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APPLICATION OF A FUZZY GOAL PROGRAMMING APPROACH WITH DIFFERENT IMPORTANCE AND PRIORITIES TO AGGREGATE PRODUCTION PLANNING

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Abstract: *This study presents an application of a fuzzy goal programming approach with different importance and priorities (FGPIP) developed by Chen and Tsai (2001) to aggregate production planning (APP), for the state-run enterprise of iron manufactures non-metallic and useful substances (Société des bentonites d'Algérie-BENTAL-). The proposed model attempts to minimize total production and work force costs, carrying inventory costs and rates of changes in work force. The proposed model is solved by using LINGO computer package and getting optimal production plan. The proposed model yields an efficient compromise solution and the overall levels of Decision Making (DM) satisfaction with the multiple fuzzy goal values.*

Key words: *Aggregate production planning; fuzzy goals programming; fuzzy linguistic; membership function*

1. Introduction

Aggregate production planning (APP) is concerned with matching supply and demand of forecasted and fluctuated customer's orders over the medium-time range, up approximately 3 to 18 months into future. APP determines the intermediate range capacity needed to respond to fluctuating demand. Given demand forecasts for each period of a finite planning horizon, the APP specifies production levels, work force, inventory levels, subcontracting rates, and other controllable variable for each period that satisfy anticipated demand requirements while minimizing relevant cost over that planning horizon. The fluctuations in demand can be absorbed by adopting one of the following strategies:

- The production rate can be altered by effecting changes in the work force through hiring or laying off workers.
- The production rate can also be altered by maintaining a constant labour force but introducing overtime or idle time.
- The production rate may be kept on a constant level and the fluctuations in demand met by altering the level of subcontracting.
- The production rate may be kept constant and changes in demand absorbed by changes in the inventory level.

Any combination of these strategies is possible. The concern of the APP is to select the strategy with least cost to the firm. This problem has been under an extensive discussion and several alternative methods for finding an optimal solution have been suggested in the literature.

Holt, Modigliani, and Simon (1955) proposed the HMMS rule, researchers have developed numerous models to help to solve the APP problem, each with their own pros and cons. According to Saad (1982), all traditional models of APP problems may be classified into six categories—(1) linear programming (LP) (Charnes & Cooper, 1961; Singhal & Adlakha, 1989), (2) linear decision rule (LDR) (Holt *et al.,* 1955), (3) transportation method (Bowman, 1956), (4) management coefficient approach (Bowman, 1963), (5) search decision rule (SDR) (Taubert, 1968), and (6) simulation (Jones, 1967). When using any of the APP models, the goals and model inputs (resources and demand) are generally assumed to be deterministic/crisp and only APP problems with the single objective of minimizing cost over the planning period can be solved. The best APP balances the cost of building and taking inventory with the cost of the adjusting activity levels to meet fluctuating demand.

In practice, the input data in the problem of APP and data of demand, resources and cost, as well as the objective function are frequently imprecise/fuzzy because some information is incomplete or unobtainable. Traditional mathematical programming techniques clearly cannot solve all fuzzy programming problems. In 1976, Zimmermann first introduced fuzzy set theory into conventional LP problems.

Since then, fuzzy linear programming (FLP) has been developed into several fuzzy optimization methods for solving APP problems. Additional references to the use of FLP to solve APP problems include Masud and Hwang (1980), Lee (1990), Tang , Wang and Fung. (2000), Wang and Fang (2001), Reay-ChenWang and Tien-Fu Liang (2005), Abouzar Jamalnia and Mohammad Ali Soukhakian (2008).

In practical production planning systems, many functional areas in an organization that send inputs to the aggregate plan are typically motivated by conflicting goals with respect to the use of the organization's resources. The decision maker (DM) must simultaneously optimize these conflicting goals in a framework of fuzzy aspiration levels. Zimmermann (1976) first extended his FLP approach to a conventional multi-objective linear programming (MOLP) problem. For each of the objective functions in this problem, the DM was assumed to have a fuzzy goal, such as "the objective function should be substantially less than or equal to some value." Subsequent works on fuzzy goal programming (FGP) included Leberling (1981), Hannan (1981), Luhandjula (1982), Sakawa (1988) and Chen and Tsai (2001).

