A FORECASTING MODEL WITH CONSISTENT ADJUSTMENTS FOR ANTICIPATED FUTURE VARIATIONS

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Abstract: Due to the limitation of most statistical forecasting models ignoring contextual information, judgmental adjustment is a widespread practice in business. However, judgmental adjustment still suffers with many kinds of biases and inconsistency inherent in subjective judgment. Our approach uses an adjustment mechanism concerning only with critical cue factors evaluated with genetic algorithm to alleviate problems caused by collinearity and insignificant sporadic variables usually arising in least square type estimators, and to derive more realistic parameter estimation. In case there are anticipated variations in the forecasting horizon and can’t be handled by the model alone, this adjusting mechanism, formulated in a set of equations, can be used to assess mixed effect of cue factors consistently and effectively without subjective judgment involved. Empirical results reveal that this adjustment mechanism could significantly reduce MAPE of forecasts across seasons with improvement mainly coming from large size adjustments.

Key words: judgmental adjustment; seasonal index realignment; genetic algorithm; calendar effect

1. Introduction

Owing to the limitations of statistical forecasting methods generating forecasts solely based on historical data [1-3], or not including critical explanatory variables, as pointed out in [4], judgmental adjustments taking advantage of contextual information or non time series information [5] become a widespread practice in business to improve forecasting accuracy [5, 6-8], especially for model-based forecast in variable environments. As [9] put it, the benefits should be greatest where series are subject to high noise and/or where the signal is relatively complex. Blattberg and Hoch [10] also argue that forecasters
can use econometric models effectively only if they have a built-in adjustment mechanism to capture the changing environment. Also see [4, 11, 12].

Most researchers in judgmental adjustment agree that, if the contextual information used is reliable, the performance of judgmental adjustments will be better than that of judgmental adjustments using unreliable one [5, 13, 14]. Hence, the eliciting of contextual information from experts using quantitative techniques such as Delphi and cross impact analysis [15], the screening [4], and classification such as the use of 5 structured types of causal forces proposed by [16], as well as processing, such as [17, 18], among many others advocating the use of decomposition method, of relevant contextual information, all are important aspects of judgmental adjustment.

However, judgmental adjustments still have all kinds of bias like cognitive bias, double-counting bias, political bias, and so on inherent in judgmental forecasting and still have the issue of inconsistency [19–21].

The objective of this study focuses on proposing an adjustment mechanism capable of handling and reflecting, without subjective judgement, detailed changes anticipated in the forecasting horizon but could not be handled by the regression model alone, which incorporating all the critical variables estimated with GA in such a way as to be more realistically conformed with the real world. Thus, this adjustment mechanism, a natural extension of the model, consisting of seasonal index realignment and proportional adjustment in a set of equations, is able to make appropriate adjustments consistently and effectively improving the forecasting accuracy of initial forecasts of the model compared favorably to other alternative like Box-Jenkins ARIMA [22-23] with adjustment.

The remainder of this paper is organized as follows. Section 2 describes our forecasting model in detail, including formulation of a regression model, a brief introduction to the feature and process of GA in model fitting, subsequent model checking, and the process of e-composition, as well as the adjusting mechanism consisting of seasonal index adjustment and proportional adjustment, as well as a combination of the former two. Section 3 portrays the background and design of our empirical research. Section 4 depicts the empirical results of model fitting and model checking, and a comparative analysis of forecasting results of various adjustment methods assessed with MAPE on per item basis, percentage of correct direction adjustment, and IMP on per adjustment basis, from the perspective of adjustment size and lead time, as well as an illustration with two graphical examples, and discussions. Finally, a conclusion is drawn in section 5.

2. The forecasting model

2.1. Formulation of a regression model

The first objective in our forecasting model involves decomposing the promotional sales of products of a company into simple components easy to handle. Eq. (1) of our regression model is motivated by Dick R. Wittink et al.'s analytical models in [24-26]. The model can be formulated as

\[
S_{it} = \lambda_{it} \left( P_{it} / \hat{P}_{it} \right) \prod_{n=1}^{n} \mu_{i,n} \prod_{o=1}^{o} W_{i,o} \epsilon_{it}, \quad \forall \ t \in Q
\]
Where, \( i \) denotes an item number, \( i = 1, 2, 3, \ldots, I \); \( t \) denotes specific number of period referenced, \( 1 \leq t \leq T \). \( T \) is the total number of normal periods. While \( I \) is the total number of items involved.

\( Q \) denotes the set of referenced periods.

\( Z \) denotes the set of periods to be forecasted.

\( S_{it} \) is the total unit sales of the item \( i \) in period \( t \) under a retailer, for weekly sales, \( t \) actually represents a certain week in the referenced periods.

\( \lambda_{it} \) denotes the normal unit sales (base sale) of the item \( i \) in period \( t \) without any promotion under a retailer.

\( \hat{P}_i \) is the list price of item \( i \).

\( p_{it} \) is the discount price of item \( i \) during period \( t \) under a retailer.

\( \theta_{it} \) denotes the coefficient of price elasticity of item \( i \) during period \( t \) under a retailer.

\( D \) denotes an indicator parameter (or dummy variable) of non-price promotion mix.

\( D_{1it} \) is the \( l \)-th component of a vector of \( n \) indicator parameters of non-price promotion mix \((D_{1it}, D_{2it}, \ldots, D_{nit})\) of item \( i \) in period \( t \). \( D_{1it} = 1 \) denotes a promotion mix of type \( l \) arises, the default value of \( D_{1it} = 0 \).

\( \mu_{it} \) denotes the non-price promotion effect parameter of type \( l \) non-price promotion mix \((D_{it})\), a combination of certain non-price promotion activities, of item \( i \) during normal period \( t \) under a retailer.

