

RELIABILITY SAMPLING PLANS: A REVIEW AND SOME NEW RESULTS

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Abstract: *In this work we present a large area of aspects related to the problem of sampling inspection in the case of reliability. First we discuss the actual status of this domain, mentioning the newest approaches (from a technical view point) such as HALT and HASS and the statistical perspective. After a brief description of the general procedure in sampling inspection, we offer what we did call here as „personalized procedures”: this means that we take into consideration the specific statistical law for time-to-failure.*

The original part refers to the construction of $(n, 0)$ sampling plans and the use of ISO standard 2859 (MILSTD 105 E, the American original) in order to derive sampling plans by linking the AQL indicator (a defective fraction, in fact) to the hazard rate function.

Some examples are given and necessary tables are provided also.

Key words: *reliability sampling plans; AQL; HALT; HASS; personalized procedures; $(n,0)$ – plans; power distribution; Rayleigh law*

1. Preliminaries: the actual status of the problem

Reliability and its main component – durability – are considered as **dynamical** quality characteristics in the sense that possible nonconformities of the underlying entities (products, components, systems) are put into light in the process in which these ones perform their functions. For instance, we may observe the exploitation of the object by a client or may organize specifical lab durability/reliability tests.

A truck company for instance, records the mileage of its „moving units“ up to their first failure and these records may constitute a database for the reliability analysis of that peculiar truck brand.

Laboratory tests are more specific since the experimenter must choose a certain strategy for them: to use complete or (multi) censored samples, a time-truncation procedure (all performed under normal or accelerated conditions) a.s.o.

The cost element must be involved too in such test designs.

A huge monograph of Blischke and Murthy (2000, [3]²) describes new research directions in this field and provides a detailed list of ISO, NATO and from other professional associations (e. g. SAE – Society of Automotive Engineers, U.S.A.) documents regarding reliability and related domains (see also an earlier work of Kochendarfer and Pabst, Jr. (1971) [18]).

It is important to mention that in the TQM (Total Quality Management) frame work or in the recent SIX SIGMA approach (see Gupta, 2004 [12]) reliability is often considered as a specific technical aspect since:

- (i) does not appear directly in the manufacturing process;
- (ii) is imbedded in the design stage of an entity in a provisional form;
- (iii) the acceptance (receiving inspection) of final products is usually performed on the so-called static characteristics such as length, volume, pressure, voltage, hardness a.s.o.;
- (iv) to test and to evaluate reliability requires an adequate logistics and a documented statistical know-how (that is a qualified personnel).

It is also necessary to remember that most of the complex products generated by nowadays industries are „mechatronic entities“ – that is their structure is a combination between mechanical, electrical and electronic parts each one with its own degree of automation and self-adjustment: this structural and functional complexity needs high engineering techniques for testing as well as an appropriate methodology for data analysis since the components of a mechatronic system have their specific behaviour in exploitation.

In this sense, the classical techniques – such as running in of various vehicles at the highest level of stress intensity allowed by specifications – are combined now with two new procedures known as HALT and HASS. Proposed by dr. Gregg Hobbs in 1995 at QUALMARK Corp. in Santa Clara (California) and registered as a **federal mark** these represent advanced methodologies for accelerated life-testing in order to determine as quick as possible the item reliability: this „quick evaluation“ is needed to evaluate the possibility of a rapid intervention in the design stage, the goal being functional improvement (see also Dowling, 1999, [7]).

HALT or **Highly Accelerated Life Test** means an experiment in which the objects are submitted to a highly accelerated test in order to estimate the durability of that kind of object that is its failure-free working period.

The method is used in the design stage in order to detect weak points of the intended item to be mass produced, such as those probable nonconformities be eliminated before launching: the corrective action consists here in **re-designing** the product.

HASS – or **Highly Accelerated Stress Screening** means an experiment in which an item is submitted under accelerated stress in the manufacturing stage in order to detect and eliminate the so-called „hidden defects“, thus, preventing them to be genetically transmitted by the process to future batches.

