leaving that fragment. The step-by-step procedure of developing an aggregated network is as follows:

Given:

- activities  $(i, j)$  entering the PERT initial network  $G(N, A)$  ;
- random activity durations  $t_{ij}$  with pregiven density distribution.

Simulate random durations  $t_{ij}, (i, j) \in G(N, A)$ .

Step 1.

On the basis of simulated values  $t_{ij}$  calculate for each  $i \in N$  the earliest moment of the event's realization,  $T^{\xi}(i)$ , where  $\xi$  denotes the index of the simulation run.

Step 2.

Repeat Steps 1-2  $M$  times in order to obtain representative statistics.

Step 3.

Step 4. Calculate

$$T_{ear}(i) = \min_{1 \leq \xi \leq M} T^{\xi}(i);$$

$$T_{lat}(i) = \max_{1 \leq \xi \leq M} T^{\xi}(i).$$

Step 5.

By using decomposition methods [9, 10] subdivide the initial set into enlarged fragments. Each fragment comprises a list of detailed activities together with all links connecting activities entering the list ("internal" links) as well as all "external" links connecting the fragment with other fragments.

Steps 6-11 have to be realized for each fragment  $F \subset G(N, A)$  separately.

Step 6. Determine two events  $i_{st}^F$  and  $i_{fin}^F$  which we will henceforth call the start and the finish events of fragment  $F$  :

$i_{st}^F \in F$  delivers the minimum to  $Min_{i \in F} \{T_{ear}(i)\}$  and

$i_{fin}^F \in F$  delivers the maximum to  $Max_{i \in F} \{T_{lat}(i)\}$ , where  $T_{ear}(i)$  and  $T_{lat}(i)$

have been calculated on Step 4.

Step 7. For both events  $i_{st}^F$  and  $i_{fin}^F$  calculate the earliest and the latest time

moments (see Step 4):  $T_{ear}(i_{st}^F)$ ,  $T_{lat}(i_{st}^F)$ ,  $T_{ear}(i_{fin}^F)$ ,  $T_{lat}(i_{fin}^F)$ .

Step 8. Calculate the minimal fragment's duration

$$\tau_F^{min} = T_{ear}(i_{fin}^F) - T_{lat}(i_{st}^F).$$

Step 9. Calculate the maximal fragment's duration

$$\tau_F^{max} = T_{lat}(i_{fin}^F) - T_{ear}(i_{st}^F).$$

Step 10. Assume that the fragment's duration  $\tau_F$  is a random variable with a  $\beta$ -distribution density function

$$p_F(x) = \frac{12}{(\tau_F^{max} - \tau_F^{min})^4} (x - \tau_F^{min})(\tau_F^{max} - x)^2$$

with the mathematical expectation

$$\tilde{\tau}_F = (3\tau_F^{min} + 2\tau_F^{max}) \cdot 0.2$$

Such a distribution has been successfully used over a long time in various network projects [2].

Step 11. External links (arrows) entering and leaving fragment  $F$  are determined [9, 10]. For each external arrow the corresponding receiver (emitter) is calculated in percentage of the fragment's duration.

After realizing Steps 6-11 for each fragment  $F \subset G(N, A)$  the enlarged aggregate network with random fragments' durations is determined. As outlined above the aggregated network must be of a small or a medium size. The model enables applying on-line control techniques to introduce proper control actions.

### 3. On-Line Control Problems

For most network projects under random disturbances the progress of the project cannot be inspected and measured continuously, but only at preset inspection points. An on-line control determines both inspection points and control actions to be introduced at those points in order to alter the progress of the project in the desired direction. Such control actions may be as follows:

- a) to redistribute the budget among the project activities in order to enhance the project's speed,
- b) to introduce additional shifts, etc., to change the speed of the progress of the project without using additional resources, etc.

Such control actions [3, 4] usually have the tendency to minimize either the number of inspection points, or the average project's speed subject to a chance constraint to meet the project's due date on time. The corresponding control algorithms are described in [3]. They have been applied to medium size construction projects [4].

After realizing the control actions the modified aggregated network is transformed to the initial network [8].

Consider a medium-size PERT type network model with a due date  $D$ . A desirable probability  $p^*$  that in practice enables completion of the project on time is pre-given. At each control moment  $t_g$  the project management may introduce several possible alternative speeds  $v_{t_g}$  to proceed with until the next control point. Let  $V_t$  be the project's output (project volume) observed at control point  $t > 0$  and let the project's target (goal) be  $V^*$ . Denote  $\Pr(t_g, v_{t_g})$  the confidence probability to accomplish the project on time after introducing speed  $v_{t_g}$  and control point  $t_g$ .

