SYSTEMIC RISK OF NON PERFORMING LOANS MARKET. THE ITALIAN CASE[1]

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Abstract

The paper considers the systemic risk due to the Non-Performing Loans in the balance sheets of banks. Using empirical data on Non-Performing Loans of Italian Banks and following the proposal of securitization of problematic loans, we propose the use of a bipartite network to simulate a hypothetical market of asset classes and investors. A default cascade dynamic runs when asset classes are hit by multiple shocks and propagation increases losses faced by investors through both direct and indirect exposure. Our results show that the degree of differentiation of the market, with a parameter that controls the sensitivity to losses of investors, is crucial to determine the systemic risk of this kind of market.

Key words: Overlapping portfolios; Systemic Risk; Non-Performing-Loans

1. Introduction

The 2007-09 financial crisis, originated in the US credit system, overwhelmed intermediaries and markets all over the world leading to a sharp drop in both financial and economic activity. One of the main outcomes of the crisis was the propagation of shocks across countries and sectors, so as to depict it as the worst financial meltdown since the worldwide economic depression that took place during the 1930s. Subsequent to the onset of the financial crisis, the turmoil developed into liquidity and solvency crises, involving real economy. Narrowing the focus on the commercial banking system, a significant number of European banks were burdened with a high share of Non-Performing Loans (NPLs) to total gross loans: loans to public administrations, financial and non-financial firms, families for consumption, mortgages, and other types of loans suffered the economic stagnation and became problematic, revealing their financial weight on bank's balance sheets.

The buildup of this problematic amount of NPLs on the balance sheets of banks is a common feature of many countries: NPLs stood at about 1 trillion in the European Union (over 9 percent of EU GDP) at the end of 2014, more than double the 2009 level (Aiyar *et al.*, 2015). A recent keynote speech by Constancio (Constancio, 2017) asserts that "NPLs are a problem with a clear European dimension, as even those countries where banks do not struggle with asset quality, are likely to be affected by spillovers, both financial and real". Nevertheless, NPL levels rest a major concern in the southern part of the euro area, as well as in several Eastern and Southeastern European countries. This outcome was the result of several related issues: although the widespread increase of credit risk undertaken, it is evident that in some countries like Italy (and other peripheral economies) there were other causes that affected the market. Many structural causes contributed to the protracted recession phase: for instance inadequate incentives for financial institutions to write off or to sell problematic loans, low provisioning that create pricing gaps between book valuations and market values of loans and wide bid-ask spreads, the reliance on collaterals -which compress the incentives to sell-, and tax disincentives to provisioning and write-offs. From the demand side, it is worth to signal the lengthy and inefficient judicial process (more than seven years to complete a bankruptcy procedure and three years to foreclose on real estate collateral) and the lack of equity capital. All the causes have negatively influenced the demand for bad loans and the subsequent profitability and efficiency of the market (Kang & Jassaud, 2015). In this framework, the impact of negative interest rates, due to monetary policies aimed at reducing other problems, could somehow worsen the situation. This combination of causes produced a strong obstacle towards the efficient disposal of NPLs compared with the rest of developed countries. In a first stage of the crisis, Italian financial institutions were not unscathed by the turmoil, to the point that no public funds had to be injected to sustain financial institutions. However, after some years the consequences of the crisis became evident. In detail, losses due to NPLs impact on banks' profits, deteriorate capital structure determining lack of capital and drives up the regulatory capital charge, limiting the credit provision to the firms: the effect on firms' performances and on whole economy is therefore negative.

The main countermeasures proposed by the Italian State had the specific target to help the banks to get rid of bad loans, so to break the credit crunch and to sustain the economic growth. One of these proposals is the constitution of a distressed securities closed end fund, designed to invests or co-invests in securitization structures that can quicken the disposal of NPLs, favoring the creation of an efficient market for bad loans. The NPLs-market framework is driven by a securitization process where a SPV (special purpose vehicle) creates Asset-Backed-Securities (ABS) sold in the market to finance the purchasing of NPLs from the banks. The intrinsic and implicit interconnections among agents in the market, due to the securitization process that overlaps asset portfolios to build ABS, are manifestly clear. Market operators discount the whole creditworthiness evaluation process, so a shock on borrowers underlying the securities causes an increasing detected riskiness of the securities itself. Funds operating in this market do not know the specific firms and people underlying these credits and their creditworthiness. Investors trust in securitization because pricing the worst NPL gives the lower bound of the other securities price in the market.

The aim of the paper is to highlight the possibility of growth in the systemic risk of a hypothetical Italian NPLs market due to the mechanisms planned to build such market, as a leading factor for the optimal market configuration and pricing process of securities. Our proposal is a model, based on a bipartite network model of asset classes and funds, in which we build the indirect interconnections between asset classes via correlation matrices. We statistically construct direct interconnections between asset classes and funds through the Bipartite Configuration Model (BiCM) (Squartini, 2016), assuming that if there is an increasing of NPLs for a specific sector or area the value of ABS on NPLs decrease.

Bipartite networks have been mostly used in financial literature for the representation of networks of banks and assets and the consequent dynamics of overlapping portfolios. Works like Caccioli *et al.* (2014) have developed asymptotic models to study the riskiness of a distress spreading dynamics on a bipartite network, reaching the conclusion that such networks suffer the so-called "robust yet fragile" behavior. Connectivity and leverage are the main features that drive the system from a stable to an unstable region. Other works such Huang *et al.* (2013) instead, focus on empirical Bank-Assets networks, trying to reproduce the bankruptcy dynamics of recent financial crises (2007-2008) in order to develop a new systemic risk-detecting framework. Others like Miranda (2013) apply systemic risk models to identify the riskiness of different bipartite empirical networks like Brazilian Firm-Bank networks.

These researches highlight a crucial role of banks in triggering distress through financial networks. Our hypothesis is, otherwise, different. In fact, banks do not belong to our model, as the transfer of problematic loans to the funds removes them from any active role in the model and its simulations[2]. The ultimate aim is to identify the way to detect systemic risk in such bipartite empirical stylized NPLs market of Funds and asset classes. The research has been inspired by the proposal of Quaestio Capital Management SGR S.p.A., a financial advisory firm appointed to create and manage the Atlante Fund with the aim to create an NPLs Italian market.

2. Data and Model assumptions

This section describes the data and the models used for the analysis of the propagation of stress due to the non-performing loans (NPLs) of a sample of 25 Italian banks[3] that joined the closed fund Atlante[4] in April 2015.

Our analysis is based on an extensive dataset at the bank-firm level. Data on gross and net "bad loans" for individual Banks were manually collected from the notes in the Annual Report of each institution for the period 2008-2015[5]. The list of 19 Investors in the securitized NPLs has been retrieved gathering information from C&W Loan Sales 2014-2016, Debtwire, CNBC, Apax, Deloitte NPL Outlook 2014-2015, Italy24, KPMG Loan Sales Feb 2016, Quaestio.

