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THE COIN CHANGING PROBLEM AS A MATHEMATICAL MODEL

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Abstract: *The coin changing problem is a well-known problem to the general public and to operations research specialists and computer scientists. It is a popular topic for both discussion and programming assignments (Ghosh, 2008). The problem is a good example of the recursive nature of many problem solving techniques. Dynamic programming and greedy heuristics are perhaps the most common approaches to the coin changing problem (Johnsonbaugh and Schaefer, 2004).*

Key words: *Mathematical Model; Coin Changing Problem; Excel Solver Function*

Statement of the Coin Changing Problem

In simple terms, the coin changing problem is stated below. Given a finite set of monetary value coins (i.e. 1¢, 5¢, 10¢, 25¢, 50¢ and $1), make change for any amount utilizing the minimum number of coins possible. The recursive nature of the problem can be illustrated in a model where one chooses the largest denomination coin and multiplies the value of that coin by the maximum number which can be divided into remaining change to be made until the final change remaining is zero. Figure 1 is a model diagram of the problem where the change to be made is $2.83. All change is to be made in coins of values 1¢, 5¢, 10¢, 25¢, 50¢ and $1.

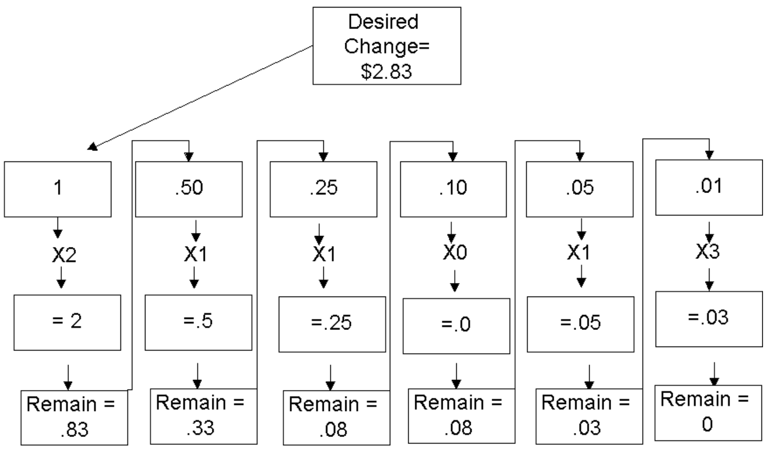

Figure 1. Schematic Model of Coin Changing Problem Desired Change = $2.83

Excel Spreadsheet Formulation

Microsoft Excel possesses a powerful add-in tool in solving optimization problems. The tool is called Solver. This Excel add-in was developed by Frontline Systems, which has also developed more powerful versions of the optimization tool for more complex problems such as non-linear, quadratic programming and discontinuous problems. The Frontline web site (Frontline Systems, n.d.) provides an in-depth discussion of the various optimization tools the company markets as well as of optimization in general. The basic Solver, which is a part of Excel, was utilized for this model. The initial spreadsheet formulation is displayed in Figure 2. Cells B2:G2 hold the values of each of the six coins. Cells B4:G4 are the cells which will be changed during the optimization process and will hold the number of each coin to be used to make the desired change. Cell I5 is the cell where the desired change should be entered. Cell I3 will hold the calculated change upon optimization which must be equal to the value entered into I5. Cell I7 is the cell which will be optimized to minimize the total number of coins used in making change and will equal the sum of cells B4:G4. Figure 3 displays the formula view of the model.

