DYNAMIC VALUATION OF WRONGFUL DEATH COMPENSATION WHEN THE GROWTH RATE OF THE WAGE FOLLOWS VASICEK MODEL

Hong MAO[1]
Associate Profesor, Economics and Management School,
Shanghai Second Polytechnic University, China

E-mail: maohong@sspu.edu.cn

Abstract
In this article, dynamic valuation of wrongful death compensation with single and annual level payment are discussed. The annual growth rate of the wage and the risk-free interest rate are assumed to follow multi-Vasicek model. The formula of calculating time-varying correlation coefficient between the wage and the discounting rate are deducted. Empirical data from U.S, and China shows that multi-Vasicek model is a good model to be fitted to display the change patterns of the annual growth of the wage and risk-free interest rate.

Keywords: *Wrongful death; Growth of the wage; Interest rate; multi-Vasicek model; Compensation*

1. Introduction

Wrongful death is a cause of action, or type of claim, that can be brought when one person or entity wrongfully causes someone's death. A wrongful death claim may be out of a number of circumstance such as in the following situations (http://findlaw.com/):

- Medical malpractice that results in decedent's death;
- Automobile or airplane accident;
- Occupational exposure to hazardous conditions or substances;
- Death during a supervised activity.

When wrongful death occurs due to any of above causes, one very important thing is to estimate the damage in a wrongful death accurately. Pecuniary or financial, injury is the main measure of damages in wrongful death action which includes the loss of support, services, lost prospect of inheritance, and medical and funeral expenses. Usually, the main consideration in award damages is the loss of income if wrongful death occurs to an adult wage earner with dependants dies. The lost income should include the decedent's earnings at the time of death, the last known earning if unemployed, and potential future earnings. Johnson and Flanigan (1984) discuss the steps to estimate the future lost income, other relevant benefit and discounting issues of the deceased persons. They also demonstrate how to measure the concerned primarily with the determination of lost income of the family breadwinner, and the valuation of services rendered by non-employed homemaker in traditional family. Mao, et al. (2011) presents a dynamic valuation model for wrongful death compensation. They considers the case of a single payment, and a series of level payments, corresponding to common methods of payments of awards in loss litigation. They establish stochastic models to calculate the discounting value of future salary, and at the same time, they considers the affect of the survival probability of both obligators and decedents to the compensation. They apply stochastic simulation to determine the loss in case of wrongful death and consider both discounting with a fixed interest rate, and discounting when interest rate follows a stochastic model. They also illustrates that the models can be extended to injury as well as other liability cases. However, one problem exists. Mao, et al. (2011) consider that the growth rate of wage follows Geometric Brownian Motion (Similar work were discussed by Mincer, 1974, Hosek, 1982, Horvize,1986, Carrie and Shand, 1998), while both U.S. and Chinese empirical data show that the annual growth rate of the average wage follows Vasicek model. In this article, we establish dynamic valuation model of wrongful death compensation under the consideration that the growth rate of the wage is a stochastic process following Vasicek model. We use similar pricing models of traditional life insurance products based on actuarial mathematics (Bowers, JR. et al.(1997)). However, at least two aspects of our models are different from traditional life insurance products:

(1) The pricing of traditional life insurance only deals with the mortality or survival probability of the insured, while the pricing models of wrongful death compensation considers the probability that both of obligators and decedents die in other accident or illness or natural death in the calculation of level compensation.

(2) The insurance benefit given to the beneficiary of the decedent if the claim event occurs is determined by bilateral agreement between the insurer and the policyholders in traditional life insurance, while the amount of compensation is calculated based on the value of life. The value of life is an economic value used to quantify the benefit of avoiding a fatality. It is also referred to as the cost of life, value of preventing a fatality (VPF) and implied cost of averting a fatality (ICAF). In many studies, it alsoincludes the quality of life, the expected life time remaining, as potential of a given person especially for an after the fact payment in wrongful death claim lawsuit (From Wikipedia). We here evaluate the value of life only from economic perspective. We estimate the potential future earnings through forecasting the stochastic change pattern of the annual growth of the wage using historical data. We also consider the time-varying correlation between the stochastic wage and stochastic discount factor in the calculation of the present value of the compensation. It is important to notice that all models we establish are based on empirical data with small and acceptable fitting errors. Therefore, the calculated results based on the models can be directly and effectively applied in liability compensation in the legislation.

2.Models of lump sum payment

2.1. Assuming that the interest rate is constant

Similar to Mao et al. (2011), assume that a person died in life hurt at age x, the mandatory age for retirement age is m and $x < m$. That is the wrongful death occurs before retirement.The compensation paid to his legitimate successor should be equal to the present value of his cumulative income from the date of his death to the date of his retirement minus his consumption. Here we neglect the additional payment for pain and suffering to survivors. We also consider the probability that he dies in other accident or illness. Assume that the annual growth rate of the wage (nominal wage growth, which includes both real wage growth and inflation) follows Vasicek Model[2] as in equation (1):

$$\frac{dW_t}{W_t} = dr_w = a_w(b_w - r_w)dt + \sigma_w dz_w \tag{1}$$

where σ_W is the standard deviation of the percentage change in the wage rate while b_w is the expected rate of return that would be earned on a (hypothetical) security that has the same rate of change risk ($\sigma_w dz_w$) as the wage rate. Since wages, W, will not necessarily be expected to grow at the rate of an asset that bears the same risk, $b_w - r_w$ denotes the difference between this equilibrium expected asset return and the expected growth in wages.