This study presents an application A fuzzy GP with different priorities model in the national firm of iron manufactures non- metallic and useful substances for solving the problems of the APP. The proposed model minimizes total production and work force costs, cost of inventory and minimizes the degree of change in Work force.

2. Model formulation

2.1. Basic structure of fuzzy goal programming

Goal programming (GP) Models was originally introduced by Charnes and Cooper in early 1961 for a linear model. This approach allows the simultaneous solution of a system of Complex objectives. The solution of the problem requires the establishment among these multiple objectives.

The principal concept for linear GP is to the original multiple objectives into specific numeric goal for each objective. The objective function is then formulated and a solution is sought which minimizes the weighted sum of deviations from their respective goal.

GP problems can be categorized according to the importance of each objective considered Nonpreemptive GP is the case in which all the goals are of roughly comparable importance. Preemptive GP has a hierarchy of priority levels for the goals, in which goal of greater importance receive greater attention in general GP models consist of three components: an objective function , a set of goal constraints, and non-negativity requirements. However, the target value associated with each goal could be fuzzy in the real-world application

The fuzzy sets theory is recurrently used in recent research. A fuzzy set A can be characterized by a membership function, usually denoted μ , which assign to each object of a domain its grade of membership in A (Zadeh, 1965). The more an element or object can be said to belong to a fuzzy set A, the closer to 1 is its grade of membership. Various types of membership functions can be used to support the fuzzy analytical

Framework although the fuzzy description is hypothetical and membership values are subjective. Membership functions, such as linear, piecewise linear, exponential, and hyperbolic functions, were used in different analysis. In general, the non-increasing and non-decreasing linear membership functions are frequently applied for the inequalities with less than or equal to and greater than or equal to relationships, respectively. Since the solution procedure of the fuzzy mathematical programming is to satisfy the fuzzy objective, a decision in a fuzzy environment is thus defined as the intersection of those membership functions corresponding to fuzzy objectives (Zimmermann, 1978, 1985). Hence, the optimal decision could be any alternative in such a decision space that can maximize the minimum attainable aspiration levels in DM, represented by those corresponding membership functions (Zimmermann, 1985).

The integrated use GP and fuzzy sets theory has already been reported in the literature , Hannan, (1981), Leberling (1981), Luhandjula (1982), Rubin and Narasimhan (1984), Tiwari, Dharmar, and Rao (1987), Wang and Fu (1997), Chen and Tsai (2001), Yaghoobi and Tamiz (2007) further integrated several fuzzy linear and multiobjective programming techniques.

The approach chosen in this study for applied to the problem of APP is similar to the method developed by Chen and Tsai (2001)

2.2. Multi-objective linear programming (MOLP) model to APP

2.2.1. Parameters and constants definition

v_{it} : production cost for product i in period t excluding labor cost in period t (Unit).

c_{it} : inventory carrying cost for product i between period t and $t+1$.

r_t : regular time work force cost per employee hour in period t.

d_{it} : forecasted demand for product i in period t.(Units).

K_{it} : Quantity to produce one worker in regular time for product i in period t.

I_{oi} : initial inventory level for product i.(units)

T : horizon of planning.

N : total number of products

P_{it} : Quantity of i product to the period t.

I_{it} : inventory level for product i in period t (units)

H_t : worker hired in period t (man).

F_t : workers laid off in period t (man).

$I_{it.Min}$: minimum inventory level available for product i in period t (units).

W_t : total number of work force level in period t (man).

W_{Min} : The minimum work force level (man) available in period t.

W_{Max} : The maximum work force level (man) available in period t.