The elements (stresses) used in HALT and HASS procedures are rapid transitions of temperature and omni axial vibrations (with six degrees of freedom) – this combination of random vibrations and thermo-cycles being the most performing way to accelerate the failure mechanisms of any kind.

The idea to combine various types of stresses is not new: the engineers working in the „resistance des materiaux” (T. Albert, 1829 and J. V. Poncelet, 1839 in France, P. Hodge, 1850 in England or A. Z. Wöhlen and Z. Bauwesen, 1858, in Germany) have used this combination of random mechanical stresses with various temperatures in order to investigate the durability of metallic materials employed in railroad industry (details in Cioclov, 1975 [5] page 5 – 10).

Some authors believe that this study of material strength face to various random mechanical and thermo stresses can be considered as **the embryo of the reliability theory** (see Bârsan-Pipu et al, 1999, [2]).

A last remark in this context: accelerated tests are needed especially if we deal with very high reliable objects for which is almost impossible to wait their natural failure in order to evaluate numerically the underlying reliability. The exclusive usage of complete samples and normal conditions has proved to be in many cases time consuming, extremely expensive or even not very significant from a technical viewpoint (examples of such pitfalls are given in Meeker and Hamada, 1995, [20]).

As regards the receiving inspection of products for which the characteristics of interest is durability/reliability, we distinguish several approaches which are quite different since are based on distinct principles. These are:

a) Attributive approach: viewed as the oldest and simplest one, it considers the durability/reliability as a measurable characteristic which can be attributivisable (an **attribute**, as for instance conforming or nonconforming). In our framework, we submit to a reliability test a witness batch of size N_0 (non-repairable items, for example), during a fixed period of time (T_0); then, record the number (d) of failed elements over that period and compute the estimated defective fraction $\hat{p} = d/N_0$ of the lot. This value (\hat{p}) may be taken as the desired AQL needed by the ISO standard 2859 or its American variant MIL STD 105 E in order to employ for instance single sampling plans $(n, A | R = A + 1)$, where A and R are acceptance and rejection numbers (see Iliescu, 1982, [13]).

The major advantage of attributive method is its simplicity: one has to compare the number (d) of failed items in a sample of size (n) – this one given by the standard, with the acceptance number A . If $d \leq A$ the lot is accepted – otherwise, that is $d \geq R = A + 1$, the lot is rejected (this is the case of single sampling plans).

The main disadvantage in this case is the reductionism itself of the method: it does not take into account the specific law for **failure behaviour**, which is a key element in **reliability evaluation** (Cătuneanu – Mihalache, 1990, [4]).

As a consequence, the so-called personalized procedures have been devised and these acceptance schemes use effectively the statistical distribution of time-to-failure.

b) Average operating time approach: this procedure takes into consideration a specific time-to-failure law and establishes acceptable and unacceptable values for mean operating time. Using a sample of size (n) and an acceptable number A (which may be a minimum member of hours of failure-free operation, considered acceptable), the lot will be accepted or rejected in comparison with this number.

c) Hazard rate approach: the procedure is similar to the above one, the fixed values being of hazard (failure) rate.

d) Sequential approach: we have in mind Abraham WALD'S procedure which can be applied in both cases – attributive and variable ones (see Wald, 1973, [23]).

In this paper we shall propose another procedure, namely linking the fraction defective (p) with the hazard rate, obviously taking into account the specific form of time-to-failure distribution.

2. The general procedure

Attribute sampling inspection may be performed regardless the very nature of quality characteristic tested: static or dynamical one. There are considered two proportions P_1 and P_2 of product units, first acceptable with $1 - \alpha = 0,95$ probability (that is $\alpha = 0,05$) and the second one acceptable with a lot smaller probability, usually $\beta = 0,10$. We may have also $\alpha = \beta$. The problem consists in determining the sampling plan (n, A) where n is the sample size and A is the acceptance number. To find n and A we proceed as follows:

(i) from the relationship (see Baron et al, 1988, [1]).

$$\frac{P_2}{P_1} = \frac{\chi_{1-\beta; m}^2}{\chi_{\alpha; m}^2} \quad (1)$$

where $\chi_{\varepsilon; m}^2$ is the ε - quantile of the chi-square distribution with m degrees of freedom (see [13]), we approximate „the best m “ for which (1) is fulfilled.

