THE PHYSICS OF INFLATION: NEWTON'S LAW OF COOLING AND THE CONSUMER PRICE INDEX

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Abstract: *In recent years, physicists have been using tools from physics to study social phenomena, an area of study sometimes called sociophysics and econophysics. Most of this work has appeared in physics journals, and the present paper is an attempt to bring this type of work to a largely social science audience. The focus is on the application of a differential equation model, widely used in physics, to the study of the long term trend in changes in the United States' price level. This model is found to provide an excellent fit to the data, indicating that this trend is an exponential growth trend.*

Key words: *inflation; price index; Newton's law of cooling; sociophysics; econophysics*

In recent years, physicists have been using tools from physics to study phenomena typically considered to fall within the domain of the social sciences, an endeavor sometimes referred to as sociophysics and/or econophysics. The work of physicists on social networks, collective decision making, financial issues, and income distribution (Toivonen, et al., 2006; de Silva, et al., 2006; and Clement and Gallegati, 2005; Galam, 1997) are some key examples. Although some social scientists have been open to this effort (Ormerod and Colbaugh, 2006 and Keen and Standish, 2006), I think it's fair to say that the influence of this work has probably been felt more within physics than outside of it. It is my view, however, that work on social scientific questions by physicists should be encouraged because the things we social scientists study are complex enough for us to need all the help we can get. I also think this approach of using tools from physics to study social issues should become more visible in venues that seem to mainly attract social scientists. My view about the importance of work in sociophysics and of moving it more out of the confines of physics venues is the occasion for this paper.

The paper focuses on the application of a differential equation model that's been used in financial theory and theories of economic growth but which comes up very frequently in physics and physical chemistry. Models of radioactive decay, Newton's Law of Cooling, and changes in the concentration of reactants over time for a first order reaction are three examples. What all these examples have in common is that the rate of change in some quantity over time is proportional to the amount of that quantity. When used to represent decay, the model stipulates that some quantity is decreasing over time. Thus, in radioactive decay the atoms of some element are decreasing in number over time, in first order reactions the concentration of some reactant is decreasing over time, and in Newton's Law of Cooling the temperature of something is decreasing over time. However, Newton's Law of Cooling is also a "Law of Warming" when used to model increasing temperature over time. It's this last example of warming that's most relevant to the topic of this paper, since it's concerned with increase in the price level over time. In fact, inflation may best be conceived of as a kind of increase in the "temperature" of the macroeconomic system. In any case, it will be shown that the price level increases over time in a way analogous to increases in temperature in accordance with Newton's Law of Cooling. First, however, I will discuss previous work in economics on inflation.

Previous Work on Inflation

Much of the work in economics, that focuses on the dynamics of inflation, has been concerned with how changes in inflation are related to changes in other important macroeconomic variables such as unemployment, wage levels, and the money supply (Hess and Scweitzer, 2009; Christensen, 2001; Bjornsrad and Nymoen, 2008;). There has also been work on the role of expectations and inflation. Rudd and Whelan (2005) provide a nice overview of this expectations oriented work. Those concerned about expectations and inflation have been mainly concerned about so called "rational expectations," a concept associated with New Classical Economists, and how this notion might be integrated into New Keynesian models of sticky prices.

Previous work in economics on inflation is important for both practical and theoretical reasons. Practically speaking, this type of work on inflation dynamics may lead to better forecasts of inflation that could be helpful in the design of macroeconomic policy. From a theoretical point of view, this work is important because it may move us closer to resolving theoretical debates about the factors that account for changes in inflation, debates among New Classicals, Post-Keynesians, Keynesians, Monetarists, and others. The present paper, however, will "stand back" from previous work a bit and focus simply on mathematically modeling the overall long-term trend in the price level, deliberately neglecting many of the theoretical issues, that have preoccupied previous analysts. The only theoretical assumptions I make are 1) that increases in the price level lead to expectations of even higher increases in this level in the future and 2) that changes in the price level are proportional to changes in the level of aggregate demand. Based on these assumptions I propose a self fulfilling prophecy based theory of the long term trend in the price level.

If for some reason(s), perhaps including those discussed in the studies referred to above, the price level increases and if assumption 1 holds, this will lead potential buyers to expect an even higher price level rise in the future. This may result in the following outcome: aggregate demand will increase, as more buyers purchase more goods more quickly than was the case before the price level increase in an effort to act before the expected higher price level rise occurs. If assumption 2 holds, this will result in another increase in the price level. This increase will, in turn, lead to another, bigger than before, increase in aggregate demand, as, once again, more buyers purchase more goods more quickly that was the case before out of a desire to act before the next expected higher rise in the price level. What is being suggested here is what sociologists call a self fulfilling prophecy, a case where people's behavior, as a consequence of their expectations, end up causing the very thing they expected to happen (Merton, 1968). In fact, as this process unfolds repeatedly, what develops is a kind of perpetual self fulfilling prophecy. In qualitative terms, such a process results in changes in the price level over time that are proportional to the level of prices at a given time. Below I translate this qualitative model into a mathematical one that's been inspired by a similar model in physics: Newton's law of Cooling.