\( H \) denotes an indicator parameter of holiday.

\( H_{rit} \) is the \( r \)-th component of a vector of \( o \) indicator parameters of holiday \((H_{1it}, H_{2it}, \ldots, H_{oit})\) to indicate whether there is any holiday(s) in a certain period \( t \) or not. \( H_{rit} = 1 \) denotes a holiday of type \( r \) arises, the default value of \( H_{rit} = 0 \).

\( w_{rit} \) denotes holiday effect parameter of holiday type \( r \) in period \( t \) of item \( i \).

\( \varepsilon_{it} \) denotes the residual error.

Besides, additional notations listed below may be helpful in the following sections.

\( \varphi \) denotes the weekend effect, which is derived via GA based on data in mixed periods.

\( d(t_1) \) denotes the length of sub-period \( t_1 \) of \( t \), \( t \in Z \). \( 0 \leq d(t_1) \leq 7 \).

\( d(t_2) \) denotes the length of sub-period \( t_2 \) of \( t \), \( t \in Z \). \( 0 \leq d(t_2) \leq 7 \). \( d(t_1) + d(t_2) = d(t) \), because in a week there are at most two different kinds of promotion mixes held.

\( \delta \) denotes the duration of the weekend.

\( \delta_{t_1} \) denotes the duration of weekend covered by sub-period \( t_1 \).

Take natural logarithm in both sides of Eq. (1), we get the following:

\[
\ln S_{it} = \ln \lambda_{it} + \theta_{it} \ln \left( \frac{P_{it}}{\hat{P}_i} \right) + \sum_{l=1}^{n} D_{lit} \ln \mu_{lit} + \sum_{r=1}^{o} H_{rit} \ln \omega_{rit} + \varepsilon_{it}, \forall t \in Q
\]
Thus, a nonlinear model like Eq. (1) is transformed to a linear regression model [27-28], which is the underlying model to conduct model fitting and model checking in this study.

2.2. Model fitting--parameter estimation with GA

To take into account of all the influential and sporadic cue factors in various sub-periods of the training period, the number of variables may amount to such a quantity that conventional parameters estimation method like ordinary least square, maximum likelihood method, and so on may become incompetent, due to the issue of collinearity [29-30], insignificant parameters, [4, 31] or small sample size. Therefore, in this study we use a customized genetic algorithm (GA) which could estimate parameters effectively and efficiently [32].

2.2.1. Features and procedures of GA in this study

GA simulates Darwin’s biological evolution through stochastic crossover and mutation by selecting encoded individuals (solutions) in the population with higher fitness via a fitness function to generate population of individuals (reproduction) more fitted to the environment (better solutions) from generation to generation [33-35].

The initial population is randomly created in the encoded form of a binary matrix, there are \( \text{pop} \) rows, each row of binary string in the matrix is an individual (solution) which encompasses \( \beta \) chromosomes, each chromosome, representing a parameter, is composed of \( \gamma \) genes, while each gene is represented by a binary code.

Each individual is evaluated by the fitness function, check Eq. (3), in each generation, the best \( \alpha \% \ (1 \leq \alpha \leq 6) \) of the population are kept as elites to the next generation, the remaining of the population are created by a randomly selected pairs of individuals conducting a multi-point crossover [36], \( n + o + 2 \) points in total, for each one of each pair to reproduce offspring, forming a random recombination of individuals’ ingredients of genes, to search new solution space and possibly better solution.

After that, a one-bit mutation is performed [37], with a view to creating new pieces of gene originally not possessed by members of the population, through randomly selected genes within each individual, this occasional random change in genes could open the door to new possibilities of better solutions. Afterwards, each encoded individual in the population is decoded back to a string of real numbers of parameters, and each individual is evaluated by the fitness function..., the iterative process goes on and on until a termination condition is met.

In this study, parameters like crossover probability \( (P_c) \) and mutation probability \( (P_m) \) of GA are designed to vary with the number of generations processed or others, such as the minimum level of moving average percent of improvement (MAPI) in fitness function value within certain number of generations, to keep proper diversity of the population, while retaining the convergence capability, to circumvent getting stuck too early at local solutions in its search process and derive satisfying results [38-39].

In estimating parameters of complicated multivariate nonlinear models, GA is generally considered to be better than other alternatives such as nonlinear least square,
maximum likelihood estimation, and so on, due to its parallel search capability [40-41], even based on small size dataset, it is capable of deriving satisfying results.

The fitness function of GA may be formulated as

$$FV_t = \text{MAPE}_t = (\sum_{t=1}^{T} \left| \ln S_{it} - \ln \tilde{S}_{it} \right| / \ln S_{it}) / T, \forall t \in Q$$

(3)

Where the term \( \left| \ln S_{it} - \ln \tilde{S}_{it} \right| \) is the absolute value of difference between natural logarithm of the actual sales volume (\( \ln S_{it} \)) of the \( i \)-th item and natural logarithm of the estimated sales volume (\( \ln \tilde{S}_{it} \)) of the same item in period \( t \). \( T \) denotes the number of normal periods. The objective of GA is to find a solution with the minimal \( \text{MAPE}_t \). The smallest \( \text{MAPE}_t \) found is updated once a smaller one is found in the solution search process. After model fitting, every effect parameter in Eq. (2) is derived in real value.

### 2.3. Model checking

In this section, a regression diagnostics focused on normality and independence is performed to see if critical assumptions of linear regression are violated, based on Eq. (2). If these assumptions are severely violated, particularly if collinearity arises among predictor variables, bias may be a serious issue in model fitting or even in model specification.

Normality test is conducted through One-Sample Kolmogorov-Smirnov test [42], and Q-Q plot [43]. Independence test in this study consists of two parts, namely, multicollinearity test and autocorrelation test. The former is performed via condition index, whereas the latter is performed via ACF checking [44].