The gross amount of the NPLs of such banks sums up to roughly 164.5 bn euros: this amount corresponds to a net value of 68 bn euros[6].

A high amount of NPLs ties up institutional capital of banks and prevents its use in the economic banking process (first and foremost lending), with negative effects on banks funding costs, profitability and credit supply. Disposal of NPLs in Italy is too slow compared with the rest of Europe due to a series of structural factors that determine an abnormal bid/ask spread:

1. Data quality, due to the lack of an organized and complete set of data in the banks databases.

2. Servicing, due to market fragmentation, lack of critical mass, critical issues affecting recovery procedures.
3. Time to recovery, a huge uncertainty in the time of recovery due to a wide divergence among the various procedures and courts.
4. Prospective trends of the Italian economy.

Banks should therefore accept a hard discount in the selling process of NPLs to specialized investors. Our hypothesis is that banks accept to liquidate the totality of net NPLs at a discount of 27.5 percent: the value of net NPLs introduced on the market would result in 49.3 bn euros, amount that would be securitized in three tranches: senior, mezzanine, and junior. This last tranche is supposed equal to 13.6 bn euros, the 27.5 percent of the selling value of NPLs. the loss suffered by the banks is ≈ 18.3 bn euros.

We can now start our process, building a bipartite model of market expositions of funds on the junior tranches of ABS.

2.1. Main data and variable description

- a vector $a \in R^{85}$ of 85 NPLs asset classes identified listing the 2015 exposure on NPLs of the 25 banks for each of the 6 customer economic activity: General Government, Other public entities, Financial companies, Insurance companies, Not Financial companies, Other entities[7].
- the NPLs sample described above was extended to the years 2008-2015. The resulting time series was used to calculate a correlation matrix $C^6 \in R^{6\times6}$ between sectors of origins (see Appendix 3 for details) and also,
- a block-diagonal correlation matrix $C^{85} \in R^{85\times85}$ between assets belonging to the same sector. We agree on the fact that the time series is short, but there were no data available before 2008.
- to the information of each element in vector a we add the membership of each asset to a regional area (North East (1), North West (2), Center (3), South & Islands (4)), that depends on the bank originator. Such data are reported in a vector $r \in R^{85}$. Each component has a value in the set {1, 2, 3, 4}.
- based on Quaestio (2016), we assume to deal with a set of 19 Investors. Their exposure to the NPLs is not uniform. Such peculiar number was deduced from the information reported in Quaestio (2016). Table 7 in the Appendix 3 shows the list of funds and exposures, that is the monetary amount funded by each of the external investors. The vector $f^1 \in R^{19}$ contains the numbers in the Table, that were calculated dividing the total exposure 13.6bn by the percentage of the exposure of the funds as reported in Quaestio (2016).
- through f^1, we build the adiacency exposition matrix $E \in R^{19\times85}$. Each element E_{ij} is the amount of money that each investor i invests in any asset class j. To do this we use the Bipartite Configuration Model (BiCM) (Saracco *et al.*, 2015); Squartini (2016)). The Appendix 1-2 contain details on the BiCM.
- Network building. Our NPLs market can be represented as a directed weighted bipartite network, in which links correspond to exposures of Investors on asset classes. With this method we can construct the network of exposures for each $z \in (0, \infty]$, that is the density parameter of links. The BiCM belongs to a series of entropy-based models for bipartite networks. In a nutshell, the method estimates Investors' individual exposures as:

$$E_{ij} = \frac{z^{-1} + f_i^1 a_j}{C} a_{ij}, \quad a_{ij} = \begin{cases} 1 & with\ probability\ p_{ij} = (f_i^1\ a_j)/(z^{-1} + f_i^1 a_j) \\ 0 & otherwise \end{cases} \tag{1}$$

where f_i^1 is the amount that the Investor *i* invests on market (the element *i* of vector ***f***), a_j is the value of asset class *j*, $C = \sum_i f_i = \sum_j a_j$ e size of the market and E_{ij} controls the presence of the link between Investor *i* and asset class *j* relted to p_{ij} that is the probability that there is a link between node *i* and *j*.

Based on the above data, we outline the ideas underlying the dynamics.

Model dynamics

The idea underling the dynamics is that the decrease of price of an asset *j* influences the system in three different ways:

a. direct exposure. Any investor *i* that is exposed on it loses a percentage of its value. Since the exposures are gathered in the matrix *E*, let us name the decrease $\Delta E\ E_{ij}$. The effect of this decrease of value does not remain confined to the funds that are directly exposed on the asset, but propagates through two other different channels;

b. indirect exposure, through the economic category. Asset classes *l* that belong to the same economic categories and that are correlated with a_j gets a decrease proportional to the correlation matrix C^{85} $_{l,j}$ and to the value of the exposition E_{lj}. As a consequence, the values of funds exposed to each asset class *j* decreases by $\Delta E\ E_{ij}$;

c. indirect exposure, through the asset classes. Recalling that vector *a* reports the membership of each asset to a regional area, the asset classes *j* that are in the same region of a_i (although belonging to different economic categories) decrease their value. Again, as a consequence, the values of funds exposed to each asset class *j* decreases by $\Delta A E_{ij}$.

Let *L* be the list of assets which values decreased in [c]. The steps [a], [b], [c] are run again starting from an asset in *L* picked up randomly. Therefore, there is a sequence of decrease of values. If an investor overtakes the percentage losses threshold *q*, that we put equal for each fund for simplicity, it leaves the market and liquidates its portfolio, so all the assets in its portfolio experience a further decrease in their value. The procedure stops when there are no more investors that leave the market. When the procedure ends, we calculate the market impact of this portfolio's configurations of the market, through BiCM, revealing the amount of residual asset classes market value survived. We run the program for different configuration of the network, obtained through BiCM. The dynamic is run again with several value of the factor z, which gives the density of links in a wide range (0%-100%). Increasing z results in an increase of the density (connectivity) of the bipartite network.

In terms of variables, the pseudo-code of the model is as follows:

- At first we construct an exposition matrix E_{ij} for a given z that controls the density of the links in the network.