2.2.2. Objective functions

Masud and Hwang (1980) specified three objective functions to minimize total production costs, carrying and backordering costs, and rates of change in labor levels. In this study, we propose a model which will be using two strategies where they are available in the national firm of iron manufactures non- metallic and useful substances. In their multi-product APP decision model, the three objectives to the APP model can be formulated as follows:

- **Minimize total production costs** :

$$Min..Z_1 \cong \sum_{i=1}^{N}\sum_{t=1}^{T}(v_{it}P_{it}) + \sum_{t=1}^{T}(r_t W_t + h_t H_t + f_t F_t)$$

The production costs include: regular time production, overtime, carrying inventory, specifies the costs of change in Work force levels, including the costs of hiring and layoff workers.

- **Minimize carrying costs :**

$$Min..Z_2 \cong \sum_{t=1}^{T}(c_{it} I_{it}) \cdot$$

- **Minimize changes in labor levels:**

$$Min..Z_3 \cong \sum_{t=1}^{T}(H_t + F_t)$$

where the symbol $\cong$ is the fuzzified version of $=$ and refers to the fuzzification of the aspiration levels.

The objective functions of the APP model, in this study, assumes that the DM has such imprecise goals as, the objective functions should be essentially equal to some value. These conflicting goals are required to be simultaneously optimized by the DM in the framework of fuzzy aspiration levels.

2.2.3. Constraints

- **The inventory level constraints :**

$$P_{it} + I_{i,t-1} - I_{it} = d_{it}$$
$$I_{it} \geq I_{it.Min}$$

- **Constraints on labor levels:**

$$W_t - W_{t-1} - H_t + F_t = 0$$
$$W_{Min} \leq W_t \leq W_{Max}$$

- **Constraints on labor capacity in regular and overtime :**

$$P_{it} - K_{it} * W_t \leq 0$$

- **Non-negativity constraints on decision variables :**

$$P_{it}, I_{it}, W_t, H_t, F_t \geq 0$$

2.3. A fuzzy goal programming with different importance and priorities to APP (FGPIP-APP)

2.3.1. Membership function

Narasimhan (1980) and Hannan (1981-a),(1981-b) were the first to give a FGP formulation by using the concept of the membership functions. These functions are defined on the interval [0, 1]. So, the membership function for the *i-th* goal has a value of 1 when this goal is attained and the DM is totally satisfied; otherwise the membership function assumes a value between 0 and 1.

Linear membership functions are used in literature and practice more than other types of membership functions. For the above three types of fuzzy goals linear membership functions are defined and depicted as follows (Fig. 1):

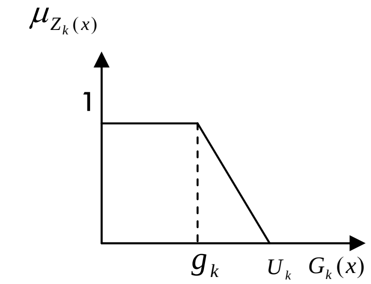
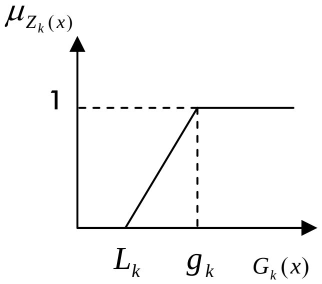
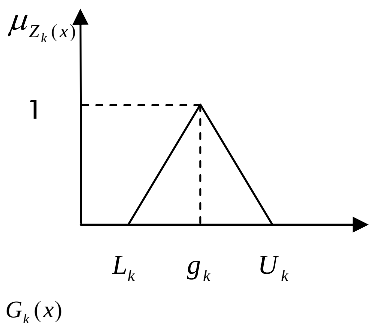
<table>
<tr><th>Membership function</th><th>Analytical definition</th></tr>
<tr><td>
</td><td>$$\mu_{Z_k(x)} = \begin{cases} 1.................if\,..G_k(x) \le g_k \\ \dfrac{u_k - G_k(x)}{u_k - g_k}..if\,..g_k \le G_k(x) \le u_k ...k = 1,...,m...(1) \\ 0...............if\,..G_k(x) \ge g_k \end{cases}$$</td></tr>
<tr><td>
</td><td>$$\mu_{Z_k(x)} = \begin{cases} 1.................if\,..G_k(x) \ge g_k \\ \dfrac{G_k(x) - L_k}{g_k - L_k}..if\,..L_k \le G_k(x) \le g_k ...k = m+1,...,n....(2) \\ 0...............if\,..G_k(x) \le L_k \end{cases}$$</td></tr>
<tr><td>
</td><td>$$\mu_{Z_k(x)} = \begin{cases} 0.................if\,..G_k(x) \le L_k \\ \dfrac{G_k(x) - L_k}{g_k - L_k}..if\,..L_k \le G_k(x) \le g_k ...k = n+1,...,l \\ \dfrac{u_k - G_k(x)}{u_k - g_k}..if\,...g_k \le G_k(x) \le u_k \\ 0...............if\,..G_k(x) \ge u_k \end{cases} \quad ....(3)$$</td></tr>
</table>