### 2.4 The re-composition of effect parameters

As the cycle length of CPG industry is about 52 weeks long, let \( t' = t + 52 \), denoting the corresponding week to be forecasted in a new year. A modified naïve sales forecasting method considering cycle length to forecast unit sales of item \( i \) of period \( t' \) in a new year, see Williams (1987), according to sales data of week \( t \) in the referenced year, would be

$$\hat{S}_{it'} = \eta, \pi, \ln \left( \frac{P_{it}}{P_t} \right) = \prod_{i=1}^{n} \mu_{it} \prod_{r=1}^{n} \omega_{it}, \quad t' = t + 52, \forall t' \in Z.$$  

(4)

Where, \( \eta \) denotes the average normal sale of item \( i \) across referenced periods. \( \pi \) denotes the seasonal index of item \( i \) in period \( t \), and \( Z \) denotes the set of periods to be forecasted.

So far, all the parameters in Eq. (4) are already derived via GA. Let \( e_{1it} \) denotes the price effect multiplier of item \( i \) in forecasting period \( t' \), \( e_{2it} \) and \( e_{3it} \) denote the effect multiplier of a non-price promotion mix and a specific holiday effect, respectively. In each group of indicator parameters at most one condition will arise in each period. We get
\[ \hat{S}_{t'} = \eta_t \pi_t \epsilon_{1t} \epsilon_{2t} \epsilon_{3t}, \quad t' = t + 52, \forall t' \in \mathbb{Z}. \] (5)

In its re-composed form, Eq. (5) can be used to forecast weekly unit sales. Actually, parameters estimated through GA, based on observations in the training periods, can be recombined as Eq. (5) in responding to expected promotional campaigns in the forecasting horizon specified in the promotion proposals to perform out of sample forecasting without any adjustment in the following empirical study.

2.5. The adjusting mechanism of this study

The mechanism of this study stresses that adjustments of forecasting are based on the anticipated changes of the context of promotions and holidays in the forecasting periods, which can not be handled by the regression model alone, in this study a set of equations are formulated to do this job, they are natural extension of the model. The objective is to improve the performance of the model, making our final forecasts more closely reflect these changes in prospect.

2.5.1. Seasonal index adjustment (SIA)

Based on domain knowledge, sales volume of the last week or average sales volume of the last few weeks (adjusted with calendar effect) in the reference periods is a better predictor to sales of the first few weeks in the forecasting periods next year than sales of the same weeks in the referenced periods in Taiwan, check Figures. 1-2. Thus, the corresponding formula can be modified from Eq. (5) and formulated as

\[ \hat{S}_{t'} = \eta_t \pi_t \epsilon_{1t} \epsilon_{2t} \epsilon_{3t} \epsilon_{4t}(t'=t+52-ws), \forall t' \in \mathbb{Z} \] (6)

Eq. (6) is an example of modified Naïve method used for multiple-step out-of-sample forecasting considering cycle length which is a year. In which, \( \eta_t \pi_t \epsilon_{1t} \epsilon_{2t} \epsilon_{3t} \) stands for normal (base) sales of the last week(s) in the training periods and is the most recent related data available. While \( \epsilon_{4t} \) stands for pre LNY (Lunar New Year) effect arises annually in a period of around 4 weeks right before LNY. In this period, sales volumes are usually much higher than usual even without any promotion. Since pre LNY effect has not been incorporated as a variable into our regression model for parsimonious purpose, it will dominate the seasonal indices in corresponding periods, hence, it’s quite intuitive to use these indices as proxy variable of pre LNY effect, denoted as \( \epsilon_{4t} \). Because there is usually a time shift of the timing of LNY from year to year, to forecast unit sales of weeks after LNY in next year, the week number referenced corresponding to the week in the forecasting horizon has to be adjusted.

Let LNY(\(t'\)) denote the week number of LNY in the year to be forecasted, LNY(\(t\)) denote the week number of LNY in the referenced year, then, let \( ws = LNY(\ t'\) – LNY(\(t\)) \) represents the number of weeks shifted between two different years as the week of LNY is concerned. If \( ws > 0 \), it means the sequence of week of LNY(\(t'\)) in the forecasting year will be \( ws \) weeks later than that of LNY(\(t\)) in the referenced year; on the other hand, if \( ws < 0 \), it means the sequence of week of LNY(\(t'\)) in the forecasting year will be \( ws \) weeks earlier than
that of LNY(t) in the referenced year. Therefore, the most right term in Eq. (6) \( e_{t(t-w)} \) will be replaced by \( \pi_{\hat{t}(t-w)} \). Thus Eq. (6) will become

\[
\hat{S}_{t'} = \eta_{t'} \pi_{\hat{t}(t-O)} e_{1t'} e_{2t'} e_{3t'} \pi_{\hat{t}(t-w)} , \forall t' \in Z
\]

(7)

We also find that sales of the same weekly order as that in a different year after LNY are more aligned than weekly sales of the same ordinary sequential order between different calendar years, check Figures 3-4, in which \( R^2 \) from sales of weeks after LNY in 2007 regressed against those of weeks after LNY of 2006 is 0.8, much better than the ordinary week \( n \) corresponding to the same week \( n \), \( n = 1,2,..,5 \), regression between different years, which only has a \( R^2 \) of 0.628, please check Figure 4.