- At the begin of each run of the program, at time *t*, we pick up randomly an asset class *l* to shock. The three means for the propagation of the shock are represented as follows:

  1. The shock has the effect to decrease of a percentage discount factor *v* the value of the asset, so any funds *i* exposed on *l* suffers a direct losses via exposition matrix E_{ij} :

  $$E_{il}(t+1) = E_{il}(t) - v \cdot E_{il}(t) \quad (2)$$

  2. then stress propagates among other asset classes of the same economic sector $\{j \neq l, j \in K\}$ via correlation matrix C_{85}, so we have:

  $$\forall_j \neq l, j \in K,\ E_{ij}(t+1) = E_{ij}(t) - v \cdot C^{85}{}_{jl} E_{ij}(t) \quad (3)$$

  Reminding that the diagonal elements of the correlation matrix are equal to one, the two steps above can be gathered into

  $$\forall_j \in K,\ E_{ij}(t+1) = E_{ij}(t) - \Delta_E E_{ij} \quad (4)$$

  Where

  $$\forall_j \in K,\ \Delta_E E_{ij} = v \cdot C^{85}{}_{jl} \cdot E_{ij}(t)$$

  3. in the third step the asset class *j* that belongs to another economic sector but comes from the same region category *R* of *l* receives a shock:

  $$\forall_j = R,\ E_{ij}(t+2) = E_{ij}(t+1) - \Delta_A E_{ij}, \quad (5)$$

where

$$\Delta_A E_{ij} = v \cdot C^{6}{}_{jl} \cdot E_{ij}(t+1)$$

- In this dynamics scenario Investor *i* suffers of incremental monetary losses. If *l*, one of the Funds, has reached the escape threshold *q* at time t^*, $\frac{\sum_l E_{lj}(t^*)}{\sum_l E_{lj}(\mathrm{t})} < q$, fund *l* sells its assets that depreciate:

$$E_{lj}(t^*+1) = E_{lj}(t^*) \cdot \left(1 - \frac{\sum_{ij} E_{ij}(t^*)}{\sum_{ij} E_{ij}(t)}\right) \quad (6)$$

- If in the previous step no one has reached the escape threshold, the dynamics stops, otherwise it restarts with an equal exogenous shock to another asset class $j \in R$ picked at random.
- When the dynamics stops in $\hat{t}$ we calculate the market impact of the cascading process in terms of monetary loss in the market at any step monitoring the devaluation of fund's exposure plus the effect of depreciation

$$AV(z) = \frac{\sum_{ij} E_{ij}(\hat{t})}{\sum_{ij} E_{ij}(\mathrm{t})} \quad (7)$$

This is the assets value at risk of default and the impact that Funds could suffer.

3. Results

The main goal of our work is to assess the systemic impact of a dynamics on NPLs default cascade on the empirical reconstruction of a hypothetic NPLs market. To do this, using BiCM, we are able to define different random overlapping portfolios configurations in the market, just varying z, the density control parameter. In financial terms, the variation of z implies the rise of market integration or, from the Investors point of view, the increase of portfolio differentiation. Given stable financial resources, it implies also the increase of the number of exposures on the market with a lower value, going from a sparse specialized market to an integrated highly differentiated one. So the maximal density of edges brings to a situation where every Investor is exposed on the same asset class: it means that everyone purchases an amount of the market portfolio. In a network perspective we have a star in the monopartite projection of the bipartite market network. In short, it is the same situation of a Fund that purchases the whole quantity of asset classes, in line with the original aim of Atlante Fund, the Investment of the whole junior tranche of the Italian NPLs securitization.

We now present the results of our model's simulations. We run 100 simulations for a discrete series of z within its range, and we vary the control parameter of the market escape $q \in \{20\%, 50\%, 80\%\}$. First, we focus on the more realistic Investors configuration reconstructing the expositions matrix E_{ij}^{1} with vector F^{1}. Looking at the dynamics of a single realization of the system, fig.1 shows the evolution of the remaining market values or conversely the percentage impact of the default cascade. The different trend trajectories refer to different model configurations on q, (figs. 1, 2, 3), in which we select randomly an asset with the exogenous shock v = 10% of initial value. The first striking observation is that the dynamics converges to a threshold around 20%. We remark that for a low connectivity (i.e. low network density of edges or low portfolio's differentiation) the value of all the assets goes to zero. Conversely, with an increasing density we recover partial value of the market. A significant assumption is the "market escape". We define a targeting clearance threshold q to reproduce the role that in usual bipartite financial networks belongs to banks. With their balance sheet constraints, at the default time, they are candidates for intensifying the stress within the network Caccioli *et al.* (2014). In our model we define q as the losses-targeting policy constraint that Investors have and that triggers portfolio liquidation: this is in line with "panic selling" situations in the markets.

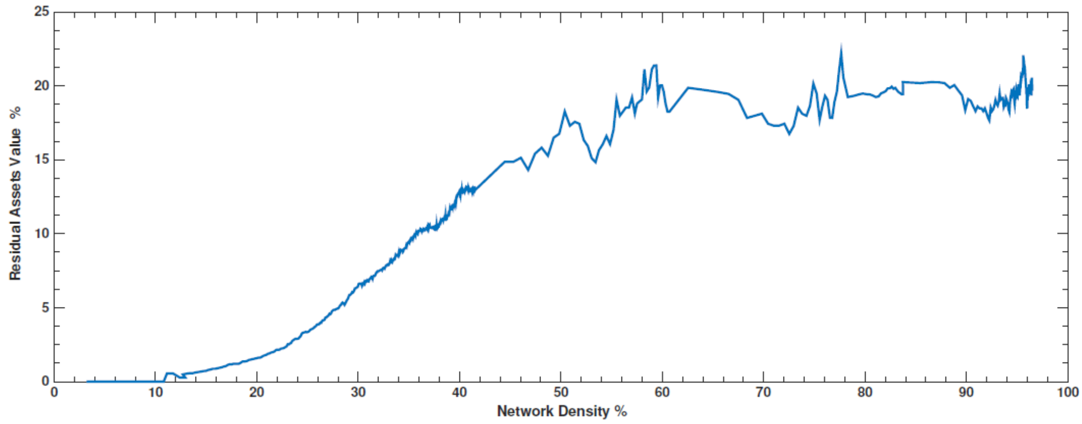

Figure 1. Final residual assets value of F^{1} in the system for $q = 20\%$

The analysis of the dynamics among the various "escape-market threshold" configurations, reveals that there is convergence of the system although there are many fluctuations. The system converges to a flat threshold of $\approx (42 - 48\%)$, where we have no more differences in systemic impact of connectivity and parameter q. Increasing the residual asset classes value, the fluctuation of the result increases. The same point emerges in fig. 4, which shows a similar dynamics of "market escape". Considering the frequencies of the dynamic's "breaks", the detection of a break indicates that there are no more "escapes", with losses at a minimal level. The dynamics of "breaks" displays exactly the same evolution of residual asset classes value because they indicate which configuration of the system is gradually more resilient to the shocks. So q is one of the main parameters that lead to different trajectories.