Figure 1. Linear membership function and Analytical definition

Where L_K (or u_k) is lower (upper) tolerance limit for $k.th$ fuzzy goal $G_k(x)$ They are either subjectively chosen by decision makers or tolerances in a technical process (Chen & Tsai, 2001; Yaghoobi & Tamiz, 2007).

2.3.2. FGPIP-APP formulation

We will use the method that was developed by Chen & Tsai,(2001) for formulated the APP problem in the fuzzy gaols , which allows decision makers to determine a desired achievement degree and importance (or weight) of each of the fuzzy goals, The complete FGPIP-APP model can be formulated as follows.

$$Max..f(u) = \sum_{k=1}^{l} \mu_k$$

Subject to :

$\mu_1 \le \mu_{z_1}$ (Minimize total production costs).

$\mu_2 \le \mu_{z_2}$ (Minimize carrying costs).

$\mu_3 \le \mu_{Z_3}$ (Minimize changes in labor levels).

$$X_{it} + I_{i,t-1} - I_{it} = d_{it}$$

$$I_{it} \ge I_{it.Min}$$

$$W_t - W_{t-1} - H_t + F_t = 0$$

$$W_{Min} \le W_t \le W_{Max}$$

$$P_{it} - K_{it} * W_t \le 0$$

$$\mu_1 \le \alpha_1$$

$$\mu_2 \le \alpha_2$$

$$\mu_3 \le \alpha_3$$

$$P_{it}, I_{it}, W_t, H_t, F_t \ge 0$$

Where $\alpha_1, \alpha_2, \alpha_3$ is the desirable achievement value for the i -th fuzzy goal.

2.3.3. Fuzzy linguistic for determing the degree of achievement

The determination of a desirable achievement degree for a goal could be a difficult task for a DM in a fuzzy environment when using method by Chen & Tsai,(2001) . For assessing desirable achievement degrees imprecisely, a useful method is to use linguistic terms such as ''*Low Important*", ''*Somewhat High Important*", and ''*Very High Important*" and so on to verbally describe the importance of each fuzzy goal. the associated membership function are then defined. We can define $\mu_I(\alpha)$ to represent the membership function of each linguistic values about the importance of different objectives, where $\mu_I(\alpha) \in [0,1]$, and α denotes the variable taking an achievement degree in the interval of $[\alpha_{\min}.\alpha_{\max}]$, $0 \le \alpha_{\min} \le \alpha_{\max} \le 1$ Then fuzzy numbers ranking methods can be used to map a membership function representing a fuzzy goal's importance to a real number in the range of [0,1]. The real number obtained can be considered as the desirable achievement degree for the fuzzy goal.

We define *I = {Very Low Important = VLI, Low Important = LI, Somewhat Low Important = SLI, Medium = M, Somewhat High Important = SHI, High Important = HI, Very High Important = VHI}* as a set of linguistic values about the importance of different goals (FIG.2). shows the $\mu_I(\alpha)$ for this linguistic values. Triangular fuzzy numbers corresponding to these linguistic values are: VLI = (0,0,10%), LI = (5%,15%,25%), SLI = (20%,32.5%, 45%), M = (40%, 50%,60%), SHI = (55%,67.5% ,80%), HI = (75%, 85%, 95%), VHI = (90%, 100%, 100%).