Assume that the random variable T_x expresses the survival time if the affected person aged x does not die in life hurt. Let ${}_t p_x = \Pr(0 \le T_x < t)$ express the probability that a person aged x survives t years. Here we also assume that T_x is a random variable that is independent of the wage and the interest rate. Assume that the risk free interest rate is r, then the discount factor calculated by continuous interest rate can be expressed as e^{-rt}. Please note that we assume that the individual's contribution before the decedent's death and corrsponding interest rate upon his(her) retirement will be returned to his (her) successor(s). To simplify analysis, we assume that the consumption for a person is h percent of his wage. Let P_{1v_1} be the present value of compensation of wrongful dearth with a single payment and we only consider the wrongful death occuring before retirement and the total compensation term includes the period between the time when wrongful death occurs and retirement time, then

$$\begin{aligned} P_{1v_1} &= E\left(\sum_{t=0}^{m-x} W_t(1-h)\,{}_t p_x e^{-rt}\right) \\ &= \sum_{t=0}^{m-x} E^p(W_t(1-h)\,{}_t p_x e^{-rt}) \end{aligned} \tag{2}$$

$$E^p(W_t) = W_0 e^{E^p\left(\int_0^t r_w(u)du / r_w(t)\right)}$$

where
$$= W_0\left(e^{E^p[\int_0^t r_w(u)du] + \frac{1}{2}\mathrm{var}[\int_0^t r_w(u)du]}\right), \tag{3}$$

$$= W_0 e^{A_t + \frac{1}{2}B_t}$$

$$A_t = E\left(\int_0^t r_w(u)du\right) = \frac{r_{w0} - b_w}{a_w}(1 - e^{-a_w t}) + b_w t \tag{4}$$

and $B_t = Var\left(\int_0^t r_w(u)du\right) = \frac{\sigma_w^2}{2a_w^2}\left(2t - \frac{3 - 4e^{-a_w t} + e^{-2a_w t}}{a_w}\right)$ (5)

(For the proof, please see Appendix), where E denotes the expected value.

Let $$\sum_{t=0}^{m-x}\left(e^{A_t+\frac{1}{2}B_t}\,{}_tp_x e^{-rt}\right)=v_1,\qquad (6)$$

where v_1 is called as the factor of the compensation of the wrongful death with a single payment. By combining equation (3) with (6), we obtain the following equation:

$$P_{1v_1}=W_0(1-h)v_1 \qquad (7)$$

The survival probability ${}_tp_x$ can be found by following (but not limited) methods:

(1) Mortality table

${}_tp_x=\prod_{s=1}^{t}(1-q_{x+s})$, where q_{x+s} is the mortality rate at time $x+s$ which can be found in the Mortality Table and x is the age of the obligator at time of policy issued.

(2) Gompertz-Makehan Model

Assume that mortality follows Gompertz-Makeham distribution (Milevsky, 2006)[3], with the instantaneous force of mortality (IFM) given as:

$$\lambda(x)=\delta+\frac{1}{b}e^{(x-g)/\beta},\ t\geq 0 \qquad (8)$$

where g is the modal value of future lifetime and β is the dispersion coefficient.

According to the equation (8), the instantaneous force of mortality is a constant δ plus a time-dependent exponential curve. The constant δ aims to capture the component of the death rate that is attributable to accidents, while the exponentially increasing portion reflects nature death. The conditional probability of survival under this Gompertz-Makeham IFM curve is equal to

$$\begin{aligned}P(t)={}_tp_x&=\exp\left\{-\int_x^{x+t}\left(\delta+\frac{1}{b}e^{(s-g)/b}\right)ds\right\}=\\&=\exp\left\{-\delta t+b(\lambda(x)-\delta)(1-e^{t/b})\right\}\end{aligned} \qquad (9)$$

2.2 Assuming that the interest rate is a stochastic process

As the cumulative calculating term of the compensation for wrongful dearth is very long, it is very important to consider the undetermined characteristic of the interest rate. It will affect the discount value of the compensation. Actually, the empirical data of long term shows that the (risk free) interest rate does be a stochastic process following Vasicek model (Vasicek (1977). Now we assume that the real interest rate can be expressed as the following stochastic differential equation:

$$dr_r=a_r(b_r-r_r)dt+\sigma_r dz_r \qquad (10)$$

where dz_r is a standard Wiener process, σ_r is the standard deviation of interest rate, b_r is the equilibrium interest rate of long term and a_r is the speed that the real interest rate recoveries to the equilibrium interest rate of long term. The factor of stochastic discount calculating with continuous interest rate can be expressed as $f(t)=e^{-\int_0^t r(u)du}$. Assume that the growth rate of the wage satisfies with equation (1), and the time-varying covariance between the wage and the interest rate at time *t* is $Y_t, t=0,1,2,\ldots m-x.$ Other assumptions are same as in section 1. Then the present value of the compensation for wrongful death is