Newton's Law of Cooling

A standard physics model attributed to Sir Isaac Newton, as a result of some experimental work he'd done, is known as Newton's Law of Cooling (Cengel, 2002). In equation form, the law states that:

$$dT/dt = k(T - T_s) \tag{1}$$

where "T" denotes the temperature of a given object, "t" denotes time, "T_s" denotes the temperature of the surrounding environment, and "k" is a constant of proportionality. Equation 1 is an example of an ordinary differential equation that can be solved by the method of separating variables (Stroud and Booth, 2005). If both sides of equation 1 are divided by $(T - T_s)$ and multiplied by dt, we get:

$$dT/ (T - T_s) = kdt \tag{2}$$

If we integrate both sides of equation 2, we get:

$$\ln(T - T_s) = kt + C \tag{3}$$

where "C" is an arbitrary constant. Taking the antilogarithms of both sides of equation 3 leaves us with:

$$T - T_s = e^{kt} + e^{C} \tag{4}$$

The rules of exponents stipulates that $e^{kt} + e^{C} = e^{kt}e^{C}$. Taking this into account and adding T_s to both sides of equation 4 gives us:

$$T = Ae^{kt} + T_s \tag{5}$$

Where $A = e^{C}$.

Equation 5 is the general solution of equation 1. If k is less than zero, equation 5 tells us how the temperature of an object, surrounded by an environment at T_s, will decrease over time until it reaches the same temperature as its environment. If k is greater than zero, equation 5 tells us how the object's temperature will increase over time, the more relevant case for the present paper. Notice in equation 5 that the increase or decrease in T is exponential to the point where the temperature of the object comes to equal that of the surrounding environment. As will be seen below, this is the feature that's most analogous to the model of inflation this paper will focus on.

The mechanism behind Newton's law of cooling has to do with the second law of thermodynamics, which, in one formulation, stipulates that heat always flows from a higher temperature object to a lower temperature object (Arieh, Ben-Naim, 2008a and 2008b). Thus, considering equation 5, along with the second law of thermodynamics, k less than zero implies that, initially, the object is at a higher temperature than its surroundings. k greater than zero implies that the object is initially at a lower temperature than its surroundings. The microscopic interpretation of the second law of thermodynamics, a hallmark of statistical mechanics, explains the second law and, by extension, Newton's law of cooling in terms of the interactions of the particles that make up the systems under investigation (Arieh, Ben-Naim, 2008a and 2008b). The price inflation version of Newton's law, to be discussed below, also has a microscopic basis in the "particles" that make up the economic system. These particles are the buyers and sellers in the various markets that make up the economy, and I assume that these particles interact with one another in such a way as to lead to the kind of self fulfilling prophecy discussed above.

The Price Inflation Version of Newton's Law of Cooling

The type of feedback process involved in price levels changes, discussed above, has implications for how to model such changes. The self fulfilling process I've described leads to the stipulation that changes in the price level over time are proportional to the level of prices at a given time. Symbolically this is:

$$dP/dt = kP \tag{6}$$

Here "P" denotes the overall price level, "t" denotes time, and k is a constant of proportionality. This equation, like equation 1, can be solved by separating variables. If both sides of equation 1 are multiplied by dt and both sides are divided by P, we end up with:

$$dP/P=kdt \tag{7}$$

If we now integrate both sides of equation 7, we get:

$$\ln P=kt + C \tag{8}$$

where "C" is an arbitrary constant. Taking the antilogarithms of both sides of equation 8 leaves us with:

$$P=e^{C}e^{kt} \tag{9}$$

If we replace e^{C} with A we get:

$$P=Ae^{kt} \tag{10}$$

which is the general solution of equation 6.

Compare equation 10 with equation 5. There are very similar in form, with the only difference being that the right side of equation 5 has another constant, while the right side of equation 10 does not. However, both equations model exponential growth or decline, depending on the sign of k.

In order to see if equation 10, and by implication equation 6, fit available data, I took the logs of both sides of equation 10, which leaves us with:

$$\ln P = \ln A + kt \quad (11)$$

Data

In an effort to determine if equation 11 fit available data, I used data from the website of the United States Department of Labor (2009). This website contains monthly data on the Consumer Price Index for urban consumers (CPI-U) from 1913 to 2008 for a total of 1,164 data points. The CPI-U is calculated on a regular basis by U.S. analysts and is widely considered to be a measure of the general price level in the U.S. Thus, the CPI-U is my measure of P seen in equations 6 through 11; therefore instead of referring to the CPI-U below, for simplicity and consistency, I'll continue referring to P. Since I used monthly data, t in equations 6 though 11 is measured in units of months.