Based on the finding mentioned above, to forecast the sales of weeks after the week of LNY in a new year, denoted as \( \hat{S}_{t'} \) for \( t' > LNY(t') \), Eq. (8) can be of use, which is modified from Eq. (7):

\[
\hat{S}_{t'} = \eta_{t'} \pi_{\hat{t}(t-w)} e_{1t'} e_{2t'} e_{3t'} \pi_{\hat{t}(t-w)} , \forall t' \in Z
\]

(8)
2.5.2. Proportional adjustment (PA)

Quite often, in the week of LNY, there is a small part of pre LNY present prior to the eve of LNY, or the last week of pre LNY is mixed with a small part of LNY in the referenced period, but the condition of the corresponding week in forecasting horizon is different, in these cases, to get a proper estimation of these effect multipliers of calendar effect in the forecasting effect, we must get them restored to regular ones (a whole week only covered by purely one kind of holiday related effect like pre LNY effect or holiday effect of LNY) first, and then proceed to calculate the changed mixed effect in the forecasting period.

The adjusting equations are used to calculate the mixed effect of pre LNY and LNY present in the same week:

\[
\phi(t) = \frac{(d(t_1) - d(\delta_{t_1})) + d(\delta_{t_1})e_{4it}}{d(t) + d(\delta)(\varphi - 1)}, \forall t \in (Q \cup R)
\]

In Eq. (9), the part \((d(t_1) - d(\delta_{t_1})) + d(\delta_{t_1})e_{4it}\) represents the sum of the effect of the last week in pre LNY in the duration of weekdays covered by sub-period \(t_1\) and the effect of the last week in pre LNY in the duration of weekend covered by sub-period \(t_1\) times the weekend effect. While the term \((d(t_2) - d(\delta_{t_2})) + d(\delta_{t_2})e_{3it}\) stands for the sum of the effect of regular LNY in the duration of weekdays covered by sub-period \(t_2\) in the referenced period and the effect of regular LNY in the duration of weekend covered by sub-period \(t_2\) in the referenced period times the weekend effect. Every parameter in Eq. (9) except \(e_{3it}\) is known, so \(e_{3it}\) can be obtained. Note that the daily effect of each normal weekday in a week is assumed to be 1. Eq. (9) actually is a daily effect weighted average formula of mixed weekly effect of pre LNY and LNY present in the same week.

It follows that the mixed effect in the forecasting period \(e_{3it_{t'}}\) could be computed through the following formula:

\[
\phi(t') = \frac{(d(t_1') - d(\delta_{t_1}')) + d(\delta_{t_1}')e_{4it_1} + (d(t_2') - d(\delta_{t_2}')) + d(\delta_{t_2}')e_{2it_2} + (d(t_2) - d(\delta_{t_2})) + d(\delta_{t_2})(\varphi)(e_{3it_2})}{d(t) + d(\delta)(\varphi - 1)}, \forall t' \in Z
\]

Where, \(e_{it_2}\) may be effect of the last week in pre LNY or just base effect equal to 1. In Eq. (10), the part \((d(t_1') - d(\delta_{t_1}')) + d(\delta_{t_1}')e_{4it_1}'\) stands for the sum of the effect of the regular LNY in the duration of sub-period \(t_1'\) in the weekdays in the forecasting period and the effect of the regular LNY in the duration of sub-period \(t_1'\) in the weekend times the weekend effect. While the part of \((d(t_2') - d(\delta_{t_2}')) + d(\delta_{t_2}')e_{2it_2}'\) represents the sum of the effect of the last week in pre LNY in the duration of sub-period \(t_2'\) in the weekdays in the forecasting period and the effect of the last week in pre LNY in the duration of sub-period \(t_2'\) in the weekend times the weekend effect.

In the same token, regular effect of the last week in pre LNY \(e_{4it_1}'\) in the referenced periods can be derived via Eq.(11). Then, the mixed effect of \(e_{4it_1}'\) could be obtained with Eq. (12):
2.5.3. Total adjustment (TA)

As the combination of both SIA and PA, TA is the most comprehensive adjustment in this study.

3. Empirical Study

3.1. The background of empirical study

This study has a focus on the adjustment of model-based forecast of weekly unit sales of several series of CPG products, manufactured by Company A, under retailer B. Company A is a leading manufacturer specialized in dehumidifier and deodorizer products in Taiwan. While retailer B is an international outlet of DIY products.

A sales data set of 10 items in 2007, aggregated from retailer B’s outlets, coupled with price promotion, non-price promotion data, as well as promotion proposals, which were set up in 2007, of the first 4 months in 2008, are used to conduct our empirical study. The training dataset covers two periods, the first period covers the whole year of 2007, the dataset of this period can be denoted as sample A, forecasting horizon is the first 6 weeks of 2008. The second period ranges from the beginning of 2007 to the 10th week of 2008, the training data of this period can be denoted as sample B, forecasting horizon ranges from 11th week to 16th week of 2008.

The underlying equation used in model fitting was Eq. (2), all the effect parameters in Eq. (2) were estimated through GA with objective function set as Eq. (3) and constraints set realistically from contextual knowledge, such as price elasticity parameter to be in the range of [0, -8], non-price promotion effect multiplier to be in the range of [1, 5], holiday effect multiplier to be in the range of [1, 2]. Besides, the number of types of non-price promotion mixes n in Eq. (2) was set to be 7, therefore, there are about 7 types of different combination of promotion activities across seasons, and the number of holiday type o was set to be 4, which means there are about 4 different types (according to duration of holiday) of holidays each year, to reflect the actual business settings. GA programs were run with Matlab 7.1. Effect parameters estimated with GA and mixed effect parameters reassessed in the mixed periods on both sample A and sample B were recomposed according to the expected variations of promotions in the promotion proposal as initial forecasts without any adjustment. The multiple-step out-of-sample forecasts with ARIMA were run with SPSS 13. In busy season, because of intensive promotion campaigns, ARIMA tends to underestimate unit sales, the rule of adjustment for forecasts of ARIMA can be formulated as

\[
\text{ARIMA ad} = \text{forecast of ARIMA} \times 1.2
\]  

However, in time of off season, ARIMA tends to overestimate unit sales, the rule of adjustment can be formulated as...
ARIMA\ ad = forecast of ARIMA * 0.8 \hspace{1cm} (14)

Thus, the performance of various adjustment methods in this study can be compared with their counterpart of ARIMA.