$$\begin{aligned}P_{2v_2} &= E\left(\sum_{t=0}^{m-x} W_t(1-h)P(t)f(t)\right)\\ &= \sum_{t=0}^{m-x}\left((1-h)P(t)W_0 E\left(e^{\int_0^t r_w(u)du-\int_0^t r_r(u)du}\right)\right)\\ &= \sum_{t=0}^{m-x}(1-h)P(t)W_0\left(e^{E(\int_0^t r_w(u)du-\int_0^t r_r(u)du)+\frac{1}{2}Var(\int_0^t r_w(u)du-\int_0^t r_r(u)du)}\right),\\ &= \sum_{t=0}^{m-x}\left((1-h)P(t)W_0 e^{A_t+\frac{1}{2}B_t+C_t+\frac{1}{2}D_t+Y_t}\right)\end{aligned} \tag{11}$$

where A_t and B_t are satisfied withe quations (4) and (5),

$$C_t = E\left(\int_0^t r_r(u)du\right) = -\frac{r_r(0)-b_r}{a_r}(1-e^{-a_r t}) - b_r t \text{ and} \tag{12}$$

$$D_t = Var\left(\int_0^t r_r(u)du\right) = \frac{\sigma_r^2}{2a_r^2}\left(2t - \frac{3-4e^{-a_r t}+e^{-2a_r t}}{a_r}\right) \tag{13}$$

$$Y_t = -\frac{\sigma_w\sigma_r}{a_w+a_r}\left(\begin{array}{c}\frac{t}{a_w}+\frac{t}{a_r}-\frac{1}{a_w^2}-\frac{1}{a_r^2}+\frac{e^{-a_w t}}{a_w^2}+\frac{e^{-a_r t}}{a_r^2}\\ -\frac{(e^{-a_w t}-1)(e^{-a_r t}-1)}{a_w a_r}\end{array}\right), \tag{14}$$

where $\frac{Y_t}{\sigma_w(t)\sigma_r(t)}$ is time-varying correlation coefficients between wage and discounting rate at time t and $t=1,2,\ldots,m-x.$

For the proof of equation (14), please see Appendix.

$$\text{Let}\quad \sum_{t=0}^{m-x} {}_t p_x\left(e^{A_t+\frac{1}{2}B_t+C_t+\frac{1}{2}D_t-Y_t}\right) = v_2 \tag{15}$$

where v_2 is referred to as the factor of compensation for wrongful death when the interest rate is a stochastic process. By combining equation (11) with equation (15), we obtain

$$P_{2v_2} = W_0(1-h)v_2 \tag{16}$$

3. Models of the compensation for wrongful death with level payments in *n* years

Because the way of a lump sum payment needs obligators to pay a lot of money one time, sometimes, it is difficult to redeem. One effective method to solve this problem is to take the form of periodical payment. Assume that the age the obligator begins to pay the claim is y, ${}_tp_y$ expresses the probability that the obligator aged x survives t years, and the value of periodical payment paid at the beginning of each year is AP.

3.1. Assuming that the interest rate is constant

The sum of present value that one dollar is paid at the beginning of each year within $k(k \le m-x)$ years equals

$$1+\sum_{t=1}^{k-1} {}_tp_ye^{-rt} \tag{17}$$

Based on the principles of actuarial science, the compensation value paid by obligator at the beginning of each year should be equal to the value paid at the form of a lump sum divided by the sum of present value that one dollar is paid at the beginning of each year within n years. That is

$$AP_{1v_1}=\frac{P_{1v_1}}{1+\sum_{t=1}^{k-1} {}_tp_ye^{-rt}}=\frac{(1-h)W_0v_1}{1+\sum_{t=1}^{k-1} {}_tp_ye^{-rt}}, \tag{18}$$

where v_1 is expressed by equation equation (6).

Let $$\frac{v_1}{1+\sum_{t=1}^{k-1} {}_tp_ye^{-rt}}=v_3 \tag{19}$$

where v_3 is called as the factor of compensation for the wrongful death when the interest rate is a constant. By combining equation (18) and equation (19), we obtain

$$AP_{1v_1}=W_0(1-h)v_3 \tag{20}$$

3.2. Assuming that the interest rate is a stochastic process

Same as in sub-section 1.2, we assume the risk-free interest rate follows Vasicek model. Then, the sum of present value of one dollar paid by the obligator at the beginning of each year within n years equals

$$1+\sum_{t=1}^{k-1} E\left({}_tp_ye^{-\int_0^t r_u du}\right)=1+\sum_{t=1}^{k-1} {}_tp_ye^{C_t+\frac{1}{2}D_t}, \tag{21}$$

where C_t and D_t are satisfied with equations (12) and (13) respectlively.

Using equation (11) divides equation (21), the compensation value paid by obligator at the beginning of each year with stochastic interest rate, AP_{v_2} can be expressed as equation (22):

$$AP_{2v_2}=\frac{P_{2v_2}}{1+\sum_{t=1}^{k-1} {}_tp_ye^{C_t+\frac{1}{2}D_t}}=\frac{(1-h)W_0v_2}{1+\sum_{t=1}^{k-1} {}_tp_ye^{C_t+\frac{1}{2}D_t}}, \tag{22}$$

where v_2 is given by equation (15).