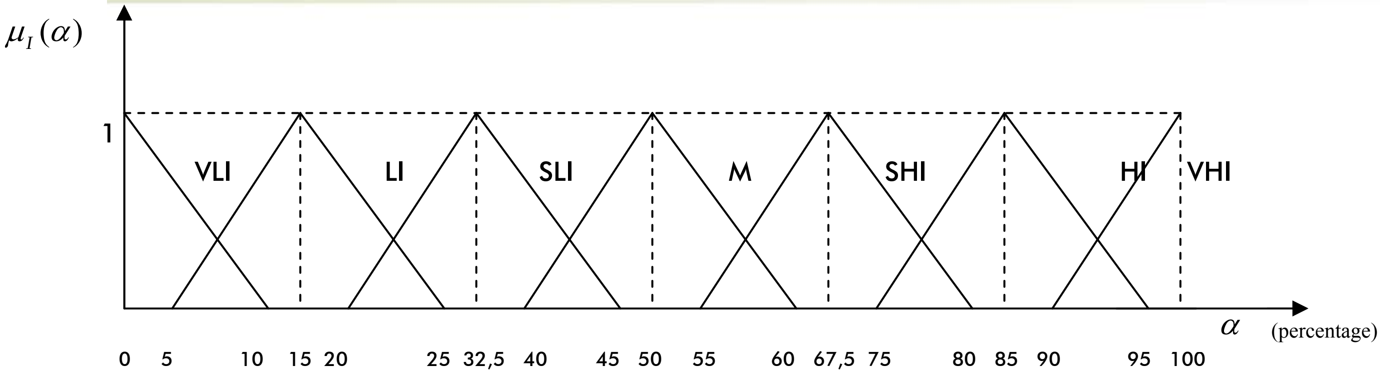


Figure 2. Membership functions for Linguistic values about the importance of different objectives

Note that subject to definition of fuzzy number, a and d corresponds, respectively, to $\alpha_{\min}$ and $\alpha_{\max}$. We use Liou and Wang (1992) approach for ranking fuzzy numbers to precisely determining the degree of achievement of different goals. As stated earlier, in $\mu_k \geq \alpha_k$ the α_k shows the degree of achievement of k th fuzzy goal. In Liou and Wang (1992) method, given $\alpha \in [0,1]$ total integral value of triangular fuzzy number $\widetilde{A} = (a,b,c)$ is:

$$
\begin{aligned}
I_T^{\alpha} &= \alpha . I_R(\widetilde{A}) + (1-\alpha) . I_L(\widetilde{A}) \\
&= \alpha \int_0^1 g_{\widetilde{A}}^{R}(y)dy + (1-\alpha)\int_0^1 g_{\widetilde{A}}^{L}(y) \\
&= \alpha \int_0^1 [c + (b-c)y]dy + (1-\alpha)\int_0^1 [a + (b-a)y]dy . \\
&= \frac{1}{2}[\alpha . c + b + (1-\alpha) . a]
\end{aligned}
$$

Where $g_{\widetilde{A}}^{R}$, $g_{\widetilde{A}}^{L}$ corresponding inverse functions the triangular membership function can be defined as :

$$
\mu .(x) = \begin{cases} 0 \ \ldots\ldots\ldots & if \ .. \ x \leq a \\ \dfrac{x-a}{b-a} .. & if \ .. \ a \leq x \leq b \\ \dfrac{x-c}{b-c} .. & if \ \ldots \ b \leq x \leq c \\ 0 \ \ldots\ldots\ldots & if \ .. \ x \geq c \end{cases}
$$