(ii) the acceptance number A is given by

$$A = \left[\frac{m}{2} - 1 \right], \text{ where } [] \text{ is the integer part} \quad (2)$$

(iii) the sample size (n) is furnished as

$$n = \frac{\chi_{\alpha; 2(A+1)}^2}{2 P_1} \text{ (rounded to the nearest integer)} \quad (3)$$

Example: if we take $P_1 = 0,25\%$ and $P_2 = 1,75\%$ and $\alpha = \beta = 0,05$, based on chi-square tables, we get

A	1	2	3	4	5	6	7	8
P_2/P_1	13	7,5	5,7	4,6	4,0	3,6	3,3	3,1

Since $P_2/P_1 = 1,75/0,25 = 7$, hence the nearest value is 7,5 for which we extract $A = 2$. If $m = 2(A + 1) = 6$, we shall obtain

$$n = \frac{\chi_{0.05; 6}^2}{2 \times 0.0025} = \frac{1.635}{0.005} = 327 \text{ (units)} \quad (4)$$

Therefore, the sampling plan is ($n = 327, A = 2$) which seems to be not very economical (since n is quite large) for an expensive testing.

3. Personalized procedures

We shall consider now the time-to-failure model, namely the mathematical object $\{T|f(t;\theta), t \in [0,+\infty), \theta = (\theta_1, \theta_2, \dots, \theta_n), \theta_j \in \mathbb{R}, j = \overline{1, k}\}$ (5)

we have $f(t;\theta \geq 0, \forall t \geq 0), \int_0^{\infty} f(t;\theta)dt = 1.$

Here, T is the continuous random variable which represents the time-to-failure behaviour. The simplest model is considered:

3.1. The exponential model

This is described by the density

$$T : f(t;\theta) = (1/\theta)\exp(-t/\theta), \quad t \geq 0, \quad \theta > 0 \quad (6)$$

where θ has the significance of average durability, since $E(T) = \theta$ and its inverse is just the hazard rate $h(t;\theta) = f(t;\theta)/R(t;\theta)$ where $R(t;\theta) = 1 - F(t;\theta) = \exp(-t/\theta)$ is the reliability function.

In this case we shall fix two values for $E(T)$, namely θ_1 (acceptable mean-life with $1 - \alpha$ associated probability, $0 < \alpha < 1$) and θ_2 (undesirable mean-life with β associated probability $0 < \beta < 1$) where usually $\alpha = 0,05$ and $\beta = 0,10$.

If we will employ an „r out of n” durability test, that is we will obtain a censored sample

$$t_{i_1} \leq t_{i_2} \leq \dots \leq t_{i_r} \quad r < n \quad (7)$$

the average life estimate is

$$\hat{\theta}_m = \frac{1}{r} \left[\sum_{i=1}^r t_{i_n} + (n-r)t_m \right] \quad (8)$$

(see [1], vol. I, page 532).

The acceptance numbers are as below

$$A_1 = \frac{\theta_1 \cdot \chi_{\alpha; 2r}^2}{2r}, \text{ if we adopt producer's risk variant or}$$

$$A_2 = \frac{\theta_2 \cdot \chi_{1-\beta; 2r}^2}{2r}, \text{ if we prefer customer's risk variant.}$$

The decision to accept the lot is taken if $\hat{\theta}_m \geq A_1$ or $\hat{\theta}_m \geq A_2$.

Example: Let $\theta_1 = 1000$ (hours), $\alpha = 0,05$ and $r = 4$ when the sample size in $n = 26$ units (we work using producer's variant). From chi-square tables ([1]) we find $\chi_{0.05; 8}^2 = 2.733$ and hence $A_1 = 1000 \times 2.733 / 8 \approx 342$.