Let $$\frac{v_2}{1+\sum_{t=1}^{k-1} {}_t p_y e^{C_t+\frac{1}{2}D_t}} = v_4 \quad (23)$$

where v_4 is called the factor of compensation for the wrongful death when the interest rate is a stochastic process. By combining equation (22) with equation (23),we obtained:

$$AP_{2v_2} = W_0(1-h)v_4. \quad (24)$$

4. Calculation of compensation for wrongful death

Assume risk-free interest rate and the annual growth rate of wage in both U.S. and China follows multi-Vasicek model. Table 1 lists the values of three parameters in Vasicek model fitted by using the historical data of U.S. from 1965 to 2014[45] and that of China from 1979 to 2016[6] and the estimation errors of three parameters in Vasicek model are rather small and all of them are acceptable. Figure 1 discribes the change patterns of time-varying correlation coefficients with time for both U.S. and China. We find from Figure 1 that the change pattern of time-varying correlation coefficient with time for U.S. is quite different from that of China. The values of time-varying correlation coefficient of U.S. is greater than that of China. And this difference is expanded with the time.

We assume that y =50, m = 65 for U.S. and m = 60 for China. We here only discuss the cases when the interest rate a stochastic process. The initial levels of the interest rate and the growth rate of the wage in both of U.S. and China are listed in Table 1. The probability that the persons aged x and y survive t years for Chinese can be found from life tables. For the situation of U.S., we use Gompertz-Makeham IFM curve with $b=9.5, g=86.34$ (Milevsky, 2006). Table 2 lists the values of the factors of the compensation for the wrongful death v_2 and v_4 calculated with equations of (15) and (23), where the death age $x=45$: 55. Figure 2 and Figure 3 dispay their change patterns with the time. Letting the initial wage level when the affected person dies W_0 multiplied by $(1-h)$ and by the factors of the compensation for the wrongful death v_2 and v_4 respectively, we can get the values of the compensation for the wrongful death P_{2v_2} and AP_{2v_2}. We neglect the discussion with fixed interest rate. We find from Table 2, Figure 2 and Figure 3 that the values of the factors of the compensation for the wrongful death v_2 and v_4 for U.S. and China are very different. The values of v_2 and v_4 of U.S. are much smaller than those of China, especially when x takes small values.

The possible explaination may be that the average growth rate of the wage of China (0.1370) is much greater than that (0.0469) of U.S. However, the total compensation of China is still much smaller than that of U.S. due to the exchange rate of China to U.S. is as high as around 1:6.3. By comparing Figure 2 and Figure 3 in this paper and Figure 1 in Mao et al. (2011), we find that the values of factors of the compensation are very different between each other. The differences are from the following possible ways. In Mao et al.

(2011), the different kinds of the stochastic processes are assumed. Mao *et.al.* (2011) assumed that the annual growth rate of the wage rate is a Geometric Browenian Motion, but it is assumed to be as a Vasicek model in this paper and also the parameters of the models were assumed very small values in Mao *et al.* (2011), for example, both of drift and volatility of the growth rate of the wage are assumed as 0.01, which is much small than those values appied in this paper (Please see Table 1), meanwhile, the parameters of interest rate used in Mao *et al.* (2011) and those in this paper are also quite different. Finally and also importantly, we consider the affect of time-varing correlations on the compensation values in this paper. Although Mao *et al.* (2011) considers constant correlation between wage and discounting rate, the value of correlation coefficient assumed is only 0.2, while the results in this paper shows that the time-varying coefficients in both U.S. and in China are as higher as over 0.92. On the whole, since all models in this paper are obtained by using historical data with small errors. Although all results are calculated in discrete ways, they are accurated enough due to successfully avoiding simulation.

Table 1. The values of parameters and asymptotic error of estimation with multi-Vasicek model of growth rate of wage and risk free interest rate of U.S. and China[7]

***MLE* U.S.**	**Growth Rate of Wage**	**Estimation error**	**Treasure Bill**	**Estimation error**
estimation(a_i)	0.4811	0.0157	0.1664	0.0085
estimation(b_i)	0.0469	0.0031	0.0509	0.0042
estimation(σ_i)	0.0215	0.00005	0.0171	9.0248×10^{-10}
$r_r(0)=0.0200, r_w(0)=0.0355$				
***MLE* China**	**Growth Rate of Wage**	**Estimation error**	**Deposit rate for three years**	**Estimation error**
estimation(a_i)	0.5780	0.2459	0.1399	0.0922
estimation(b_i)	0.1354	0.0101	0.0592	0.0048
estimation(σ_i)	0.0649	0.000553	0.0158	0.000029
$r_r(0)=0.0275, r_w=0.0820$				

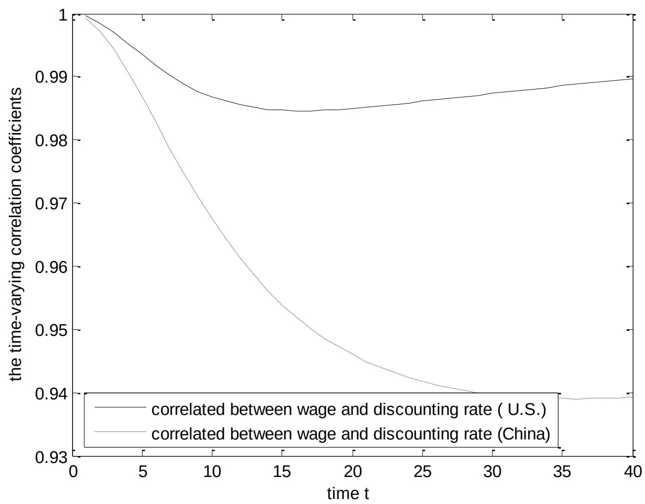

Figure 1. The time-varying correlation coefficients between the wage and the discounting rate