1/A	B	C	D	E	F	G	H	I
2	1	0.5	0.25	0.1	0.05	0.01		Change
3								0
4	0	0	0	0	0	0		Desired
5								0
6								Total Coins
7								0

Figure 2. Initial Spreadsheet Formulation of Coin Changing Problem

1/A	B	C	D	E	F	G	H	I
2	1	0.5	0.25	0.1	0.05	0.01		Change
3								=SUMPRODUCT(B4:G4,B2:G2)
4	0	0	0	0	0	0		Desired
5								0
6								Total Coins
7								=SUM(B4:G4)

Figure 3. Formula View Spreadsheet Formulation of Coin Changing Problem

A mathematical statement of the problem is displayed below.
Minimize: A + B + C + D + E + F
Where: A = number of $1 coins
B = number of 50¢ coins
C = number of 25¢ coins
D = number of 10¢ coins
E = number of 5¢ coins
F = number of 1¢ coins
Subject to the following constraints:
1.00A+.50B+.25C+.10D+.05E+.01F = Desired Change
A,B,C,D,E,F = Integers
A,B,C,D,E,F >= 0

The Excel Solver parameters which utilize the mathematical statement of the problem are displayed in Figure 4.

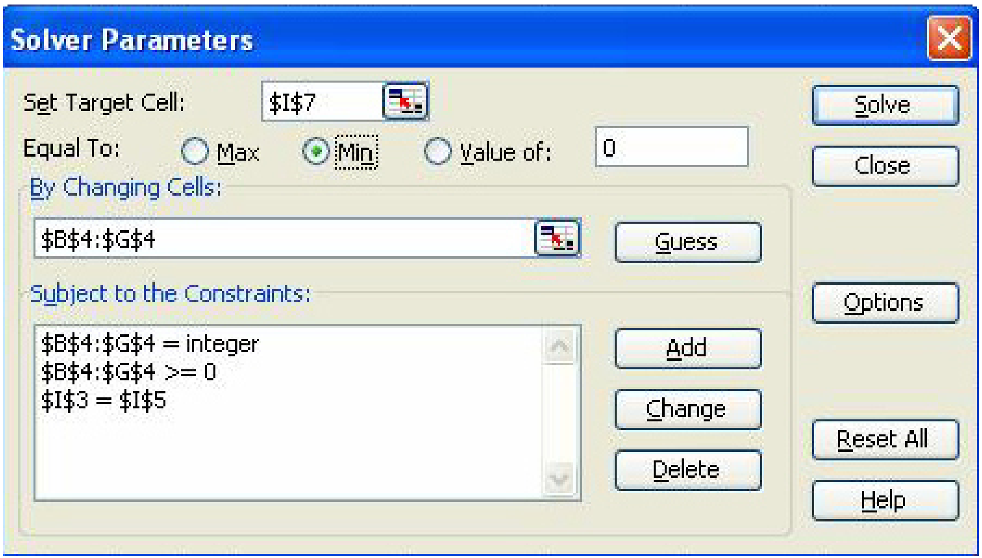

Figure 4. Solver Parameters for Coin Changing Problem

Figure 5 displays the model output solution to the previously discussed desired change amount of $2.83. Figure 6 and 7 displays output for other change amounts

1/A	B	C	D	E	F	G	H	I
2	1	0.5	0.25	0.1	0.05	0.01		Change
3								2.83
4	2	1	1	0	1	3		Desired
5								2.83
6								Total Coins
7								8

Figure 5. Solution View of Coin Changing Problem Desired Change = $2.83

1/A	B	C	D	E	F	G	H	I
2	1	0.5	0.25	0.1	0.05	0.01		Change
3								0.92
4	0	1	1	1	1	2		Desired
5								0.92
6								Total Coins
7								6

Figure 6. Solution View of Coin Changing Problem Desired Change = $.92

1/A	B	C	D	E	F	G	H	I
2	1	0.5	0.25	0.1	0.05	0.01		Change
3								3.54
4	3	1	0	0	0	4		Desired
5								3.54
6								Total Coins
7								8

Figure 7. Solution View of Coin Changing Problem Desired Change = $3.54

Summary

The well-known coin changing problem is a widely used exercise in recursive logic exercises in business and computer science classes. Most of the examples observed by the authors involved heuristic type logic, dynamic programming, greedy algorithms and backtracking techniques. The spreadsheet model in this paper is designed to illustrate what the authors believe to be a more concise and intuitive approach which will be interesting and accessible to most instructors, students, and individuals.

References

1. Frontline Systems. (n.d.) Downloaded on July 23, 2008, online source: http://www.solver.com/xlsplatform4.htm.

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