It is worth to note that these results, based on empirical data, do not confirm the thesis which states that networks with medium connectivity are more resilient to default cascade - Caccioli *et al.* (2014). The characteristics of the network (two types of nodes and two types of links) seem to be a key factor in the present analysis. Namely, the correlation matrices induce spreading of distress beyond directed expositions. So this effect of multiplicative shocks, given by the multiple cycles in our framework design, is high when we have sparse networks. With a lower node degree and higher single exposure the effect of multiplicative shocks is higher. The conclusion with a model that reconstructs links redistributing proportionally their weights, is that low density networks with hidden connections are more fragile.

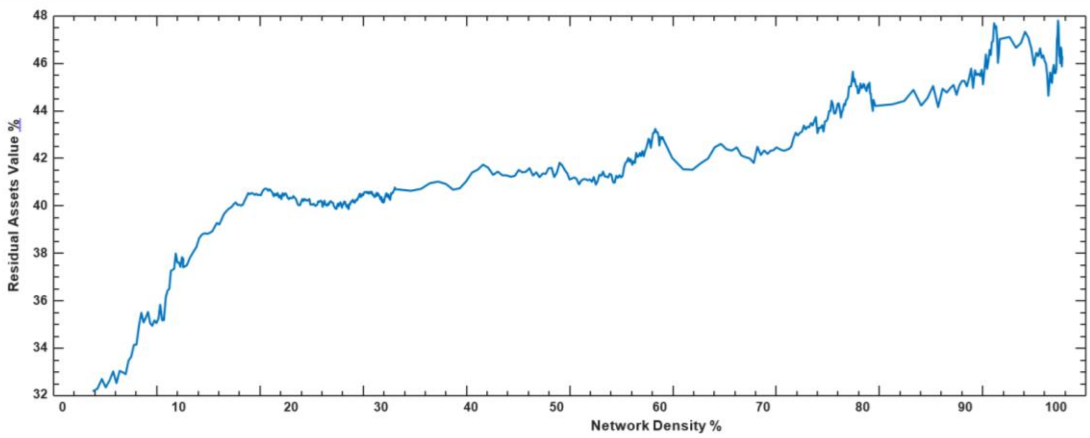

Figure 2. Final residual assets value of F^1 in the system for $q = 50\%$

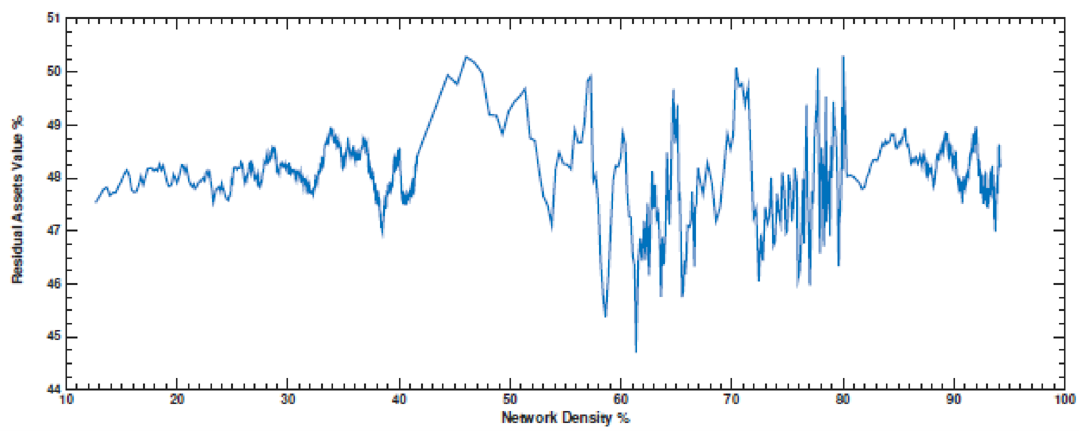

Figure 3. Final residual assets value of F^1 in the system for $q = 80\%$

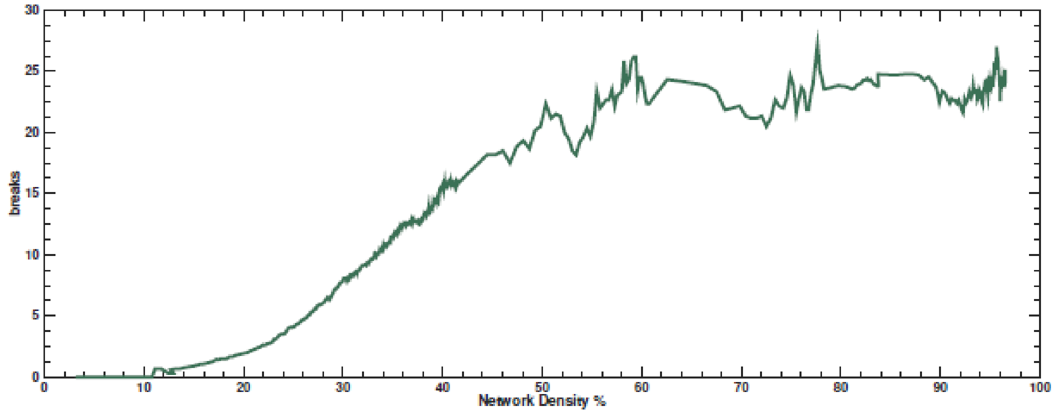

Figure 4. Frequencies of breaks of F^1 for $q = 20\%$

4. Second setting

We proceed with a different assumption on the Funds that buy the NPLs: a second hypothesis considers a large amount of investors, that invest all the same amount. Since we have 85 assets, we assumed to have 85 funds. Each component of the vector $F^2 \in R^{85}$ is equal to 159964.78. This number was calculated as a raw division 13.6bn/85=159964.78m. Of course, any other large number could have been used for the present analysis. Figs. 5, 6, 7 show the results of the dynamics when F^2 is used in order to build an exposition matrix E_{ij}^2 (85 x 85) with 85 Investors of the same dimension.

We see that there are no remarkable differences between these two configurations for both residual asset classes value and breaks. It means that the topology of the network plays the major role in the dynamics. The connectedness is, as it is well known, the main spreading channel of stress through the market.