Table 2. The values of the factor of compensation v_2 and v_4

U.S. $m=65, y=50, k=m-x$						
x	45	46	47	48	49	50
v_2	19.2494	18.2281	17.2066	17.1853	16.1853	15.1642
v_4	1.3659	1.3330	1.3005	1.2684	1.2367	1.2053
x	51	52	53	54	55	
v_2	14.1434	13.1233	12.1038	11.0853	9.0519	
v_4	1.1740	1.1428	1.1115	1.0798	1.0475	
China $m=60, y=50, k=m-x$						
x	45	46	47	48	49	50
v_2	57.7855	51.0500	45.0063	39.5833	34.7161	30.3463
v_4	3.6789	3.3820	3.1108	2.8629	2.6359	2.4280
x	51	52	53	54	55	
v_2	26.4214	22.8945	19.7243	16.8743	14.3116	
v_4	2.2372	2.0618	1.9002	1.7511	1.6129	

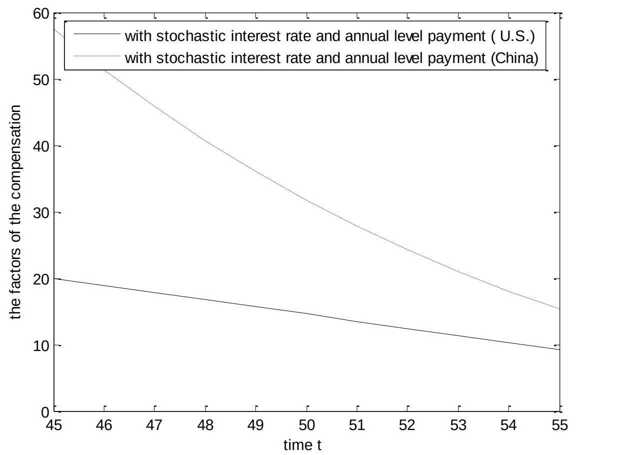

Figure 2. The values of compensation coefficients for single payment and different x values

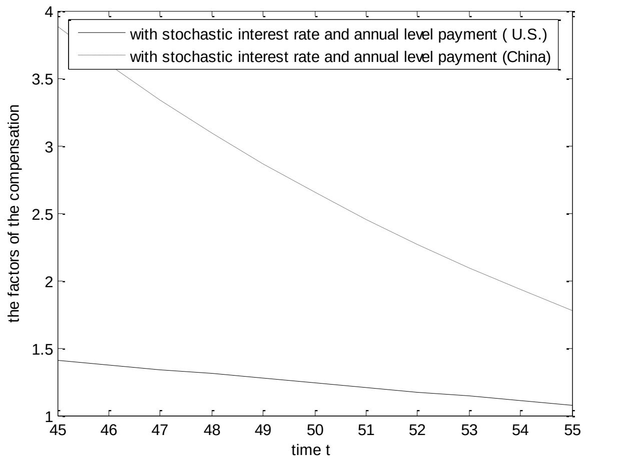

Figure 3. The values of compensation coefficients for annual level payment and different x values

5. Discussions

In above sections, all factors of compensation are calculated based on the period between the time when the wrongful death event occurs and the retirement time. It implies that the wrongful death event only occurs before retirement and the compensation after retirement is neglected. However, the wrongful death event actually can occur at any time of individual's whole life and the compensation should be based on individual's whole life term but not only is limited before retirement. In this section, we will discuss how to consider the compensation after retirement in the calculation of the factors of compensation when the wrongful death occurs at any time of individual's whole life term. One thing should be noticed is that if the wrongful death occurs close or at or after retirement, the decedent has accumulated enough retirement benefit before his (her) death, the compensation paid by the obligator to his (her) legitimate successor(s) is neglected and Social Security will return the contribution and corresponding investment return or interest accumulated before retirement to the decedent's successor(s). However, if the wrongful death occurs well before retirement, we will calculate two parts of compensations, one part is the compensation before retirement which will be paid by the obligator and another part is the compensation after retirement which will be paid by Social Security to the decedent's successor. We assume that the factor of stochastic discount calculating with continuous return rate of investment portfolio can be expressed as $f_n(t) = e^{-\int_0^t r_n(u)du}$ and the return rate of investment portfolio r_r follows stochastic differential equation (25).

$$\begin{aligned} r_n &= \sum_{i=1}^{n} \alpha_i dr_i(t) = \sum_{i=1}^{n} \alpha_i \left(a_i \left(b_i - r_i(t) \right) dt + \sigma_i dz_i \right) \\ &= \sum_{i=1}^{n} \alpha_i a_i \left(b_i - r_i(t) \right) dt + \sum_{i=1}^{n} \alpha_i \sigma_i dz_i \end{aligned} \tag{25}$$