- when $\alpha = 0$, the total integral value $I_T^0(\widetilde{A})$ which represents a pessimistic decision maker's the totale integrale value becomes :

  $$I_T^0(\widetilde{A}) = \frac{1}{2}[b + a]$$

- when $\alpha = 0.5$, the total integral value $I_T^{0.5}(\widetilde{A})$ which represents a moderate decision maker's the totale integrale value becomes:

  $$I_T^{0.5}(\widetilde{A}) = \frac{1}{2}[0.5.c + b + 0.5.a]$$

- when $\alpha = 1$, the total integral value $I_T^1(\widetilde{A})$ which represents a optimistic decision maker's the totale integrale value becomes :

  $$I_T^1(\widetilde{A}) = \frac{1}{2}[c + b]$$

3. Model implementation

3.1. An industrial case study and data description

In this section, as a real-world industrial case a data set provided by the national firm of iron manufactures non- metallic and useful substances (BENTAL) in Algeria , This company manufactures three types of products which are important, and one of the raw materials used in many industries with : Bentonite (BEN) , Carbonate of calcium (CAL) , Discoloring (TD), The Firm operates 175 workers, and the system of work in the Firm is a continuous production (8×3 hours) for all days of the week except Thursday hailed the work is only a half-day and Friday, which is rest day, and production management composed in 68 worker divide in 3 groups.

The individual firm in the production of mineral products mentioned above, the demand for their products makes is large, which may cause problems in the productive capacity of this firm, fig.3 show fluctuations in demand on the level of monthly production capacity of any production capacity (CAP).

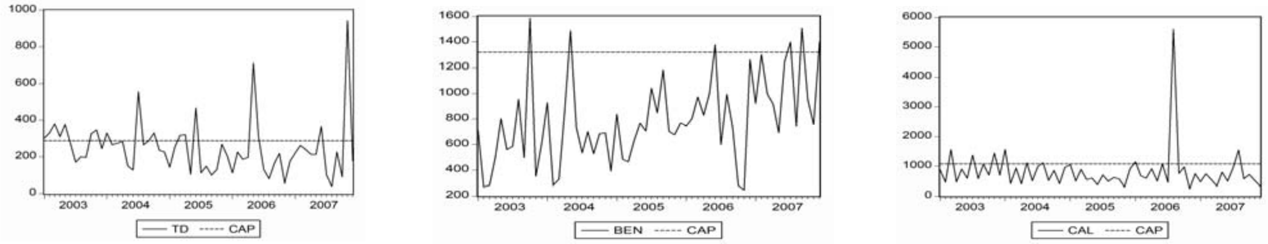


Figure 3. The fluctuation of the actual demand on the level of production capacity for TD, BEN, CAL

Therefore, fluctuations in demand on the level and volatility of productive capacity, calls for the Firm in an attempt to develop a plan of production, trying to cope with the impact that fluctuations in demand due to seasonal changes, Table 1 summarizes the basic data gathered from the firm , The proposed model implementation in the company has the following conditions:

1. There is a Six period planning horizon.
2. A three product situation is considered.
3. The initial inventory in period 1 is $I_{10} = 1857$ Tons of BEN, $I_{20} = 1029$ Tons of TD and $I_{30} = 1860$ Tons of CAL.
4. Minimum inventory must be maintained during the period t of product i is $500.Tons$
5. The costs associated with hiring and laying off, according to estimations of human resource management department per man are respectively 5178DA/man and 4155 DA/man.
6. The Linguistic values about the importance of objectives are : *Very High Important = VHI, High Important = HI, Medium = M.* respectively . and assumed that we have moderate decision maker , with $\alpha = 0.5$.

7. The cost of one worker in the production of three products during the t period is $r_t = 2694.706.DA/man$
8. The minimum work force level (man) available in each period is $W_{Min} = 55$ worker .
9. The maximum work force level available in each period is $W_{Max} = 68$ worker .
10. The initial worker level is ($W_0 = 68$).
11. the Maximum capacity of storage in 3 products in the firms is 6000 Tons.