The main control problem [3] is to determine both control (inspection) points  $t_g$  ( $g = 1, \dots, N$ ) and speeds to proceed with from that point on until the next adjacent control point  $t_{g+1}$ , in order to minimize the number  $N$  of inspection points:

$$\text{Min } N \quad (1)$$

$$\{t_g, v_{t_g}\}$$

subject to

$$\text{Pr}\{t_g, v_{t_g}\} \geq p^*, \quad (2)$$

$$t_0 = 0, \quad (3)$$

$$t_N = D, \quad (4)$$

$$t_{g+1} - t_g \geq \Delta. \quad (5)$$

Pre-given value  $\Delta$  is usually introduced to force convergence.

Note that if introducing a control action results in determining the project's speed  $v_{t_g}$  to proceed with until the next control point  $t_{g+1}$  and if several alternative speeds can be chosen, then the optimal control action enables using the minimal speed to develop the project honouring chance constraint (2) [3, 4].

Control model (1-5) is a stochastic optimization problem with a non-linear chance constraint and a random number of optimized variables. The problem is too difficult to solve in the general case. Thus, heuristic control algorithms have been developed [4] to determine the next inspection point  $t_{g+1}$ . Two algorithms are considered:

- A. Using sequential statistical analysis to maximize the time span  $\Delta t_g = t_{g+1} - t_g$ .
- B. Using the idea of a risk averse decision-maker.

Algorithm A [3, 4] solves the on-line control problem as follows: to maximize the objective  $(t_{g+1} - t_g)$  subject to (3-5) and

$$\text{Pr}\{V_t \geq V_t^*(t_g)\} \geq p^*, \quad \forall t: t_g \leq t \leq t_{g+1}, \quad (6)$$

where  $V_t^*(t_g)$  is a trajectory control curve connecting two points  $(t_g, V_{t_g})$  and  $(D, V^*)$ .

This problem can be solved by determining the maximal value  $T^*$  satisfying

$$T^* = \text{Max}_{t_g < t \leq D} \{t: \Psi(q_t) \geq p^*\} \quad (7)$$

Here

$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du, \quad q_t = \frac{\bar{H}}{S^2(H_t)}, \quad H_t = V_t - V_t^*(t_g), \quad (8)$$

and  $\bar{H}_t$  and  $S(H_t)$  are the mean value and the standard deviation of random value  $H_t$ , correspondingly. In practice,  $T^*$  can be calculated via simulation with a constant step of

length  $\Delta$ . The procedure of increasing  $t$  step-by-step is followed until (7) ceases to hold. It can be well-recognized that  $t_g + T^* = t_{g+1}$  holds.

Algorithm B is based on the concept of a risk averse decision-maker [3, 8]. Given a routine inspection point  $t_g$ , the project's output observed at that moment  $V_{t_g}$  and the speed  $v_{t_g}$  to be introduced at the moment  $t_g$  up to the next inspection point, the problem is to determine that next point  $t_{g+1}$ . Just as for Algorithm A, the objective is to maximize the time span  $t_{g+1} - t_g$ . Value  $t_{g+1}$  is determined so that even if the project develops most unfavourably in the interval  $[t_g, t_{g+1}]$ , i.e., with the minimal rate  $v'_{t_g}$ , then introducing the highest speed  $v_{max}$  at moment  $t_{g+1}$  enables the project to meet its target on time, subject to the chance constraints.

Value  $t_{g+1}$  is determined via a "risk averse" heuristic [3, 8]

$$V_{t_g} + v'_{t_g}(t_{g+1} - t_g) + \bar{v}_{max}(D - t_{g+1}) = V^* \quad (9)$$

Usually Algorithm B is more efficient than Algorithm A. Both algorithms can be applied to those projects when the output can be measured at inspection points in quantitative attitudes, e.g. in percentages of the whole target (goal). This often happens in various construction projects. Another fruitful application area is PERT-COST projects when the assigned budget defines the project's target while the remaining budget actually defines the remaining project's volume.

#### 4. Application

The outlined above methodology together with on-line techniques have been used successfully for various construction projects [4]. Note that simulating on-line control procedures for a medium-size project (40-50 activities) takes about 4 hours on PC-486 using model (6-8). A risk averse method (9) has a higher speed. Thus, controlling a project comprising several hundred activities offers quite a lot of computational time. Such projects of large-size need decomposition in order to be controlled.

#### 5. Conclusions

After introducing the control actions outlined above the modified medium-size aggregated network is transformed to the initial network [9, 10] and the project's realization proceeds.

All other procedures at the project's level, e.g. scheduling procedures, are carried out for the initial network between two adjacent control points. Although those procedures usually comprise biased estimates and errors, they are periodically corrected by introducing proper control actions. *That is why those procedures in combination with control actions are more effective than without controlling the project in inspection points.*

In conclusion, the on-line control model has to be used as an *additional tool* in order to help the project manager to realize the project on time. Implementing the model does not result in undertaking any revisions in traditional PM procedures.

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