3.2. The design of empirical study

In order to take both the busy season and off season into account to have a proper assessment of the performance of different adjustment methods, the forecasting horizon is designed to consist of two periods of equal duration, the first period includes the first 6 weeks of 2008 which covers the busiest season, ie, the LNY season in Taiwan, and can be denoted as busy season, while the second one starts from the 11th week and ends at the 16th week of 2008, which is one of the off seasons in the same year and can be denoted as off season.

As the forecasting target is concerned, 10 items of products were selected to conduct our empirical research. The relevant prices and promotion activities can be found in promotion proposals which actually are the source of anticipated variations in promotions. Another source of anticipated variations in calendar effects is the calendar.

To properly evaluate the performance of various methods in adjusting original forecasts of the model, which can be denoted as NA, made by regression model, SIA was performed first to adjust NA, followed by PA to adjust the same NA. Then, the combination of SIA and PA were used to adjust NA, denoted as TA. The busy season was the first forecasting horizon, and off season was the second forecasting horizon. In addition, for the purpose of comparative reference, forecasts with Box and Jenkins ARIMA were derived, adjustments of forecasts from ARIMA were derived with Eq. 15 in busy season, and Eq. 16 in off season, respectively.

4. Empirical results

4.1. The results of model fitting

The estimated error in terms of MAPE in general is below 3%, except item 10. Most parameters derived are consistent with our expectations, such as the effect parameters of $\mu_1$ to $\mu_5$ are increasingly bigger from 2.391 to 2.992 for sample A and from 2.382 to 2.848 for sample B, respectively, because more effort and expenditure are made for the bigger number type of promotion therein, and $\mu_5$ is larger than $\mu_4$ because non-price promotion type 5 employs direct mail in addition to what type 4 has.

4.2. The results of model checking

The normality test, consisting of one-sample Kolmogorov-Smirnov test and Q-Q plot, in which, this model passed the test with data from sample A or sample B without problem based on standardized error term $\varepsilon_i$ in Eq. (2) and natural logarithm of predicted weekly unit sales denoted as $\ln \hat{S}_i$ in Eq. (3). However, in independence test, the measures of condition index and results of ACF showed complex but interesting consequences in both samples, in which 5 out of 10 items have autocorrelation problems for sample B. As for sample A, there are 2 items have the same kind of problems, however. As collinearity is concerned, according to [44], if the condition index is above 10 and below 30, there may
have a minor problem of collinearity, if condition index is above 30 and below 100, there may have moderate to severe collinearity issue. In our empirical study, 4 out of 10 items may have moderate to severe collinearity problems, 5 items may have severe collinearity problem for sample B, the condition looks similar for sample A, check Table A1 in Appendix A. If model parameters are estimated by OLS, it is quite possible to have serious bias issues.

4.3. Comparing and analyzing results from various kinds of forecasting adjustment methods

The performance of weekly sales forecasting adjustment from various methods in terms of MAPE can be displayed in Table 1 and Table 2. Each cell with negative adjustment performance is in bold face. Among these adjustment methods, without taking advantage of any adjustment, the MAPE of sales forecasting with the regression model, that is NA, in average, is 17.96% and 37.06% in busy season and off season, respectively.

If forecasts are adjusted with SIA (seasonal index adjustment), the average performance across items in terms of MAPE is 19.51% for busy season, which seems a little worse than NA, check Table 1. However, for off season, the average figure of SIA is 24.77%, a significant improvement of 33.16% of initial MAPE, check Table 2. There are 5 items improved out of 10 because of SIA for busy season, while there are 7 items get improved due to SIA’s contribution for off season. The relatively poor performance of SIA in busy season may be attributed to the already good performance of NA compared to that of ARIMA without adjustment.

If adjustment is conducted with PA (proportional adjustment of mixed effect in mixed periods) in busy season, we see an improvement from 17.96% to 14.98%, a 16.59% improvement in average. The number of items with negative results is reduced to 3 also, check Table 1. However, for off season, it’s a different story for PA, with MAPE just slightly reduced from 37.06% to 34.37%, a small improvement of 7.26% overall in average. Nevertheless, 5 out of 7 adjustments performed gets improvement in MAPE, besides, item 3, 8, and 10 didn’t perform any PA adjustment, therefore, their MAPE are the same as that of NA, and almost only 1 out of 6 weeks needs to get adjustment with PA in off season, their contribution to the improvement of MAPE is therefore trivial, check table 2.

<table>
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<th>MAPE of various forecast adjustment methods</th>
<th>ARIMA ad</th>
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<td>SIA</td>
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<td>10</td>
<td>16.99%</td>
<td>17.01%</td>
</tr>
<tr>
<td>AVG</td>
<td>17.96%</td>
<td>19.51%</td>
</tr>
</tbody>
</table>

Note: forecasting with ARIMA^1(1, 1, 1)
If TA (total adjustment) is performed, since it combines both SIA and PA, for busy season, average MAPE reduced from 17.96% to 16.88%, the reduction of MAPE amounts to an average of 6.01% in improvement. For off season, the improvement is even more significant, the MAPE of NA improved from an average of 37.06% to an average of 22.78%, about 38.51% improvement over MAPE of initial forecasts. Note that if both SIA and PA have positive contribution in improving forecast accuracy, TA can be very effective, check item 1 and item 4 in Table 1, also item 1, 4, 5, 6, and 7 in Table 2.