Suppose that the average mean-life was $\hat{\theta}_m = 960$ (hours). Since $\hat{\theta}_m > A_1$, the underlying lot is accepted with confidence level of 95%.

3.2. The Weibull model

Proposed in 1951 (see [24]) by a Swedish military engineer, Wallodi WEIBULL (1887 – 1979), this model is considered as a generalization of exponential and Rayleigh laws (see Isaac-Maniu, 1983, [14]). Its density is

$$T : f(t; \theta, k) = (k/\theta)t^{k-1} \exp(-t^k/\theta), \quad t \geq 0, \quad k > 0 \tag{9}$$

and has the following average value

$$E(T) = \theta^{1/k} \cdot \Gamma(1 + 1/k) \text{ where from } \theta = \left[\frac{E(T)}{\Gamma(1 + 1/k)} \right]^{1/k} \tag{10}$$

Here $\Gamma(u) = \int_0^\infty e^{-t} \cdot t^{u-1} dt$ is the well-known Gamma function (see Dorin et al, 1994 [6], page 239 – 244).

If $p = p_0$ is the proportion of nonconforming objects, then

$$p_0 = 1 - \exp\left\{-\left[\frac{t \cdot \Gamma(1 + 1/k)}{E(T)}\right]^k\right\} \tag{11}$$

where θ has been replaced by its expression from (10).

Hence, we deduce

$$\frac{t}{E(T)} = \frac{[-\ln(1 - p_0)]^{1/k}}{\Gamma(1 + 1/k)} \tag{12}$$

and taking $t = T_0$ as testing time (expressed usually in hours) we can compute $T_0/E(T)$ if k and p_0 are known.

As it has been shown in [22, page 112] this approach leads often to large sample sizes, which is not always convenient.

This handicap could be eliminated by constructing $(n, 0)$ type sampling plans, that is plans where the acceptance number is zero.

3.3. The Gamma model

A random variable T has a Gamma density function if

$$T : f(t; \theta, k) = \frac{1}{\theta \Gamma(k)} \cdot \left(\frac{t}{\theta}\right)^{k-1} \exp(-t^k/\theta), \quad t \geq 0, \quad \theta, k > 0 \tag{13}$$

It is also a generalization of the exponential one (for $k = 1$), we get $f(t; \theta, 1) = (1/\theta) \exp(-t/\theta), \quad t \geq 0, \quad \theta > 0$.

The average life is in this case $E(T) = k\theta$. If the shape parameter k is known, then if we fix an acceptable mean-value $k\theta_0$ and a testing time T_0 , such that with a given probability P , the average durability be at least $k\theta_0$, using the ratio $T_0/k\theta_0$ and P , one may deduce the needed sample size to perform the test (details in Gupta and Groll, 1961 [11] or Vodă, 1981 [22], page 116 – 119).

Example: Suppose that we give $A = 0$ (the acceptance number) $P = 0.95$, $k = 2$ and the average-life $k\theta_0 = 10.000$ (hours) – that is $\theta_0 = 5.000$. If T_0 (testing time) is 1.000 hours, then $T_0/k\theta_0 = 0.10$. From the below table (reproduced from [22] page 119) we detect $n = 170$ units. Therefore

- a) a sample of $n = 170$ item are submitted to a test over the period of $T_0 = 1.000$ hours;
- b) if there are no failures during this testing period, the lot is accepted;
- c) if there exists at least one failure, the lot is rejected, since the acceptance number is zero.

$T_0/k\theta_0$	1.0	0.05	0.10	0.05
4				
0	4	10	170	639
1	6	16	269	1013
2	9	22	358	1444
3	11	27	411	1655

4. (n, 0) – type sampling plans

These plans, no matter which are the other input elements are invariantly based on the zero acceptance criterion, that is always $A = 0$.

A way to construct such plans is to fix in advance the following elements: testing time T_0 , lower bound for reliability $R(t)$ evaluated for $t = T_0$ or a lower bound for the average lifetime $E(T)$ and the consumer risk β .