Results

Equation 11 was fit to the to the P series using ordinary least squares regression. The adjusted R^2 value for the model was .92, indicating an excellent fit to the data. That is, this R^2 value provides strong evidence that equations 6 through 10 more than adequately model inflation over this period, consistent with the self fulfilling nature of price level changes referred to above. The regression model output provided .003 as an estimate of k and 2 as an estimate of lnA. Thus, the price inflation version of Newton's Law of Cooling might best be described as a law of warming, with prices trending exponentially higher over time. It follows from the regression output that $A=e^{\ln A}=e^2=7$. Thus, equation 11 takes the form:

$$\ln P = 2 + .003t \quad (12)$$

and equation 10 takes the form:

$$P = 7e^{.003t} \quad (13)$$

Figure 1 displays a graph of P against t, indicating the exponential long term trend in P.

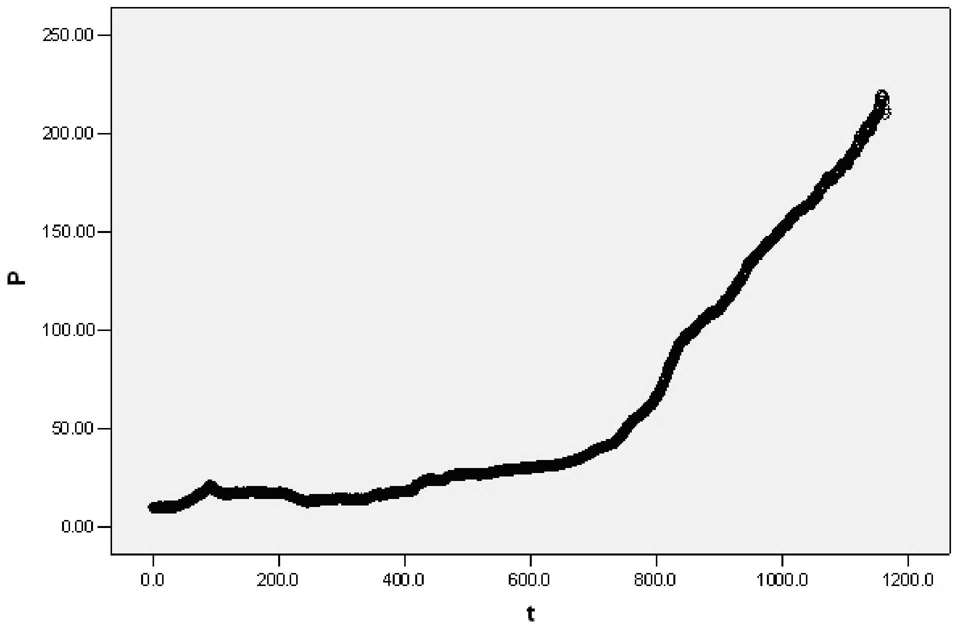

Figure 1. Graph of P against t

A key difference between this model and the physics version is that in the physics version, the laws of thermodynamics result in the temperature rise ceasing once equilibrium has been reached. There appears to be no laws of thermodynamics in the social realm to perform this function. Thus, aside from possible deflationary episodes caused by economic downturns, the upward trend in price level can apparently go on indefinitely.

The United States Department of Labor's website also has price level data for the months of January through July of 2009. I used these data to test out of sample predictions of the model spelled out in equation 10. First, I calculated the predicted values for T = 1,165 through 1,171 (the months of January 2009 to July 2009) using equation 13. I compared these to the actual values for these time points from the Department of Labor's website. The comparison was made using the percentage error formula which is:

$$(\text{actual value} - \text{predicted value})/(\text{actual value}) * 100 \qquad (14)$$

The R^2 value of .92 already provides evidence of the strong predictive power of the model, and this is reinforced by the fact that all of the absolute values of the percentage errors were less than 10%.

Discussion

This paper has focused on the analysis of changes in the price level."Standing back" from traditional debates about the correlates of inflation, involving New Keynesians, New Classicals, Monetarists, and others, I have modeled the long term dynamics of price level changes on the assumption that increases in the price level respond to increases in aggregate demand and that price level changes unfold as a kind of self fulfilling prophecy. These assumptions led to an ordinary differential equation that's popular in physics for modeling the cooling or warming of objects in a surrounding medium, and this model was found to have an excellent fit to a time series of inflation data for the United States. Evidence for this fit was the very high R^2 value of .92 and the relatively small percentage errors referred to above. Thus, Newton's Law of Cooling appears to be applicable to the dynamics of price inflation and, hopefully, this finding will provide more impetus for and acceptance of the agenda of sociophysics.

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