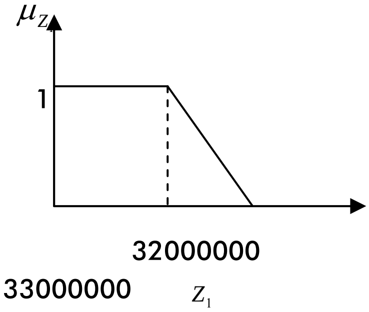
Table 1. The basic data provided by Bental firm (in units of Algerian Dinar DA ...1\$ $\cong 90$ DA)

Product	Period	d_{it}	v_{it}	c_{it}	K_{it}
BEN (P_{1t})	1	1177.225	3293.493	208.796	17.794
	2	923.021	3293.493	208.796	15.367
	3	883.342	3293.493	208.796	18.602
	4	1071.99	3293.493	208.796	16.985
	5	1379.269	3293.493	208.796	17.794
	6	1315.222	3293.493	208.796	17.794
TD (P_{2t})	1	128.620	21646.608	848.721	3.883
	2	163.777	21646.608	848.721	3.353
	3	164.617	21646.608	848.721	4.059
	4	166.005	21646.608	848.721	3.706
	5	193.317	21646.608	848.721	3.883
	6	206.662	21646.608	848.721	3.883
CAL (P_{3t})	1	1164.191	1296.109	139.149	14.558
	2	463.447	1296.109	139.149	12.573
	3	659.034	1296.109	139.149	15.220
	4	425.240	1296.109	139.149	13.897
	5	78.967	1296.109	139.149	14.558
	6	478.221	1296.109	139.149	14.558

3.2. Formulate and solving problem by FGPIP-APP

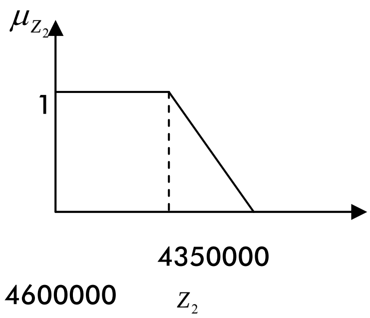
3.2.1. Construct the membership functions

The linear membership function of each objective function is determined by asking the DM to specify the interval $[g_k..u_k]$ of the objective values, and also to specify the equivalence of these objective values as a membership value in the interval [0, 1]. The linear and continuous membership function is found to be suitable for quantifying the fuzzy spiration levels. The corresponding linear membership functions can be defined in accordance with analytical definition of membership functions (Fig.1 Eq (1)). as follows.



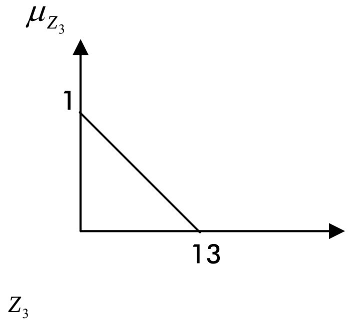
$$\mu_{Z_1}\begin{cases}1.......................................if...Z_1 \le 32000000\\ \dfrac{33000000 - Z_1}{33000000 - 320000000}......if...32000000 \le Z_1 \le 33000000.\\ 0.......................................if...Z_1 \ge 33000000.\end{cases}$$

Figure 4. Membership function of Z_1 (Minimize total production costs)



$$\mu_{Z_2}\begin{cases}1.....................................if...Z_2 \le 4350000\\ \dfrac{4600000-Z_2}{4350000-4600000}......if...4350000 \le Z_2 \le 4600000.\\ 0...................................if...Z_2 \ge 4600000.\end{cases}$$

Figure 5. Membership function of Z_2 (Minimize carrying costs)



$$\mu_{Z_3}\begin{cases}1.........................if...Z_3 \le 0\\ \dfrac{13-Z_2}{13}..............if...0 \le Z_3 \le 13.\\ 0.........................if...Z_3 \ge 13.\end{cases}$$

Figure 6. Membership function of Z_3 (Minimize changes in labor levels)