5.1. Lump sum compensation

5.1.1. The wrongful death occurs before retirement

Assume that the wrongful death occurs at time $x(x \le m)$, where m is retirement age. Assume that all compensation is paid at the time that wrongful death occurs. Let the present value of the compensation for wrongful death in the decedent's whole life term be $P_{2v_2}^{(1)}$. The future value of the compensation at time $x - a_0$ be $P_{ar}^{(1)}$, which is equal to the future value of the contribution accumulated by the decedent before his (her) wrongful death and it is compensated by Social Security. Then we have:

$$P_{ar}^{(1)} = E\left(\sum_{0}^{x-a_0} c_t^{(1)} W_t P(t) / f_n(t) \right) = \sum_{0}^{x-a_0} c_t^{(1)} W_0 e^{A_t + \frac{1}{2}B_t + C_t^{(n)} + \frac{1}{2}D_t^{(n)} + Y_t^{(n+1)}}, \tag{26}$$

and $$P_{2v_2}^{(1)} = P_{2v_2} + P_{ar}^{(1)}, \tag{27}$$

where $c_t^{(1)}$ is the contribution rate at time $t, t = 0,1,\text{L}, x - a_0$, a_0 is the age of the decedent begin to collect contribution, A_t, B_t and Y_t are satisfied with equations (4), (5) and (14).

$$C_t^{(n)} = E\sum_{i=1}^{n}\alpha_i r_i(t) = \int_0^t r_n(u)du = \sum_{i=1}^{n}\alpha_i\left(\frac{r_i(0)-b_i}{a_i}\left(1-e^{-a_it}\right)+b_it\right), \tag{28}$$

$$D_t^{(n)} = Var\left(\int_0^t r_n(u)du\right) = \sum_{i=1}^{n}\alpha_i^2\cdot\frac{\sigma_i^2}{2a_i^2}\left(2t-\frac{3-4e^{-a_it}+e^{-2a_it}}{a_i}\right), \tag{29}$$

$$Y_t^{(n+1)} = \sum_{\substack{i=1\\ i\neq j}}^{n+1}\frac{\sigma_i\sigma_j}{a_i+a_j}\left(\frac{t}{a_i}+\frac{t}{a_j}-\frac{1}{a_i^2}-\frac{1}{a_j^2}+\frac{e^{-a_it}}{a_i^2}+\frac{e^{-a_jt}}{a_j^2}-\frac{(e^{-a_it}-1)(e^{-a_jt}-1)}{a_ia_j}\right) \tag{30}$$

where $a_{n+1}=a_w, b_{n+1}=b_w$ and $\sigma_{n+1}=\sigma_w$. P_{2v_2} is defined by equation (11). Please note that $f_n(t)$ here is the return rate of investment portfolio and $Y_t^{(n+1)}$ expresses the covariance among risky assets invested and among the wage and risky assets invested.

5.2.2. The wrongful death occurs after retirement

Assume that the wongful death occurs at time $x\,(x>m)$. Assume that the compensation is paid by Social Security and its compensation value is $P_{ar}^{(2)}$, which is the future value of the total contribution at time $x-a_0$ accumulated by the decedent before his (her) wrongful death deducted the future value of retirement benefit the decedent obtains before he(she) is hurted, which will be paid by Social Security and is written as:

$$P_{ar}^{(2)} = E\left(\sum_{0}^{x-a_0}c_t^{(1)}W_tP(t)/f(t)\right)-E\left(\sum_{t=1}^{x-m}\theta W_tP(t)/f_n(t)\right) = \max\left\{\sum_{0}^{x-a_0}c_t^{(1)}W_0e^{A_t+\frac{1}{2}B_t+C_t^{(n)}+\frac{1}{2}D_t^{(n)}+Y_t^{(n+1)}}-\sum_{t=1}^{x-m}\theta W_0e^{A_t+\frac{1}{2}B_t+C_t^{(n)}+\frac{1}{2}D_t^{(n)}+Y_t^{(n+1)}},0\right\}, \tag{31}$$

where θ is the replacement rate, A_t and B_t are satisfied withe quations (4) and (5), and $C_t^{(n)}$, $D_t^{(n)}$ and $Y_t^{(n+1)}$ are satified with equations (28), (29) and (30) respectively. For the determination of the values of $c_t^{(1)}$ and θ, please see Mao and Wen (2017).

5.2. Annual level compensation

5.2.1. The wrongful death occurs before retirement

Assume that the compensation is annually given to the legitimate successor of the decedent at the beginning of each year after wronful death occurs and assume that the wongful death occurs before retirement. Then the total annual compensation includes two parts. One part is annual compensation paid by the obligator and another part is paid by Social security which is the contrbution collected by the decedent during his alive time. Let the present value of annual compensation be

$$AP_{2v_2}^{(1)} = \frac{P_{2v_2}}{1+\sum_{t=1}^{k-1} {}_t p_y e^{C_t^{(n)}+\frac{1}{2}D_t^{(n)}+Y_t^{(n)}}} + \frac{P_{ar}^{(1)}}{1+\sum_{t=1}^{o-1} e^{C_t^{(n)}+\frac{1}{2}D_t^{(n)}+Y_t^{(n)}}}, \tag{32}$$

where $$Y_t^{(n)} = \sum_{\substack{i=1 \\ i\neq j}}^{n} \frac{\sigma_i\sigma_j}{a_i+a_j} \left(\frac{t}{a_i}+\frac{t}{a_j}-\frac{1}{a_i^2}-\frac{1}{a_j^2}+\frac{e^{-a_i t}}{a_i^2}+\frac{e^{-a_j t}}{a_j^2} - \frac{(e^{-a_i t}-1)(e^{-a_j t}-1)}{a_i a_j} \right), \tag{33}$$

${}_t p_y$ is the probability of the obligator alive from time age y to $y+t, t=1,2\ldots k-1$, k is the term of compensation by the obligator, o is the term of compensation by Social Security and P_{2v_2} is defined by equation (11). Please note that here the value of $Y_t^{(n)}$ is the sum of the covariance among risky assets invested, but it not includes the covariance between wage and investment.