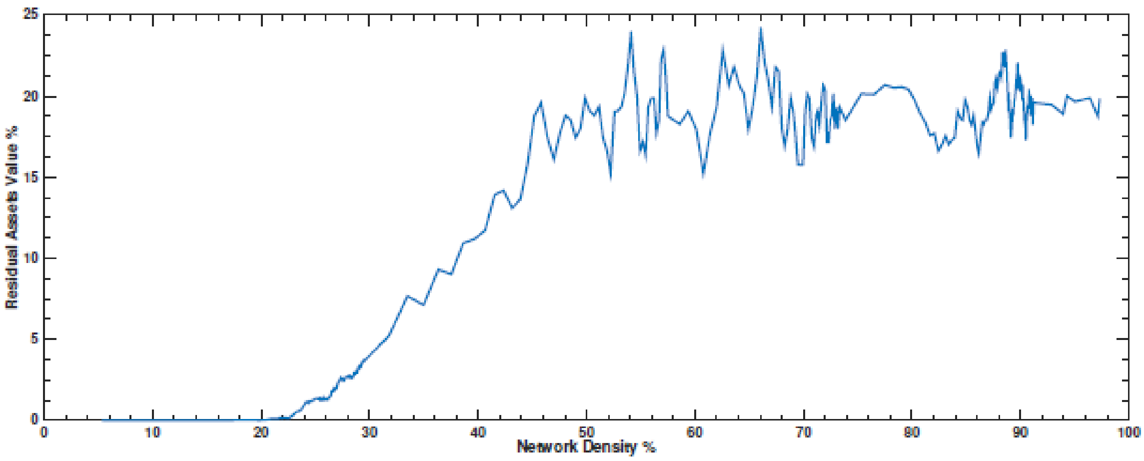

Figure 5. Final residual assets value of F^2 for $q = 20\%$

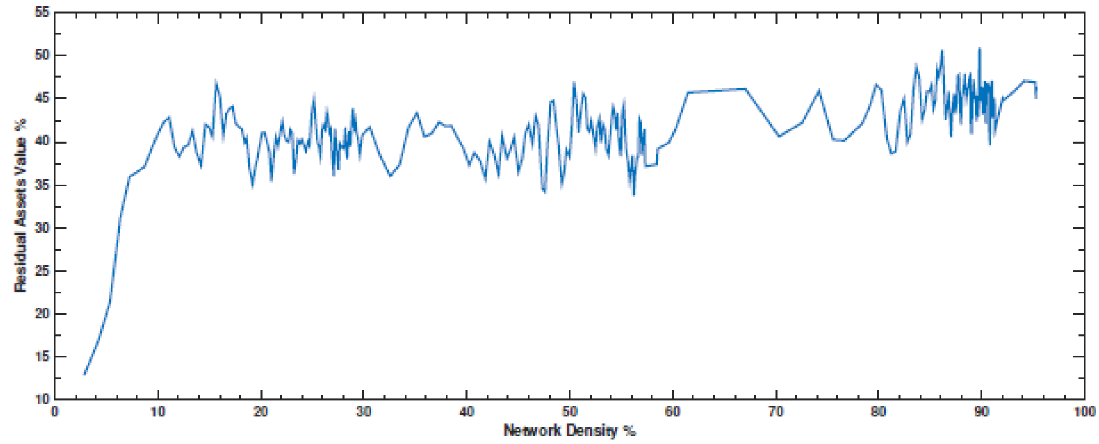

Figure 6. Final residual assets value of $F^2$for q = 50%

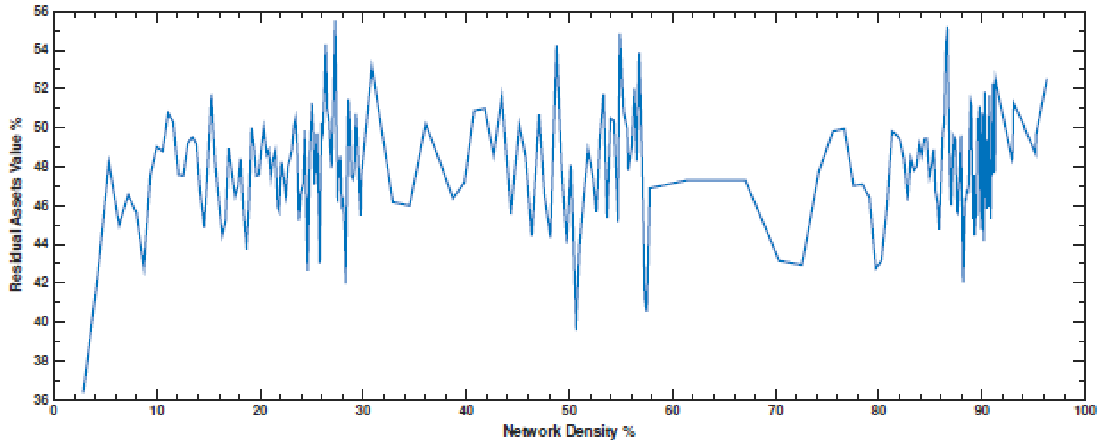

Figure 7. Final residual assets value of F^2 for q = 80%

5. Conclusions

In our work we have designed an innovative empirical model to mimic the dynamics of a cascade defaults of assets in a hypothetical NPL market. The model relies on Italian banks NPLs data source coming from banks balance sheets, and is built on simple assumptions on Funds strategic behavior during periods of financial distress on portfolios assets. What emerges is that the main leading factors of stress are: the "market escape" dependence of paths, the fragility of a system with too many similar units where there are multiple connections of different nature and not relevant differences between systems of actors of various wealth. These features of NPL market are of the main relevance w.r.t. the capability of defining and quantifying the Systemic Risk of this market. We have focused on the dynamics of NPLs because of their crucial role due to the fact that they are becoming a major concern at policy level and about the correct monetary policy transmission and so for the economic growth. The results show that the control parameters of the connectivity and "market escape", v and q, lead the dynamics path. For instance, v could be managed with a scheme like Atlante where a Fund purchases all the asset classes, because it is the same situation of many funds that purchase a fraction of all the asset classes and where the bipartite network would have the maximum density.