If the adjustment of ARIMA is of concern, 6 out of 10 items get improved in busy season, check Table 1, but average percent of improvement from adjustment is a negative -0.942%, overall performance of ARIMA adjustment seems to be worse than that of PA and TA, however, it still is better than SIA in busy season.

Table 2. Comparison of the accuracy of various forecast adjustment methods in off season 2008

<table>
<thead>
<tr>
<th>item</th>
<th>NA</th>
<th>SIA</th>
<th>PA</th>
<th>TA</th>
<th>ARIMA</th>
<th>ARIMA ad</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33.03%</td>
<td>20.93%</td>
<td>23.27%</td>
<td>12.12%</td>
<td>46.95%</td>
<td>18.95%</td>
</tr>
<tr>
<td>2</td>
<td>12.77%</td>
<td>33.43%</td>
<td>16.58%</td>
<td>36.78%</td>
<td>21.23%</td>
<td>20.80%</td>
</tr>
<tr>
<td>3</td>
<td>36.16%</td>
<td>12.37%</td>
<td>36.16%</td>
<td>12.37%</td>
<td>40.07%</td>
<td>52.05%</td>
</tr>
<tr>
<td>4</td>
<td>39.28%</td>
<td>23.41%</td>
<td>38.52%</td>
<td>24.32%</td>
<td>38.86%</td>
<td>17.87%</td>
</tr>
<tr>
<td>5</td>
<td>90.76%</td>
<td>46.36%</td>
<td>85.31%</td>
<td>41.77%</td>
<td>29.58%</td>
<td>23.08%</td>
</tr>
<tr>
<td>6</td>
<td>59.41%</td>
<td>8.83%</td>
<td>56.82%</td>
<td>10.43%</td>
<td>15.94%</td>
<td>26.55%</td>
</tr>
<tr>
<td>7</td>
<td>37.50%</td>
<td>32.12%</td>
<td>25.04%</td>
<td>19.38%</td>
<td>32.02%</td>
<td>28.37%</td>
</tr>
<tr>
<td>8</td>
<td>24.50%</td>
<td>19.55%</td>
<td>24.50%</td>
<td>19.55%</td>
<td>16.56%</td>
<td>8.38%</td>
</tr>
<tr>
<td>9</td>
<td>17.09%</td>
<td>19.03%</td>
<td>17.36%</td>
<td>19.44%</td>
<td>11.51%</td>
<td>11.93%</td>
</tr>
<tr>
<td>10</td>
<td>20.13%</td>
<td>31.63%</td>
<td>20.13%</td>
<td>31.63%</td>
<td>34.87%</td>
<td>47.90%</td>
</tr>
<tr>
<td>AVG</td>
<td>37.06%</td>
<td>24.77%</td>
<td>34.37%</td>
<td>22.78%</td>
<td>28.76%</td>
<td>25.59%</td>
</tr>
</tbody>
</table>

In off season, the adjustment of ARIMA displays a performance obviously better than its counterpart in busy season, the number of items get improved in MAPE remains the same as that in busy season. However, overall percentage of improvement from adjustment amounts to 11.02%, a performance much better than its counterpart in busy season.

4.3.1. Analysis of various adjustment methods from the perspective of adjustment size

In this subsection, the percentage of correct direction in adjustments over total adjustments is used to measure the performance of various adjustment methods. Whether the direction is correct or not depends on the comparison between initial forecast and the actual sale, if initial forecast is under-forecast, the correct direction of adjustment should be adjusted upwards, regardless of adjustment size. On the other hand, if initial forecast is over-forecast, the correct direction of adjustment should be adjusted downwards, regardless of adjustment size. However, if the initial forecast is within the range of [actual unit sales - 3%*actual unit sales, actual unit sales + 3%*actual unit sales], any subsequent adjustment with result less than or equal to initial over-forecast or any adjustment with result more than or equal to the under-forecast is perceived as adjustment in the correct direction.

In this study, any adjustment with result within less than 10% range of the initial forecast, regardless of adjustment direction, is regarded as small adjustment, otherwise, it's a large adjustment. In Table 3, all the small adjustment, in both busy season and off season,
has the ratio of adjustment with correct direction regardless of adjustment method, to be less than 60%.

On the other hand, large size adjustments seem to have a much more consistent and better performance in average than that of small ones, with the average correct ratio at least over 70% in off season except ARIMA adjustment, particularly, in busy season, an average ratio of correct direction around 86% is recorded for methods proposed in this study, whereas for adjustment of ARIMA, the percentage is 73.33%, still is not bad. This result is not surprising, in the literature, there are considerable similar evidences (Fildes and Goodwin, 2007; Syntetos et al., 2009).

Among three adjustment methods proposed in this study, SIA seems to have the best performance in terms of the ratio of correct direction adjustment in both small and large adjustments in off season, the overall ratio of correct direction adjustment on the basis of per adjustment is about 80%, but it does not necessarily mean that SIA provides the most positive contribution to improvement of forecast accuracy, because in Table 4, whether the adjustment is over-adjusted or not is not taken into account. Table 4 may offer some remedies in this regard. However, in busy season, PA seems to be the winner, with an overall ratio of correct direction adjustment close to 80%, whether it offers the most positive contribution to forecast accuracy improvement or not, still have to be crosschecked with other criteria like IMP in Table 4 to have an adequate assessment.

A measure called IMP, which can be used to evaluate the adjustment improvement, may be formulated as

\[ IMP = APE_{ini} - APE_{ad} \] (15)

Where, APE denotes absolute percentage error, APE_{ini} denotes APE of initial forecast, while APE_{ad} denotes APE after adjustment.