5.2.2. The wrongful death occurs after retirement

If the wrongful death occurs after retirement, the obligator's legitimate successor can only get the compensation expressed in the second term of right hand side of equation (27), which will be paid by Social Security. Therefore, the present value of annual compensation is expressed as:

$$AP_{2v_2}^{(2)} = \frac{P_{ar}^{(2)}}{1+\sum_{t=1}^{o-1} e^{C_t+\frac{1}{2}D_t+Y_t^{(n)}}}, \tag{34}$$

where $P_{ar}^{(2)}$ is defined by equation (31), $C_t^{(n)}$, $D_t^{(n)}$ and $Y_t^{(n)}$ are defined by equations (28), (29) and (33) respectively.

5.2.3. An example

In the following, we will give an example with the data from U.S. to illustrate the application of the compensation valuation models presented in this section. Using the optimization results in Mao and Wen (2017), we have the values of optimal contribution rate and replacement rate as $c_t^{(1)} = 0.11$ and $\theta = 0.80$. We assume that the rate of consumption for the decedent if he (she) is alive is $h = 0.4$. and the age of the decedent participating Social Security plan is $a_0 = 25$. For other values of parameters, please see Table 3 (Also see Mao Wen (2017). Putting them into equations of (26), (27), (31), (32) and (33), we can get the lump sum or annual level compensation values paid by the obligator and Social Security in situations of wrongful death occurs before and after retirement when initial wage is one dollar. Please see Table 4 and Table 5. If one wants to know the actual compensation value, What he (she) should do is to use the initial wage times the values in Table 4 and Table 5. Table 4 shows that the compensation values are much greater due to the fact that we consider the accumulated contribution and corresponding investment return and annual compensation value is also much greater than those without considering the compensation after retirement. It indicates that considering the compensation after retirement is very important. Table 5 shows that the compensation is zero when the decedent's age is greater than 70.

Because the difference between the future value of contribution and the future value of retirement benefit is less than 0, we set the compensation value as zero, which means that the legislate successors of the decedent will have no any compensation if decedent's wrongful death occurs at the age greater than 70.

Table 3. The values of parameters and asymptotic error of multi-Vasicek model estimated using historical data in U.S. from 1965 to 2014

MLE	Growth Rate of Wage	Estimation error	S&P	Estimation error	Treasure Bond	Estimation error	Treasure Bill	Estimation error
estimation(a_i)	0.4811	0.0157	4.9677	0.2389	2.4594	0.0654	0.1664	0.0085
estimation(b_i)	0.0469	0.0031	0.1134	0.0240	0.0985	0.0221	0.0509	0.0042
estimation(σ_i)	0.0215	0.00005	0.5348	0.2624	0.3473	0.0248	0.0171	9.0248×10^{-10}

Table 4. The future annual value of cumulated contribution for unit initial wage ($x \le m$)

U.S. $m = 65, y = 50, k = 15$						
x	45	46	47	48	49	50
$P_{ar}^{(1)}$	37.6978	42.6423	48.6890	55.9894	64.6528	74.7746
$AP_{2v_2}^{(1)}$	2.0620	2.3228	2.6412	3.0245	3.4786	4.0071
x	51	52	53	54	55	
$P_{ar}^{(1)}$	86.3451	99.2272	113.0934	127.3654	141.1708	
$AP_{2v_2}^{(1)}$	4.6089	4.7485	4.9687	5.2279	5.4866	

Table 5. The future annual value of cumulated contribution for unit initial wage ($x > m$)

U.S. $m = 65, y = 50, k = 15$						
x	65	66	67	68	69	70
$P_{ar}^{(2)}$	75.8141	61.0358	47.7183	35.4138	23.5824	11.7068
$AP_{2v_2}^{(2)}$	2.4571	1.9710	1.5461	1.1557	0.7777	0.3915
x	71	72	73	74	75	
$P_{ar}^{(2)}$	0	0	0	0	0	
$AP_{2v_2}^{(2)}$	0	0	0	0	0	

6. Conclusions

In this article, we present the dynamic valuation of wrongful death compensation with single and annual level payment. The annual growth of the wage and the risk-free interest rate are assumed to following multi-Vasicek model. Time-varying correlation is considered in the valuation model. Empirical data from U.S. and from China is used to fit the Vasicek model and to calculate the present value of the wrongful death compensation. The comparison of the compensation values between China and U.S. is also carry out. Finally, we discuss the calculation of total compensation values paid by the obligator and Social Security if considering the compensation after retirement. The application is illustrated with an example using the data from U.S.