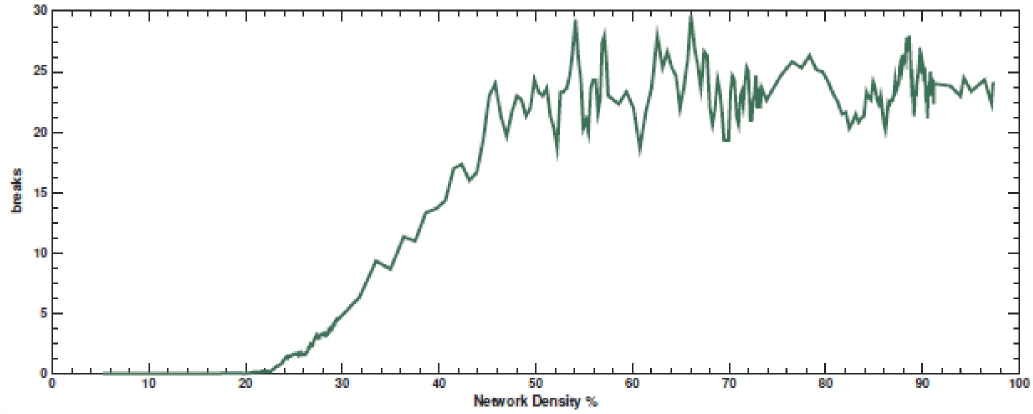

Figure 8. Frequencies of breaks of F^2 for q = 20%

Otherwise, policy makers could regulate a parameter of differentiation avoiding concentration of few portfolios on the same asset class. The parameter q could be managed by policy makers and calibrated in view of the institutions that would purchase the assets. For example, there are investors, like some Hedge funds with a short term investment duration policy, that are more sensitive to immediate losses. Investors with a lower q would exacerbate a potential fire-sale. Instead other investors, for instance Pension funds, that have a long term investment horizon, would be less sensitive to sudden losses and do not accelerate the fire-sale betting in a recovery of the investment. Therefore, policy makers could define some mechanisms to select investors and other ones to freeze the market avoiding panic-selling. This could be of interest for Policy makers, Authority and operators involved in the market optimal design and interested in evaluate the systemic impact of NPLs default cascades.

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[2]This implies that banks play no active role anymore in the system after the divestiture of NPLs.
[3] Intesa Sanpaolo, Unicredit, UBI Banca, Banca Popolare dell'Emilia Romagna, Banca Popolare di Milano, Credito Valtelli- nese, Banca Mediolanum, Banca Popolare di Sondrio, Banco Popolare, MPS, Iccrea Banca, Popolare di Bari, Carige, Banca Sella, CR Asti, CR Bolzano, Popolare di Puglia e Basilicata, Banco Desio, Banca di Piacenza, Banca Valsabbina, Popolare Alto Adige, Popolare Pugliese, Banca di Credito Popolare, CR Ravenna, Popolare di Cividale.

[4] Atlante is a closed-end alternative investment fund created in April 2016 by Quaestio Capital Management SGR S.p.A. The fund, regulated by Italian law and reserved for professional investors such as banks, insurance companies, banking foundations and the Cassa Depositi e Prestiti, invests at least 30% of its funds in Non-Performing Loans (NPLs) from several Italian banks. The purpose of the Fund was to promote the creation and development of a large and efficient secondary market of distressed assets in Italy.

[5] Bank of Italy classified non-performing loans in four categories: a) Bad loans ("sofferenze"), exposure to any borrower in a position of insolvency (even if insolvency is not legally ascertained) or a substantially similar situation, regardless of any loss estimate made by the bank and irrespective of any possible collateral or guarantee b) Substandard loans ("incagli"), exposure to any borrower experiencing temporary payment difficulties defined on the basis of objective factors - that the lender believes can be resolved within a reasonable period of time c) Restructured loans ("ristrutturati"), exposures in which a pool of banks or an individual bank, as a result of the deterioration of the borrowers financial situation, agree to change the original conditions (interest rate, reduction in capital, rescheduling of monthly payments, etc.), giving rise to a loss. In the event of a partial restructuring, exposure remains in its original category. d) Past due ("scaduti" or "sconfinanti"), exposure to any borrower whose loans are not included in other categories and who, at the date of the balance sheet closure have past due amounts or unauthorised overdrawn positions of more than 90 days. A new classification of loans by Bank of Italy (1/2015), transposing the technical standard published by European Banking Authority (EBA) (CE Reg. 2015/227), refers to non-performing exposures and forbearance exposures and has no impact on our sampled data. For details, see Cimini (2016).

[6] Net NPLs are defined as gross NPLs less loan loss provisions (reserves). The normative references and administrative provisions issued by the Bank of Italy (Circ.262) on the layout and presentation of Income Statement for Banks can be found in - Vicari & Berselli (2016); Cimini (2016).

[7] This classification has been extracted from the Tables related to the "Breakdown and concentration of credit exposures" in the Notes of the Annual report of each and from the Pillar III of the 25 Banks. Each bank is exposed only to some of the six customer economic activities, and the complete list has a lenght of 85.

APPENDICES

Appendix 1: Exponential Random Graph Model

To introduce the Exponential Random Graph Model (ERGM) it is useful to recall the concept of Entropy due to its fundamental role in defining Information and the way to use it to build models of networks generation.

Entropy

By definition, the amount of self-information contained in a probabilistic event depends only on the probability of that event - Shannon (1948): the smaller its probability, the larger the self-information associated with receiving the information that the event indeed occurred. Furthermore, by definition, the measure of self-information is positive and additive and the proper choice of function to quantify information, preserving this additivity, is logarithmic. If an event C is the intersection of two independent events A and B, then the amount of information at the proclamation that C has happened, equals the sum of the amounts of information at proclamations of event A and event B respectively: $I(A \cap B) = I(A) + I(B)$. Taking into account these properties, the self-information $I(wn)$ associated to the outcome $wn$with probability $P\ (wn)$ is

$$\mathrm{I}\ (w_n) = \log\left(\frac{1}{P\ (w_n)}\right) = -\log(P\ (w_n))$$

This definition complies with the above conditions. In the definition above, the base of the logarithm is not specified: if using base 2, the unit of $I(wn)$ is bits. Shannon defined the entropy H of a discrete random variable X with possible values $\{x1, \ldots, xn\}$ and probability mass function P(X) as:

$$H(X) = E[I(X)] = E[-\ln(P\ (X))]$$

Here E is the expected value operator, and I is the information content of X. I(X) is itself a random variable. When taken from a finite sample, the entropy can explicitly be written as

$$\mathrm{H(x)} = \sum_{\mathrm{i}} \mathrm{P(x_i)\, I\,(x_i)} = -\sum_{\mathrm{i}} \mathrm{P(x_i)\, \log_b P(x_i)}$$

The Exponential Random Graph Model

Now we consider a specific class of Configuration model. Consider a set G of graphs - Park & Newman (2004). Suppose to have a collection of graph observables $\{xi\}$, $i = 1\ldots r$, that we have measured in empirical observation of some real-world networks. In practice it is often the case that we have only one measurement of an observable. In this case, however, our best estimate of the expectation value of our variable of interest is simply equal to the one measurement that we have. Let $g \in G$ be a graph in our set of graphs and let P(g) be the probability of that graph within our ensemble. We would like to choose P(g) so that the expectation value of each of our graph observables $\{xi\}$ within that distribution is equal to its observed value. The best choice of probability distribution is the one that maximizes the Gibbs-Shannon entropy

$$S = -\sum_{g=G} P(g)\ \ P(g) \ln P(g)$$

subject to the constraints

$$\sum_g P(g)\ x_i\,(g) = \langle x_i \rangle$$

plus the normalization condition

$$\sum P(g) = 1$$

Here $xi(g)$ is the value of xi in graph g. Introducing Lagrange multipliers α, $\{\vartheta i\}$, we then find that the maximum entropy is achieved satisfying