3.2.2. Transform FGPIP-APP problem to linear programming(LP)

Transform FGPIP-APP problem to equivalent LP with one objective that maximizes the summation of achievement degrees. The LP model for FGPP-APP problem is constructed as follows:

$$Max..f(u)=\sum_{k=1}^{3}\mu_k$$

Subject to :

$$\mu_1 \le (33000000 - Z_1)/1000000.$$

$$\mu_2 \le (4400000 - Z_2)/250000.$$

$$\mu_3 \le (13 - Z_3)/13.$$

$$P_{it} - K_{it} \times W_t \le 0$$

$$P_{it} + I_{i,t-1} - I_{it} = d_{it}$$

$$W_t - W_{t-1} - H_t + F_t = 0$$

$$W_{Min} \le W_t \le W_{Max}$$

$$\sum_{i=1}^{3} I_{it} \le 6000$$

$$I_{it} \ge 500$$

$I_{10} = 1856.25$

$I_{20} = 1029$

$I_{30} = 1860$

$W_0 = 68$

$\mu_1 \geq 0.725$

$\mu_2 \geq 0.850$

$\mu_3 \geq 0.50$

$P_{it}, I_{it}, W_t, H_t, F_t, \mu_1, \mu_2, \mu_3 \geq 0 \qquad i = 1.,2.,3 \qquad t = 1.,2.,......,6$

W_t, H_t, F_t (integers).

3.2.3. Solve the FGPIP-APP Problem

The LINGO computer software package was used to run the Linear programming model. Table 2 presents the optimal aggregate production plan in the industrial case study based on the current information:

Table 2. Optimal production plan in the BENTAL firm case with FGPIP-APP model

Period	Product	P_{it} (Tons)	I_{it} (Tons)	W_t (man)	H_t (man)	F_t (man)
0	1 (BEN)	-	1865.25	68	-	-
	2 (CAL)	-	1029			
	3 (TD)	-	1860			
1	1 (BEN)	0	679.025	68	0	0
	2 (CAL)	0	900.38			
	3 (TD)	0	695.809			
2	1 (BEN)	743.996	500	68	0	0
	2 (CAL)	0	736.603			
	3 (TD)	267.638	500			
3	1 (BEN)	1074.857	691.515	68	0	0
	2 (CAL)	0	571.986			
	3 (TD)	659.034	500			
4	1 (BEN)	1154.980	774.505	68	0	0
	2 (CAL)	94.019	500			
	3 (TD)	425.24	500			
5	1 (BEN)	1209.992	605.228	68	0	0
	2 (CAL)	193.317	500			
	3 (TD)	78.967	500			
6	1 (BEN)	1209.992	500	68	0	0
	2 (CAL)	206.662	500			
	3 (TD)	478.221	500			

Using FGPIP to simultaneously minimize total production costs (Z_1), carrying costs (Z_2), and changes in Work force levels (Z_3), yields total production cost of 32032504.2 DA, carrying cost of 4375292.99 DA, and changes in Work force levels of 0. and resulting achievement degrees for the three fuzzy goal (μ_1 , μ_2 and μ_3) are 0.9682679 , 0.8975380 and 1 respectively , all of which satisfy the requirements of decision makers.

Despite the good results that were obtained through the proposed model , but remains very much sensitive to the accuracy of the information and data provided by the Organization,

4. Conclusions:

The APP is concerned with the determination of production, the inventory and the workforce levels of a company on a finite time horizon. The objective is to reduce the total overall cost to fulfill a no constant demand assuming fixed sale and production capacity.

In this study we proposed an application of a fuzzy goal programming approach with different importance and priorities developed by Chen and Tsai (2001) to aggregate production planning, The proposed model attempts to minimize total production and work force costs, carrying inventory costs and rates of changes in Work force so that in the end, the proposed models is solved by using LINGO program and getting optimal production plan.

The major limitations of the proposed model concern the assumptions made in determining each of the decision parameters, with reference to production costs, forecasted demand, maximum work force levels,, and production resources. Hence, the proposed model must be modified to make it better suited to practical applications. Future researchers may also explore the fuzzy properties of decision variables, coefficients, and relevant decision parameters in APP decision problems.

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