In Table 4, with the only exception of SIA applied in busy season, large adjustments consistently and significantly outperform small adjustments in terms of IMP, regardless of the adjustment method. The only exception is SIA which implies that most large size SIA adjustments with correct direction in Table 3 are actually over-adjusted. Note that in busy season, all three adjustment methods using small adjustment, the average IMP are all negative, among them, more than half of small adjustments made by SIA and TA are in correct direction, this means that there are serious issue of over-adjustment in the small size adjustments of these two adjustment methods.

Table 3. Comparing the performance concerning direction of adjustment of various adjust

<table>
<thead>
<tr>
<th>AD method</th>
<th>busy season 2008</th>
<th>off season 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ratio of small ad</td>
<td>% of correct direction in small ad</td>
</tr>
<tr>
<td>SIA</td>
<td>30/60</td>
<td>53.33%</td>
</tr>
<tr>
<td>PA</td>
<td>5/20</td>
<td>40.00%</td>
</tr>
<tr>
<td>TA</td>
<td>25/60</td>
<td>56.00%</td>
</tr>
<tr>
<td>ARIMA ad</td>
<td>0/60</td>
<td>--</td>
</tr>
</tbody>
</table>
Table 4. Comparing IMP of various forecast adjustment methods in either small or large adjustments

<table>
<thead>
<tr>
<th>AD method</th>
<th>busy season 2008</th>
<th>off season 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>avg IMP from small adjustments</td>
<td>avg IMP from large adjustments</td>
</tr>
<tr>
<td>SIA</td>
<td>-0.21%</td>
<td>-5.63%</td>
</tr>
<tr>
<td>PA</td>
<td>-3.86%</td>
<td>14.37%</td>
</tr>
<tr>
<td>TA</td>
<td>-0.395%</td>
<td>4.44%</td>
</tr>
<tr>
<td>ARIMA ad</td>
<td>--</td>
<td>-1.036%</td>
</tr>
</tbody>
</table>

As large size adjustments are concerned, in Table 3, even though PA is not the best performer in terms of percent of correct direction adjustment, however, in Table 4, it does have the best performance in terms of IMP, this implies that the number of over-adjustments of PA is the smallest. In busy season, PA still is the best performer in terms of IMP, it is the best one even from the viewpoint of percent of correct direction adjustment, crosscheck with Table 1, obviously, it provides the most consistent and the largest contribution to the improvement of forecast accuracy in busy season among various methods. However, in off season, because of the relatively less frequency of PA adjustments made, even though it still offers the best performance in terms of IMP, check Table 4, its overall contribution to improvement of forecast accuracy in terms of MAPE is not impressive.

On the other hand, the performance of TA in terms of IMP in off season though is not the best among these methods, due to its highest ratio of large size adjustment, check Table 3, TA still provides the most positive contribution to improvement of forecast accuracy in terms of MAPE, check Table 2.

4.3.2. Analysis of various adjustment methods from the perspective of lead time

In Figure 6, each forecasting horizon is divided into two parts, namely, the first 3 weeks and the second 3 weeks in both busy season and off season. Obviously, in busy season, the performance of various adjustment methods is relatively more stable than that of its counterpart in off season. Among different adjustment methods, PA seems to have the best performance in terms of IMP across different seasons, TA ranked second, and SIA still is the worst performer. Note that PA is only conducted in the second half of the forecasting horizon, check Figure 5, unlike other methods, it appears as a point in each season.

The performance of adjustment of ARIMA looks not so stable in different lead time of busy season, however, in off season, its performance is parallel to others in shape but obviously inferior to others.

4.4. Illustration of typical adjustment of different methods with two examples

To further present the detailed results of each adjustment method compared to initial forecast and Box and Jenkins ARIMA’s forecasts and their simple adjustments as a yardstick, two graphs are drawn, see Figures 6-7. Note that to expose the detailed results of adjustments more clearly, all data from week 11 to week 50 of 2007 are cut off. The forecasting horizon is the first 6 weeks of 2008 (the busy season) in Figure 6.

Note that the original forecasts of the regression model are expressed in purple square dots in purple line, after adjustment of SIA, which are expressed in yellow triangle dots with yellow line, in weeks 3-4, the direction of adjustment are not correct, therefore, SIA
actually made obvious negative improvements. There is no adjustment made in the first 4 weeks for PA, However, in weeks 5-6, due to excellent adjustments of PA, which are expressed in empty blue diamonds with bold blue line, the forecast points are moved much closer to the actual sales, check TA in weeks 5-6 in Figure 6. Also note that down in the bottom are points formed by ARIMA, after 20% upward adjustment, these points are moved closer to the reality.

To illustrate the adjustment performance of different adjustment methods and initial forecasts as well as forecasts and adjustment of ARIMA in a bigger scope, the same item is used in Figure 7, in which, initial forecasts of the model are expressed in purple dots which are not close to the reality, after adjustments of SIA, check yellow triangle dots in weeks 11-16. In weeks 15-16, owing to the adjustment of PA, the forecasts are moved much closer to the reality. Because of the relative poor performance of NA in the first 4 weeks in Figure 7, and PA doesn’t make any adjustment in this period, its overall performance is unlike its performance in Figure 6. TA, due to its combination of SIA and PA, particularly SIA, which has a good performance and therefore push overall TA closer to the reality.

The points of ARIMA stays in relative high positions from week 11 to week 16 and are also forming the very one most far away from the reality, after a 20% downward adjustment, they are moved much closer to the actual sales in average, check the bold green line with empty circles in Figure 7.

![Figure 5. Comparison of average IMP of various adjustment methods on different lead times](image)

**4.5. Discussions**

From the above analysis and explanations, on a per adjustment basis, PA offers the most improvement in terms of both MAPE and IMP in busy season, check Table 1 & 4, it also has the highest percentage in correct direction adjustment in busy season. In off season, since it is rarely used (the mixed effect condition is rare in comparison), its total contribution is not impressive.