This procedure has been described in a series of research reports of SVÚSS (Státný Výzkumný Ústav pro Stavbu Strojů/Běchovice, ČSSR – State Research Institute for Machine Construction/ Běchovice, former Czechoslovakya – see Drimlová (1970 [8], 1973 [10]) and Drimlová and Žaludová (1971) [9]) and it was applied for the exponential law. In this case a lower bound for the reliability $R(t; \theta) = e^{-t/\theta}$, $t \geq 0$, $\theta > 0$ may be easily obtained from a limit fixed for the mean durability, since $E(T) = \theta$.

The sample size is given as

$$n = \frac{-\ln \beta}{\exp(-T_0/\theta_0)} \tag{14}$$

where θ_0 is the acceptable value for $E(T)$.

Example: Take $T_0 = 100$ (hours) and $R_{\text{lower}}(100) = 0.90$ and $\beta = 0.10$ (or 10%). Since $-\ln \beta \approx 2.302$, we get easily $n \approx 2.3/0.10 = 23$ units. Hence, the sampling plan is ($n = 23, A = 0$).

For the Weibull law, taking account the results from § 3.2. and § 3.3., we get

$$n = \frac{-\ln \beta}{\left[\frac{T}{E(T_{\text{lower}})} \cdot \Gamma(1+1/k) \right]^k} \tag{15}$$

where $E(T)_{\text{lower}}$ is the mean lifetime which we wish to accept with β probability. If the sample exhibits a smaller value for $E(T)$ then $E(T)_{\text{lower}}$, the lot is rejected with $(1 - \beta)$ probability (in this model, the shape parameter k is assumed to be known).

Example: Let us consider a Rayleigh distribution (which is a Weibull one for $k = 2$ – see for details the present authors 1998 [16]). We wish to reject batches with a mean

durability less than 2000 (hours) with a 90% probability (that is $\beta = 0.10$). The testing time was fixed as $T_0 = 500$ (hours) and obviously, $A = 0$.

In the next table, we present some values of (n) for various β and $T_0/E(T)$, using formula (15).

β	$T_0/E(T)$			
	0.1	0.2	0.4	0.8
0.05	382	95	24	6
0.10	293	73	18	5

In our case, $T_0/E(T) = 500/2000 = 0.4$ and we have for $\beta = 0.10$ the sample size $n = 18$. Therefore the sampling plan is $(n = 18, A = 0)$.

For the power distribution, namely

$$T : F(t; \delta, b) = \left(\frac{t}{b}\right)^\delta, \quad 0 \leq t \leq b, \quad \delta > 0 \tag{16}$$

which is a generalization of the uniform one (which is obtained for $\delta = 1$) or a peculiar case of Sedrakian's one (see Sedrakian, 1968 [21] or the present authors, 1995 [15]):

$$T : F(t; b, c, \delta, k) = 1 - \left[1 - \left(\frac{t-c}{b-c}\right)^\delta\right]^k \tag{17}$$

where $0 \leq c \leq t \leq b$, $\delta, k > 0$ (the power form is recovered if $c = 0$ and $k = 1$), we have immediately for (16):

$$E(T) = \frac{\delta b}{\delta + 1} \quad \text{and} \quad \text{Var}(T) = \frac{\delta b^2}{(\delta + 1)^2 (\delta + 2)} \tag{18}$$

Hence, if δ is known, if we fix a lower acceptable bound for $E(T)$, we get

$$b_{\text{lower}} = \frac{1 + \delta}{\delta} \cdot E(T)_{\text{lower}} \tag{19}$$

and taking into account the results from [22 page 114], we finally obtain

$$n = \frac{\ln \beta}{\ln \left[1 - \left(\frac{\delta \Delta}{\delta + 1}\right)^\delta\right]} \tag{20}$$

where $\Delta = T_0/E(T)_{\text{lower}}$

In the below table we offer some values for n given some Δ and $\beta = 0.01; 0.05; 0.10$ and $\delta = 3/2; 2$.