References

1. Bowers, JR. N., H. U. Gerber, J. C. Hickman, D. A. Jones and C. J. Nesbitt **Actuarial Mathematics,** Second Edition Printed by IPC Publishing Co., St. Joseph, Mich., 1997
2. Carriere, J. F. and K. J. Shand **New salary functions for pension valuation,** *North American Actuarial Journal,* Vol. 2, 1998, pp. 18-26.
3. Hoser, W. R. **Problems in the use of historical data in estimating economic loss in wrongful death and injury cases,** Journal of Risk and Insurance, Vol. 40, 1982, pp. 300-318.
4. Horvitz, S. A. **Implications of projecting future losses of earning capacity with deterministic models,** Journal of Risk and Insurance, Vol. 53, 1986, pp. 530-551.
5. Joseph E. Johnson, J.E. and G. B. Flanigan **Economic Valuation for Wrongful Death,** Campbell Law Review, Vol. 8, 1984, pp. 47-73.
6. Mincer, J. S. **Experience and earning,** National Bureau of Economic Research, New York, 1974
7. Mamon, R.S. **Three ways to solve for bond prices in the Vasicek Model,** Journal of Applied Mathematics and Decision Science, Vol. 8, 2004, pp.1-14.

a. Mao, H., K. M. Ostaszewski and Y. Zhang **Dynamic models of wrongful death compensation,** Investigacions en seguros y gestion del riesgo: Riesgo, 2011, pp. 73-81.

8. Mao, H. and Z. Wen **Optimal contribution and investment in a defined benefit pension plan when the return rate of risky assets is time-varying correlated and cyclical change,** Working paper, 2017
9. Pennacchi, G.G. **The valuation of guarantees on pension fund return,** Journal of Risk and Insurance, Vol. 66, 1999, pp. 219-237.
10. Vasicek, O.A. **An equilibrium characterization of the term structure,** Journal of Financial Economics, Vol. 5, 1977, pp. 77-88.

---

[1] Hong Mao is an associate professor of Economics and Management School of Shanghai Second Polytechnic University. During her teaching years, she has got many honors including Distinguished Teacher of Shanghai Municipal. Distinguished Teacher of Shanghai Second Polytechnic University, and Distinguished Teacher Most Liked by Students. She has published more than 30 papers in international important journals, such as Insurance: Mathematics and Economics, Geneva Papers: Practice and Issues; Journal of Insurance and Regulation; Journal of Insurance Issues. One of her paper published in 2008 has got second class awards conferred by Pudong Science Association of Shanghai. Her paper with other two coauthors published in Korea Journal of Finance and Insurance, 2016 has got Best Paper Prize. She also published about 30 articles in China's important journals.

[2] Equation (1) is similar to that in Pennacchi (1999).

[3] Lee R, Carter L model (1992) is also a very important and widely used model for forecasting mortality besides GM survival function used in this paper. Other survival functions include exponential survival function, Gamma survival function, lognormal survival function, and Weibull survival function and so on.

[4] Sources: http://www.ssa.gov/oact/cola/AWI.html

[5] Sources: Stocks, Bonds, Bills, and Inflation Yearbook. The data is reported annually.

[6]Table of historical data of deposit interest rate-infoport of banks https://www.yinhang123.net/lltz/llxg/141289.html . The historical data of more long term in China has no continuity due to the the great reform beginning at 1978. Therefore, we use more recent data to fit Vasicek model.

[7] The estimation of errors in Table 1 expresses the mean absolute error of estimation of parameters of a_i, b_i and σ_i for $i = r, w$ in multi-Vasicek model.

Appendix

The proof of the formula calculating B_t :

Based on Mamon (2004), we have:

$$Cov[X(t), X(u)]$$
$$= \sigma_i^2 e^{-a_i(u+t)} E(\int_0^t e^{a_i s} dw_s \int_0^u e^{a_i s} dw_s)$$
$$= \sigma_i^2 e^{-a_i(u+t)} \int_0^{t \wedge u} e^{2a_i s} ds$$
$$= \frac{\sigma_i^2}{2a_i} e^{-a_i(u+t)} (e^{2a_i(t \wedge u)} - 1)$$

$$Y_t = Cov[\int_0^t r_{iu}(u)du \ , -\int_0^t r_{ju}(u)du]$$
$$= -\int_0^t \int_0^t Cov(r_i(u), r_j(s))duds$$
$$= -\frac{1}{a_w + a_r} \int_0^{u \wedge t} \int_0^{u \wedge t} \sigma_w \sigma_r e^{-a_w u - a_r s} (e^{(a_w + a_r)(s \wedge u)} - 1)duds$$
$$= -\frac{1}{a_i + a_j} \left[\int_0^t \int_0^s \sigma_w \sigma_r e^{-a_w u - a_r s} (e^{(a_w + a_r)u} - 1)duds \right]$$
$$- \frac{1}{a_w + a_r} \left[\int_0^t \int_s^t \sigma_w \sigma_r e^{-a_w u - a_r s} (e^{(a_w + a_r)s} - 1)duds \right]$$
$$= -\frac{\sigma_w \sigma_r}{a_w + a_r} \begin{pmatrix} \frac{t}{a_w} + \frac{t}{a_r} - \frac{1}{a_w^2} - \frac{1}{a_r^2} + \frac{e^{-a_w t}}{a_w^2} + \frac{e^{-a_r t}}{a_r^2} \\ - \frac{(e^{-a_w t} - 1)(e^{-a_r t} - 1)}{a_w a_r} \end{pmatrix}$$