TA, on the other hand, is more comprehensive in off season, and provides the most contribution in improving MAPE, even though in terms of percentage of correct direction adjustment, it is not the best performer. Crosscheck Table 3 and Table 4, it is easy to see that, the correct direction is the prerequisite for an adjustment of any kind to improve forecasting accuracy, but due to the issue of over-adjustment, many correct direction adjustments still have negative contributions to forecasting accuracy.
As adjustment of ARIMA is concerned, its performance in terms of MAPE is negative in average in busy season, nevertheless, in off season, the average MAPE after adjustment is a not bad 25.59%, a performance quite close to that of TA in the same season. Therefore, if efficiency is an important issue in forecasting, ARIMA with adjustment is a good alternative in off season. Otherwise, TA is the best tool of adjustment in off season. In busy season, PA is the best choice in forecast adjustment, due to its significantly much better performance.

Besides, with the only exception of SIA in busy season, if the performance is measured in terms of both percentage of correct direction adjustment and IMP in average, large size adjustment has a significant advantage over small size adjustment, regardless of season. Our model assumes that there will be no big difference between actual promotion activities and those specified in promotion proposals in forecast horizon, if this is not true,
there will be larger MAPE incurred in the original forecasts and various types of forecast adjustments. The relatively more accurate performance of original forecasts in busy season than in off season may due to the fact that promotion and calendar effects are so strong that they dominate unit sales in busy season, while in off season, these effects are much less obvious and much less frequent as in busy season, other factors like seasonal index realignment, competitors’ actions and so on may have critical impacts on unit sales therein.

5. Conclusions

The forecasting adjustment mechanism proposed in this study only concerning with realignment of seasonal indices and the anticipated mixed effect of certain variables, such as the multiplier of the effect of promotion mix, and the multiplier of holiday effect, already incorporated in the regression model and assessed with GA which is more flexible and is capable of deriving more realistic coefficient of variables than most other conventional alternatives. Therefore, adjustment mechanism proposed in this study is a necessary and natural extension of the regression model. And in the process of forecast adjustment, subjective judgement based on contextual information is minimized.

Among three adjustment methods embedded in the adjustment mechanism of this study, SIA focuses on the realignment of seasonal index in forecasting horizon in a different year than the year of referenced periods, PA provides the necessary reassessment of mixed effect in mixed periods and is capable of offering the most contribution to the improvement of forecasting accuracy on per adjustment basis and is also the best performer in average in busy season. However, in off season, since both SIA and PA provide positive contribution in improving forecasting accuracy, and TA combines the above two adjustment methods, it offers the most comprehensive and most reliable adjustment in off season of this study, even though it’s not necessarily the best performer in improving initial model-based forecast on a per adjustment basis in average.

In off season, ARIMA with adjustment, which just moves down the original forecast by 20%, also provides very good forecast accuracy close to that of TA in average, besides, ARIMA is embedded in most statistical software package and is very handy, in case efficiency is an important requirement in forecasting, ARIMA with adjustment intuitively is a good alternative.

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2. Armstrong, S. Long-range forecasting: from crystal ball to computer, New York; Wiley, 1985


[24. Hendry, D. F. Econometric modeling, Lecture notes for the PhD course in econometric modelling and economic forecasting, Department of Economics, University of Oslo, 2000]


38. Sanders, N. and Manrodt, K. B. Forecasting practices in US corporations: Survey results, Interfaces 24, 1994, pp. 92-100
44. Wang, C. L., and Wang, L. C. A GA-based sales forecasting model incorporating promotion factors, Working Paper., Department of Industrial Engineering and Enterprise information, Tunghai University, Taiwan, ROC, 2009
### Table A1. Results of model checking

<table>
<thead>
<tr>
<th>Item</th>
<th>Sample A</th>
<th>Sample B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normality</td>
<td>CI (mean of max)</td>
</tr>
<tr>
<td>1</td>
<td>O&lt;sup&gt;a&lt;/sup&gt;</td>
<td>105.341</td>
</tr>
<tr>
<td>2</td>
<td>O</td>
<td>118.517</td>
</tr>
<tr>
<td>3</td>
<td>O</td>
<td>116.247</td>
</tr>
<tr>
<td>4</td>
<td>O</td>
<td>51.337</td>
</tr>
<tr>
<td>5</td>
<td>O</td>
<td>67.519</td>
</tr>
<tr>
<td>6</td>
<td>O</td>
<td>21.686</td>
</tr>
<tr>
<td>7</td>
<td>O</td>
<td>77.332</td>
</tr>
<tr>
<td>8</td>
<td>O</td>
<td>40.015</td>
</tr>
<tr>
<td>9</td>
<td>O</td>
<td>195.611</td>
</tr>
<tr>
<td>10</td>
<td>O</td>
<td>374.608</td>
</tr>
</tbody>
</table>

**Note:**
- CI denotes condition index.
- O denotes a success to pass the test.
- X denotes a failure to pass the test.

---

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Dr. Li-Chih Wang has consulted for a number of high tech industries and served as Chief Advisor for developing DigiChains’s supply chain planning (SCP) and APS software/system which has been implemented in a number of high tech industries. He also actively promotes SCM concept and system in government, research institute, academy and industry for the last 15 years.

---

<sup>c</sup>Codification of references:

[1] Armstrong, S. *Forecasting by extrapolation: Conclusion from 25 years of research*, Interfaces 14, 6, 1984, pp. 52-66


<table>
<thead>
<tr>
<th>Reference</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>[31] Hendry, D. F., Econometric modeling, Lecture notes for the PhD course in econometric modelling and economic forecasting, Department of Economics, University of Oslo, 2000</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>Author(s)</td>
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<td>-----------</td>
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