$$\frac{\partial}{\partial P(g)}\left[S+\alpha\left(1-\sum_g P(g)) + \sum_i \theta_i(< x_i > - \sum_g P(g)\ x_i(g)\right)\right] = 0$$

for all graphs g this gives

$$-\ln P(g)\left(\ P(g)\ \frac{1}{P(g)}\right) - \alpha - \sum_i \theta_i\, x_i\,(g) = 0$$

or equivalent

$$e^{\ln P(g)} = \quad e^{(-1-a)}\, e^{-\sum_i \theta_i x_i (g)}$$

where we define graph Hamiltonian H and partition function Z;

$$H(g) = \sum_i \theta_i\, x_i\,(g), \qquad Z = e^{(1+a)} = \sum_g e^{(-H(g))}$$

for normalization, and then

$$P(g) = \frac{e^{-H(g)}}{Z}$$

The last equation defines the exponential random graph model - Park & Newman (2004). The Exponential Random Graph is the distribution over a specified set of graphs that maximizes the entropy subject to the known constraints. The expected value of any graph property x within the model is simply

$$< x \quad > = \sum_g P(g)\, x(g)$$

Suppose we have the complete set $\{ki\}$, the degree sequence of the network. The Exponential Random Graph model appropriate to this set is the one having Hamiltonian $H = \sum_i \theta_i k_i$ where we now have one parameter θ_i for each vertex i. Noting that $k_i = \sum_j \sigma_{ij}$, this can also be written

$$H = \sum_{ij} \theta_i \sigma_{ij} \quad = \sum_{i<j} (\theta_i + \theta_j)\sigma_{ij}$$

The partition function is

$$Z = \sum_{\sigma_{ij}} \exp\left(-\sum_{i<j}(\theta_i+\theta_j)\sigma_{ij}\right) = \Pi_{i<j}\left(\sum_{\sigma_{ij}=0}^{1} e^{-(\theta_i+\theta_j)\sigma_{ij}}\right) =$$

$$= \Pi_{i<j}\left(1 + e^{-(\theta_i+\theta_j)}\right)$$

More generally, we could specify a Hamiltonian

$$H = \sum_{i<j} \Theta_{ij}\, \sigma_{ij}$$

with a separate parameter Θij coupling to each edge. Then $Z = \Pi_{i<j}\,(1 + e^{-\Theta_{ij}})$ and, defining $F = -\ln Z$, we have

$$F = -\sum_{i<j} \ln(1 + e^{-\Theta_{ij}})$$

This allows us to calculate the probability of the occurrence *pij* of an edge between vertices *i* and *j*

$$p_{ij} = <\sigma_{ij}> = \frac{1}{Z}\sum_{G} \sigma_{ij} e^{-H} = -\frac{1}{Z}\frac{\partial Z}{\partial \Theta_{ij}} = \frac{\partial F}{\partial \Theta_{ij}} = \frac{e^{-\Theta_{ij}}}{1+e^{-\Theta_{ij}}}$$

Appendix 2: The configuration model

The Configuration Model (CM) (Cimini et al. (2015a,b)) investigates if it is possible to estimate topological properties of a network starting from limited information. The Fitness based Configuration model can be seen as a specific case where the set of properties {*Ci*} is the degree sequence {*ki*}, *i* = 1 . . . *n* of the nodes of the network, where the values of <*ki*> for all nodes i are fixed and each node can be identified by its control parameter (or Lagrange multiplier) *ϑi* . Fixing the values of {*ϑi*} is equivalent to fix the mean values of {*ki*}. In order to further clarify the role of {*ϑi*} in controlling the topology, let us define in general *xi* = e−*ϑi* . From aprevious equation, modified for directed networks, we have now two kinds of control parameters {*xi, yj* } and so *xiyj* = e−Θi e−Θj . Now, knowing the set {*ϑ*} for all nodes, the ensemble is such that, for each network in Ω, two nodes *i* and *j* are directly connected with a probability given by

$$p_{i\to j} = \frac{x_i\, y_j}{1+ x_i\, y_j}$$

where *xi*(*yi*) quantifies the ability of node i to receive incoming (form outgoing) connections. Suppose to have incomplete information about the topology of a given network G.

In particular one have the in-degree sequence $\{k_i^{in}\}_{i\epsilon I}$ and the out-degree sequence $\{k_i^{out}\}_{i\epsilon I}$ only for a subset $I \subset G$ of the nodes and conversely a pair of fitness a pair of fitness $\{\chi_j\}\epsilon$ V and $\{\psi_j\}\epsilon$ V where

$$s_i^{in} = \sum_{i\epsilon V} w_{j\to i} \equiv \chi_i \quad \text{and} \quad s_i^{out} = \sum_{j\epsilon V} w_{i\to j} \equiv \psi_i \text{ for all the nodes.}$$

The CM defines the probability distribution over Ω subject to Lagrange multipliers $\{x_i, y_i\}$ (two for each node), whose values must satisfy the equivalence $< k_i^{in} > = k_i^{in}$ and $< k_i^{out} > = k_i^{out}$, ∀i.

The assumption that the fitnesses χ_i *and* ψ_i are proportional to the in-degree-induced and out-degree-induced Lagrange multipliers $\{x_i\}$ and $\{y_i\}$ through universal (unknown) parameters *α* and *β*:

$x_i \equiv \sqrt{\alpha\chi_i}$ and $y_i \equiv \sqrt{\alpha\psi_i}$ leads to

$$p_{i\to j} = \frac{\sqrt{\alpha\chi_j}\sqrt{\alpha\psi_i}}{1+\sqrt{\alpha\chi_j}\sqrt{\alpha\psi_i}} = \frac{z\,\chi_j\psi_i}{1+z\,\chi_j\psi_i}$$

where z≡ $\sqrt{\alpha\beta}$. We can now impose the condition that ensures the likelihood

$$\sum_{i\epsilon I}[< k_i^{in} >_\Omega + < k_i^{out} >_\Omega] = \sum_{i\epsilon I}[k_i^{in} + k_i^{out}]$$

where $< k_i^{in} >_\Omega = \sum_{j(\neq i)} p_{j\to i}$ and $< k_i^{out} >_\Omega = \sum_{j(\neq i)} p_{i\to j}$ so now we have an algebraic equation in z to solve numerically

$$\sum_{i\epsilon V}\sum_{j\neq i}\left[\frac{z\chi_i\psi_j}{1+z\chi_i\psi_j} + \frac{z\chi_j\psi_i}{1+z\chi_j\psi_i}\right] = \sum_{i\epsilon V}[k_i^{in} + k_i^{out}]$$

Using z and the fitnesses $\{\chi_i, \psi_i\}$, we generate the ensemble *Ω* by placing a direct link from *i* to *j* with probability $p_{i\to j}$ with a weight

$$w_{i\to j} = \frac{\chi_j\psi_i a_{i\to j}}{\sum_{l,m\epsilon V}\chi_m\psi_l a_{l\to m}} W$$

Where $W = \sum_i \chi_i$, and l and m are referred to the edges present in the bootstrappeted network with the relative strength in the real net. Then, we compute the estimate of the variable of interest X(g) on the networks ensemble *Ω* as $< X >_\Omega \pm \sigma_X^\Omega$ averaging the results.