α	β	$\Delta = T_0/E(T)_{\text{lower}}$			
		0.2	0.4	0.6	0.8
3/2	0.01	54	18	9	5
	0.05	70	23	12	7
	0.10	108	36	18	11
2	0.01	128	31	13	6
	0.05	167	40	17	8
	0.10	256	62	26	13

Example: Assume that $\delta = 2$ and we wish to reject batches with a mean lifetime less than 3000 (hours) with a 99% probability (hence $\beta = 0.01$). The testing time T_0 is 600 (hours).

We have $\Delta = 600/3000 = 0.2$ and for $\beta = 0.01$ and $\delta = 2$ we read in the table $n = 128$. Therefore the plan is ($n = 128, A = 0$).

5. The use of MILSTD 105 E

As we said in § 1, the attributive sampling in practice makes use of the well-known document MILSTD 105 E or ISO variant ISOSTD 2859 (see Iliescu, 1982, [13, page 162 – 180]).

This standard does not refer to purely reliability elements such as testing time, failure/hazard rate, MTBF (Mean Time Between Failures), EOT (Effective Operating Time) a.s.o. (for specific English acronyms used in reliability theory, see Kovalenko, 1975 [19, page 437 – 466]). The items are simply divided into two classes: conforming and nonconforming/defective ones – no matter what indicator is considered.

In this paragraph we shall construct sampling plans by linking the lot defective fraction ($p = AQL$) with the specific hazard rate of those objects, using some input elements of MILSTD 105 E such as lot size (N), code-letter (CL) which will lead to the sample size (n).

Our approach assumes that the failure model is of the type

$$f(t; \theta) = \varphi'(t; \theta) \exp[-\varphi(t; \theta)] \quad (21)$$

where $\varphi(t; \theta) \geq 0$ for every $t \geq 0$ and $\theta > 0$ and

$$\int_0^{\infty} f(t; \theta) dt = 1 \quad (22)$$

The form (21) provides the distribution function

$$F(t; \theta) = 1 - \exp[-\varphi(t; \theta)] \quad (23)$$

the defective fraction p being just $p = 1 - e^{-\varphi(t; \theta)}$.

Taking logarithms, we have

$$-\ln(1 - p) = \varphi(t; \theta) \quad (24)$$

and since the hazard rate is $h(t; \theta) = f(t; \theta)/R(t; \theta)$

where $R(t; \theta) = 1 - F(t; \theta) = \exp[-\varphi(t; \theta)]$, we obtain $h(t; \theta) = \varphi'(t; \theta)$ and therefore (24) becomes

$$-\varphi'(t; \theta) \ln(1 - p) = h(t; \theta) \varphi(t; \theta) \quad (25)$$

If we take now $-\varphi(t; \theta) = \theta t^2$, then $h(t; \theta) = 2\theta t$, $t \geq 0$ $\theta > 0$ which is just the classical Rayleigh hazard rate (see Bârsan-Pipu et al, 1999 [2, page 91]).

In this case (25) has the form

$$-2 \ln(1 - p) = t \cdot h(t; \theta)$$

Consider now the simplest situation when we know N (the lot size), T_0 (the testing time – assumed to be the life span of the items) and the acceptable hazard rate $h(t; \theta)$ for $t = T_0$, expressed in failures per hour.

In order to ease the computations, we present some values of $100t \cdot h(t;\theta)$ linked with some **preferential AQL values** listed in MILSTD 105 E (table II A) – see also Kirkpatrick, 1970 [17, page 363]).

Table (*)

AQL (%)	0.15	0.25	0.40	0.65	1.0	1.5	2.5	4.0
$100t \cdot h(t;\theta)$	0.30	0.50	0.80	1.30	2.01	3.02	5.6	8.16

In these instances, the sampling procedure is the following:

1) Knowing N (lot size – let’s say N = 930 units) and using the general inspection level II (suggested in most of the cases by the standard – see Table I „Sample size and code letters“, [17, page 362]), we find the code letter CL = J; we shall denote **IL** as inspection level.