Appendix 3: Vector F[1]:

Fund - hypothetical purchasing of NPLs asset classes expressed in thousand of Euro. It has been calculated dividing the total exposure 13.6bn by the percentual exposure of funds listed in Quaestio - Quaestio (2016). This last data were calculated as usual, summing the exposure listed in Quaestio, and dividing each single exposure by the sum.

Anacap	2583430
Fortress	2583430
Prelios	2311490
Deutsche bank	1541899,8
Eurocastle	1284916,5
Pimco	770949,9
GWM	770949,9
Pve Capital	770949,9
Cerberus	726079,8
Lone Star	648576,9
Credito Fondiario	513966,6
TPG	403830,9
Algebris	285537
Morgan Stanley	142768,5
Poste Vita	142768,5
Baml	142768,5
Beni Stabili	142768,5
GS	141408,8
Ares Managment	135970

Correlation Matrix C6 between origin sectors:

	GOV	ADM	FIN	INS	NFF	OTH
GOV	1	0,6571	0,5612	0,4498	0,5850	0,6423
ADM	0,6571	1	0,4985	0,1742	0,6637	0,5461
FIN	0,5612	0,4985	1	0,6533	0,6475	0,7125
INS	0,4498	0,1742	0,6533	1	0,5920	0,7640
NFF	0,5850	0,6637	0,6475	0,5920	1	0,9215
OTH	0,6423	0,5461	0,7125	0,7640	0,9215	1

Appendix 4: Supplementary figures

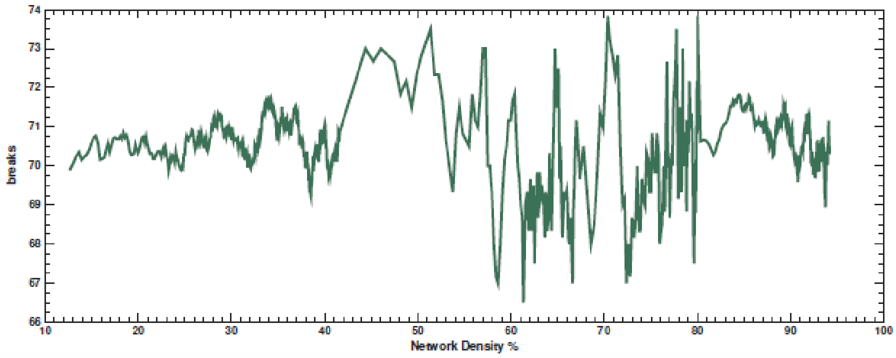

Figure 9. Frequencies of breaks of *F* 1 for *q* = 80%

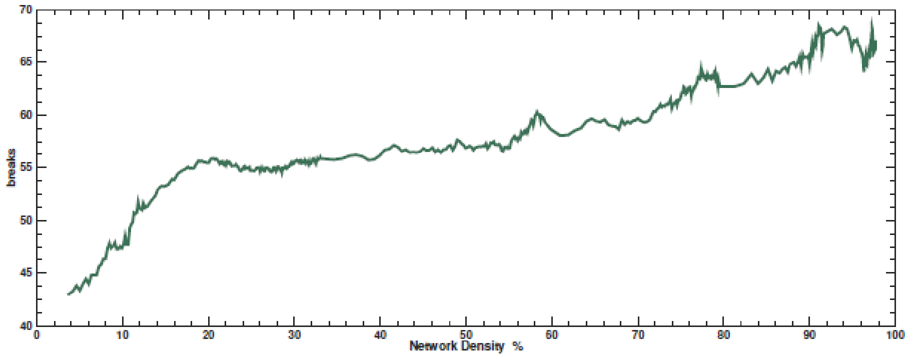

Figure 10. Frequencies of breaks of *F* 1for *q* = 50%

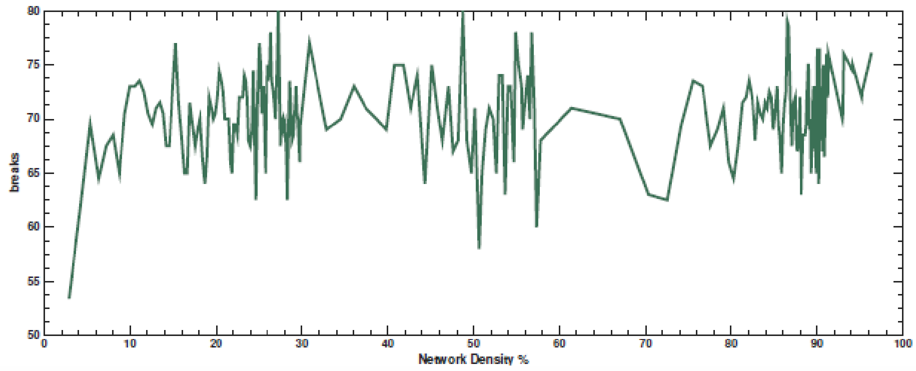

Figure 11. Frequencies of breaks of F 2 for q = 80%

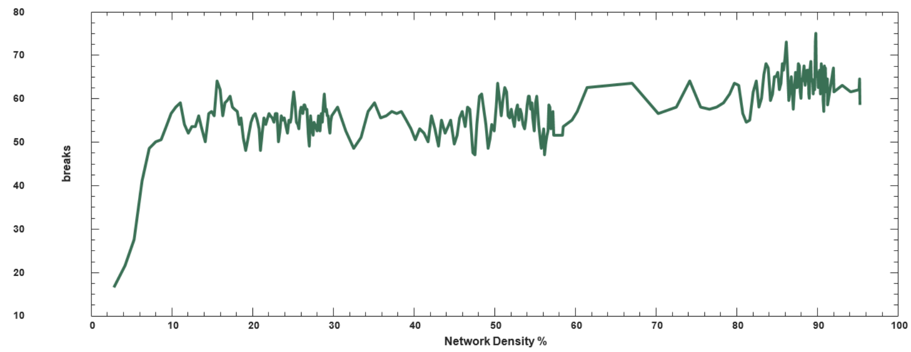

Figure 12. Frequencies of breaks of F 2 for q = 50%