2) from Table II A „Single sampling plans for normal inspection“ (already mentioned) we have to draw from the lot a sample of size n = 80 units which will be submitted to the test.

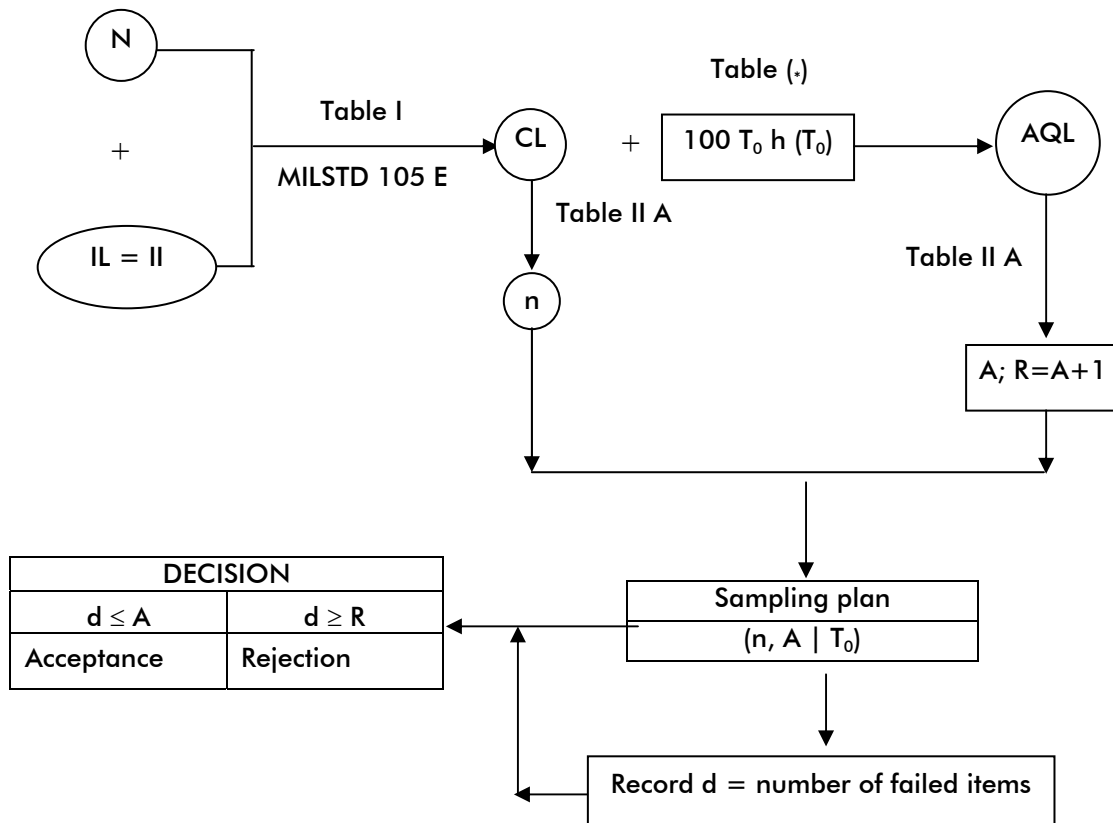
3) assuming that the testing time is $T_0 = 1000$ (hours) and the acceptable hazard rate for T_0 is $h(1000) = 0.000003$ failures/hour, we deduce

$$100 T_0 h(T_0) = 100 \times 1000 \times 0.000003 = 3.0$$

and from the table (*) we see that the nearest value is 3.02 wich indicates AQL = 1.5%.

4) with CL = J and AQL = 1.5%, from the same table II A we read the acceptance number A = 3 (consequently, the rejection one is R = A + 1 = 4).

Therefore, during the testing period $T_0 = 1000$ (hours) we shall observe the number (d) of failed elements from the sample (n = 80 units). If $d \leq A = 3$, the lot is accepted – otherwise (that is $d \geq R = 4$) the lot is rejected. See the below scheme:



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¹ With deep regrets we announce that our colleague Viorel Gh. VODĂ passed away in the last part of May 2009.

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